SPREADING SEQUENCE DESIGN VIA PERFECT-RECONSTRUCTION FILTER BANKS

Eleftherios Kofidis

Department of Informatics
and Telecommunications,
University of Athens,
15784 Athens, Greece.
E-mail: kofidis@di.uoa.gr

Phillip A. Regalia

Département Communications,
Images et Traitement de l' Information,
Institut National des Télécommunications,
91011 Évry, France.
E-mail: Phillip.Regalia@int-evry.fr

ABSTRACT

Digital transmultiplexers, i.e., synthesis/analysis filter banks, traditionally employed for TDMA to FDMA conversion and vice-versa, provide also a multirate model for channel distortion free DS-CDMA systems, with the synthesis filter impulse responses playing the role of spreading sequences. Orthogonal filter banks constitute therefore an appropriate framework for designing new orthogonal codes and have been shown to give rise to codes with improved performance compared to classical ones (Gold). This paper investigates the possibility for further improvements via exploiting the additional freedom present in biorthogonal filter banks.

1. INTRODUCTION

Multirate filter banks (FB's) [27] have been shown to provide attractive models for a number of communication systems [19, 30, 4, 1]. Their applicability stems from their inherent flexibility in processing signals in the composite time-frequency space. The central role played by digital transmultiplexers (TMUX's), i.e., synthesis/analysis FB configurations [27, 1, 16], in converting between TDMA and FDMA is well known. As revealed in [26], a Direct Sequence - Code-Division Multiple Access (DS-CDMA) system (free of channel distortion) [22] is also a TMUX, where the synthesis filter impulse responses play the role of spreading sequences. The upsampling (spreading) factor, P, of the TMUX coincides with the filters' length [26], that is, there is no overlap between successive spread symbols. However, an appropriately designed TMUX may allow the multiplexed signals to be perfectly recovered even when the filter sequences are longer than P.

The perfect-reconstruction (PR) ability of an orthogonal TMUX lies on the fact that the analysis and synthesis filter sequences have (auto)correlations that vanish for all lags that are (nonzero) multiples of P. However, as demonstrated in [28], any phase inaccuracy due to uncoordinated user transmissions and/or distortion introduced by the transmission medium will damage the orthogonality and hence the PR property of the TMUX.[1] This makes necessary to consider also the (auto)correlations between filters (codes) in lags that are not multiples of P. The question thus arises whether one can design orthogonal TMUX's whose filters are approximately orthogonal with respect to shifts other than multiples of P. Several works have been reported dealing with this problem [29, 10, 11, 12, 13, 14, 2, 3, 24], and demonstrated the versatility of PRFB's as a spreading sequence design framework. The resulting codes have been shown to outperform Gold codes [8, 23] of similar length in combating both multiuser (MU) and multipath (MP) interference.

All the works mentioned above dealt with orthogonal (O) TMUX design. This implies that synthesis and analysis filters are identical modulo a reversal in the order of coefficients. Nevertheless, since it is the PR property we are interested in for this problem, any PR, not necessarily orthogonal, synthesis/analysis system could be used. This would require the receiver to employ a different spreading sequence from the transmitter. A deviation from using orthogonal TMUX's was presented in [5] in the context of OFDM. In that work, biorthogonal demultiplexing is proposed, that is, though the filters, that are modulated versions of a single prototype window, in each of the banks are orthogonal to each other, a different window is used in the receiver. The results obtained confirmed the plausibility of such an approach.

In this paper, the PRFB-based code design problem is revisited and the possible gains from using general, biorthogonal (BO) FB's are investigated. Transmit and receive codes are different but of the same length. Complete (canonical)

[1] Although PR TMUX's have been studied in the presence of channel distortion on the composite CDMA signal (corresponding to the forward (down) link in cellular communications) and adaptive algorithms for compensating for that distortion have been reported (e.g., [20]), the more general problem, where each user transmission goes through a different channel (as in the reverse (up) link) with no synchronization among users, is far more challenging.

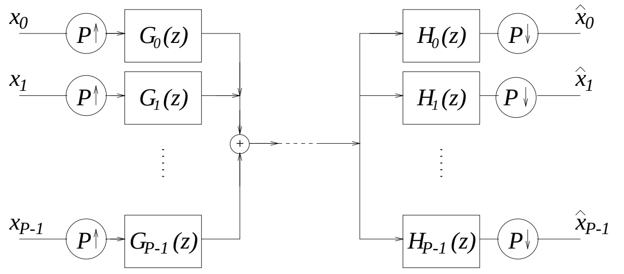

Figure 1: Multirate model for a CDMA system with spreading factor P.

realization structures for BOFB's are employed to transform the optimization problem to an unconstrained one. The greater freedom available in BOFB's is shown, via design examples, to allow the construction of spreading sequences with improved correlation properties compared to the Gold codes and those designed via OFB's.

2. PR TREE-STRUCTURED FB'S FOR ASYNCHRONOUS CDMA IN THE PRESENCE OF MULTIPATH

Fig. 1 shows the structure of a general, P-channel, critically-sampled TMUX. In an OTMUX, the synthesis filter G_i is given by $G_i(z) = z^{-(L-1)}H_i(z^{-1})$, with $L \geq P$ being the filter's length. Any such TMUX, applied to a CDMA system with no channel distortion (including asynchronism), would suffice for perfectly recovering the transmitted signals. However, asynchronous transmissions will prohibit PR by introducing to the signal $\hat{x}_i$ interference (MU) from other sources unless the analysis filter, H_i, has a sufficiently small correlation with all possible delayed versions of synthesis filters G_j, $j \neq i$. In the time domain, this means that $\sum_n h_i(n)\tilde{g}_j(n+d) \approx 0$, where $\tilde{g}$ denotes g reversed and d is any integer delay within the interval of delays under consideration. It is readily seen that this type of orthogonality is obtained if the frequency responses of the filters do not overlap, thus leading to an FDMA-type system.

On the other hand, to cope with MP interference, involving several delayed versions of a single source, H_i should have negligible correlation with G_i for all but one of the possible lags. Again in the time domain this means that $\sum_n h_i(n)\tilde{g}_i(n+d) \approx 0$, for all $d \neq d_0$. A solution for this could be a set of filters with Kronecker delta impulse responses, which would result in a TDMA-type system.

Hence the requirements for MU and MP interference elimination promote respectively the selection of sequences with high frequency (FDMA) and time (TDMA) localization. CDMA lies somewhere in between and asks for sequences that are well spread in both time and frequency.

Performing the optimization of the correlation properties of the filters in the general, parallel FB model of Fig. 1 turns out to be a formidable task for large values of P. Hence, binary tree-structured FB's have been instead adopted in the relevant literature [9]. Fig. 2 depicts the hierarchical structure imposed on the FB's. Note that PR for the overall system is guaranteed by ensuring PR on each of the 2-channel FB's. The resulting solutions will of course be suboptimal, yet much easier to obtain, and, as we will see, sufficiently good. The equivalent parallel FB's readily result from the noble identities [27]. Thus, the product filters for the two-stage synthesis FB of Fig. 2 are $G_0^I(z)G_0^{II}(z^2)$, $G_0^I(z)G_1^{II}(z^2)$, and so on. In general, for an S-stage tree-structured bank of filters of length M, there will be 2^S filters in its equivalent parallel FB, of length $(2^S-1)(M-1)+1$. To further simplify the optimization task, the 2-channel FB's are optimized one after another, in the order suggested in Fig. 2, taking the filters already computed as fixed. Thus, in Fig. 2, one would first optimize FB I, then compute FB II such that the corresponding product filters are optimized, and so on. This procedure appeals to the fact that the lower stage filters are more critical to the overall system's properties and is termed *progressive optimization* in [25].

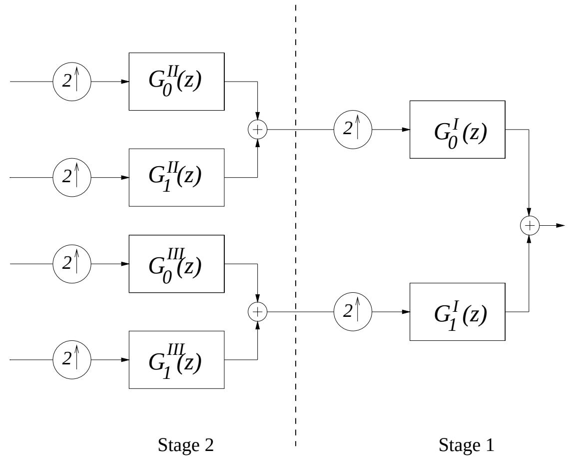

Figure 2: Tree-structured transmultiplexer (only synthesis part is shown).

Let us now define the criterion to be employed. Assume all filters of the tree-structured TMUX have (even) length M. Then the (product) filters involved in a step of the above procedure will be of the same length, K. Denote by $\boldsymbol{h}_i = [\, h_i(0)\, h_i(1)\, \cdots\, h_i(K-1)]^T$ the vector of coefficients for the ith analysis filter and similarly for the synthesis filters. For an OTMUX, $\boldsymbol{g}_i$ is just $\boldsymbol{h}_i$ reversed: $\boldsymbol{g}_i = \tilde{\boldsymbol{h}}_i$. Consider the *periodic* correlations [23] $r_{ij}(d) = \boldsymbol{h}_i^T \boldsymbol{\mathcal{Z}}^d \tilde{\boldsymbol{g}}_j$, $i \neq j$, and $\rho_i(d) = \boldsymbol{h}_i^T \boldsymbol{\mathcal{Z}}^d \tilde{\boldsymbol{g}}_i$, where, using a notation similar to that employed in [15],

$$\boldsymbol{\mathcal{Z}} = \begin{bmatrix} \mathbf{0} & 1 \\ \boldsymbol{I}_{K-1} & \mathbf{0} \end{bmatrix}$$

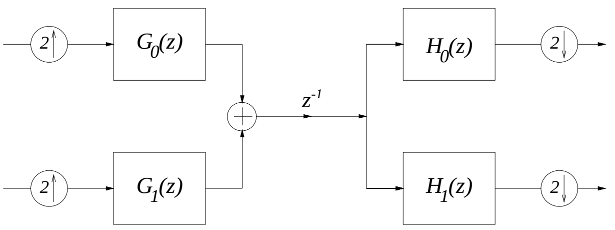

Figure 3: Two-channel transmultiplexer.

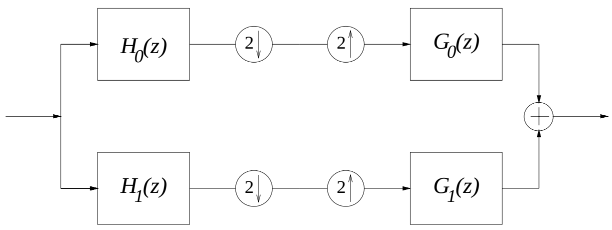

Figure 4: Two-channel analysis/synthesis system.

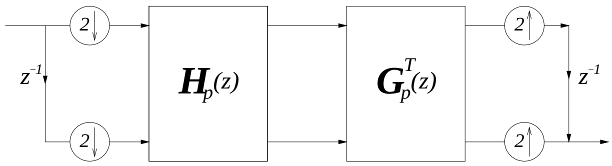

Figure 5: Polyphase equivalent of Fig. 4.

and $0 \leq d \leq K-1$. Notice that, because of the PR property, $r_{ij}(kP) = 0$ for all integers k and $\rho_i(d) = 0$ for all $d \neq d_0$ that are multiples of P. Moreover, we should emphasize here that the above holds for the analogous aperiodic correlations [23] as well. To cope with MU and MP interference one has to take also into account the 'intermediate' correlations, i.e., $r_{ij}(d)$, $d \neq kP$, and $\rho_i(d)$, $d \neq d_0, kP$. Let us collect these latter quantities into the vectors $\boldsymbol{r}_{ij}$ and $\boldsymbol{\rho}_i$ respectively. In the case of antipodal BPSK transmissions, to cope with the case of different bit polarity, we must also consider the following correlations: $r_{ij}^{-}(d) = \boldsymbol{h}_i^T \boldsymbol{\mathcal{Z}}_-^d \tilde{\boldsymbol{g}}_j$ and $\rho_i^{-}(d) = \boldsymbol{h}_i \boldsymbol{\mathcal{Z}}_-^d \tilde{\boldsymbol{g}}_i$, where

$$\boldsymbol{\mathcal{Z}}_- = \begin{bmatrix} \mathbf{0} & -1 \\ \boldsymbol{I}_{K-1} & \mathbf{0} \end{bmatrix}.$$

Defining analogously the vectors $\boldsymbol{r}_{ij}^{-}$ and $\boldsymbol{\rho}_i^{-}$, one can see that minimizing the functions

$$F_{MU} = \sum_{i \neq j} (\|\boldsymbol{r}_{ij}\|_2^2 + \|\boldsymbol{r}_{ij}^{-}\|_2^2) \quad (1)$$

and

$$F_{MP} = \sum_{i} (\|\boldsymbol{\rho}_i\|_2^2 + \|\boldsymbol{\rho}_i^{-}\|_2^2) \quad (2)$$

amounts to minimizing the effects of MU and MP interference, respectively. In order to come up with filters that are neither frequency nor time localized, a combined cost needs to be optimized, i.e.,

$$F = \alpha F_{MU} + (1-\alpha) F_{MP} \quad (3)$$

where the constant $\alpha \in [0, 1]$ plays the role of a weighting factor trading-off time- for frequency-resolution [9].

Of course, the above optimization should be performed subject to the PR constraint. Consider the 2-channel TMUX, as shown in Fig. 3. The delay factor z^{-1} needs to be inserted to properly synchronize subsamplers with upsamplers, if PR is desired. For the corresponding analysis/synthesis system, shown in Fig. 4, the PR condition will read, in the polyphase domain (see Fig. 5),

$$\boldsymbol{H}_p(z) \boldsymbol{G}_p^T(z) = z^{-l} \boldsymbol{I}, \quad (4)$$

l being a nonnegative integer. Structures for the realization of Fig. 5 that guarantee the satisfaction of the above condition regardless of the value of their parameters can be employed to obtain an unconstrained optimization problem.

3. ORTHOGONAL FB STRUCTURES

A complete parameterization of a 2-channel OFB with filter lengths M is given by [27, 24]

$$\boldsymbol{H}_p(z) = \left(\prod_{m=1}^{\frac{M}{2}-1} \begin{bmatrix} c_m & s_m \\ -s_m & c_m \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix} \right) \times \begin{bmatrix} c_0 & s_0 \\ -s_0 & c_0 \end{bmatrix} \quad (5)$$

where $c_m = \cos\theta_m$, $s_m = \sin\theta_m$, $m = 0, 1, \ldots, \frac{M}{2} - 1$ for the analysis part and similarly for the synthesis part. $\frac{M}{2}$ parameters completely describe the FB. In this case, $l = \frac{M}{2} - 1$ in (4) and $d_0 = 0$.

4. BIORTHOGONAL FB STRUCTURES

4.1. Lifting Structure

Based on the properties of the Euclidean division for polynomials, one can show that the matrices $\boldsymbol{H}_p(z)$ with determinant equal to $\pm z^{-1}$ are *generically* described by the factorization [6, 18]

$$\boldsymbol{H}_p(z) = \begin{bmatrix} 1 & 0 \\ \epsilon & z^{-1} \end{bmatrix} \left(\prod_{m=1}^{\frac{M}{2}} \begin{bmatrix} q_m(z) & 1 \\ 1 & 0 \end{bmatrix} \right) \begin{bmatrix} q_0 & 1 \\ 1 & 0 \end{bmatrix} \quad (6)$$

where ϵ, q_0 are scalars and $q_m(z)$ are first-order polynomials. There are $M+2$ parameters in total. In (4), $l = 1$, and $d_0 = 4$. The synthesis filters are given as $G_0(z) = (-1)^{M/2} H_1(-z)$ and $G_1(z) = -(-1)^{M/2} H_0(-z)$.

4.2. Padé Table-based Structure

Based on properties of normal Padé tables, a generic parameterization for 2-channel BOFB's is derived in [17], which in the polyphase domain takes the form

$$\boldsymbol{H}_p(z) = \begin{bmatrix} 1 & 0 \\ \sum_{j=0}^{(M-2)/2} \beta_{j+1} z^j & \beta_1 z^{(M-2)/2} \end{bmatrix} \times \left(\prod_{m=2}^{M-1} \begin{bmatrix} b_m & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix} \right) \begin{bmatrix} b_1 & \frac{1}{2} \\ 1 & 0 \end{bmatrix} \quad (7)$$

where b_m, β_j are scalars. A causal filter H_1 results when the parameters $\beta_2, \ldots, \beta_{M/2}$ assume the (unique) solution of the equation:

$$\begin{bmatrix} h_0(0) & 0 & \cdots & 0 \\ h_0(1) & 0 & \cdots & 0 \\ h_0(2) & h_0(0) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ h_0(M-4) & h_0(M-6) & \cdots & h_0(0) \\ h_0(M-3) & h_0(M-5) & \cdots & h_0(1) \end{bmatrix} \begin{bmatrix} \beta_{M/2} \\ \vdots \\ \beta_2 \end{bmatrix} = - \begin{bmatrix} h_1'(0) \\ h_1'(1) \\ \vdots \\ h_1'(M-3) \end{bmatrix} \beta_1$$

where H_1' is the (length-$(M-1)$) second filter of the FB given at the second row of (7). We thus have M parameters, $b_1, \ldots, b_{M-1}, \beta_1$. As in the orthogonal case, we have $d_0 = 0$ and $l = (M-2)/2$. The synthesis filters are given by $G_0(z) = \frac{2(-1)^M}{\beta_1} H_1(-z)$ and $G_1(z) = -\frac{2(-1)^M}{\beta_1} H_0(-z)$.

5. DESIGN EXAMPLES

It is seen that in a BOFB there are about twice as many degrees of freedom as in an OFB of the same size. Better codes are then expected to be possible to construct by relaxing the orthogonality constraint. This has been verified in our experiments and is illustrated in the following examples.

Example 1 We designed an orthogonal and a biorthogonal 2-channel TMUX minimizing (3) with $\alpha = 0.5$, relying on the structures (5) and (6), respectively, with $M = 8$. Fig. 6 shows the periodic autocorrelation samples for the first channel, without taking into account polarity changes (i.e., ρ_0). The autocorrelation for a length-7 Gold sequence [22] is also shown. The improvement on the autocorrelation properties of the Gold code is much more evident in the BO case.

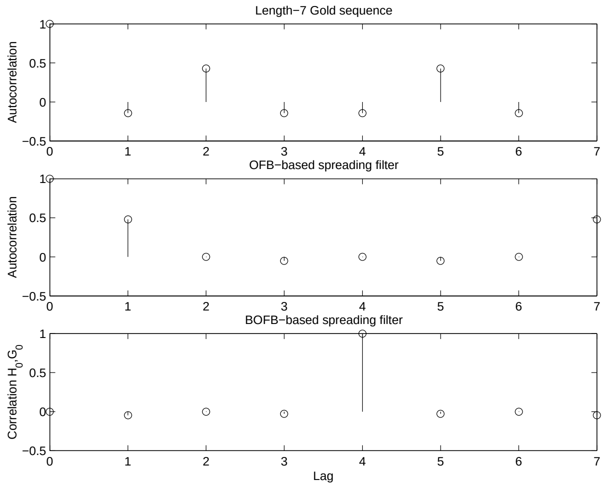

Figure 6: Periodic autocorrelation for a length-7 Gold code, a length-8 OFB-based and a length-8 BOFB-based spreading filter, both designed with $\alpha = 0.5$.

The frequency responses of the two OFB filters are shown in Fig. 7 for three values of α, namely 0, 0.5, and 1. As expected, a TDMA (resp. FDMA) system results for $\alpha = 0$ (resp. $\alpha = 1$).

Example 2 Two 4-stage TMUX's were designed with $\alpha = 0.5$, based on the structures (5) and (7), with $M = 4$. The 16 resulting sequences have length 46. The correlations for every combination of two codes in each set, including different bit polarities, were calculated over every possible delay. Their normalized histograms are plotted in Fig. 8. An analogous plot for the length-31 Gold codes [22] is also included. Its spike-like appearance should be expected since the Gold sequences are binary valued.[2] Again the BOFB-based design results in sequences of a superior performance. The variance of the correlations for the BOFB-based codes turns out to be about 10 times smaller than that for the OFB-based ones.

6. CONCLUDING REMARKS

Orthogonal multirate FB's have been recently demonstrated to constitute a versatile tool for constructing new DS-CDMA spreading sequences with improved performance. The work presented in this paper is a preliminary investigation of the possibility for further improvements via exploiting the additional freedom present in biorthogonal FB's. The design was based on the minimization of a composite cost combining MU and MP interference. Two structures for realizing BOFB's were employed to guarantee PR. The resulting sequences have been shown to outperform OFB-based and

[2] The fact that the Gold codes appear to have more than three correlation values is due to the inclusion of BPSK modulation [9].

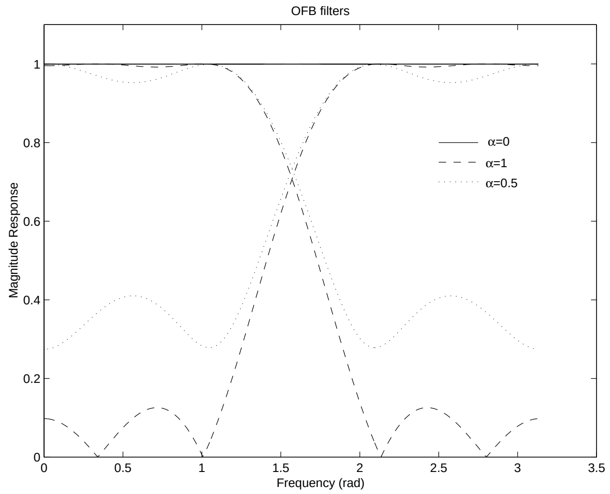

Figure 7: Magnitude responses of the filters in the OFB, for $\alpha = 0, 0.5, 1$.

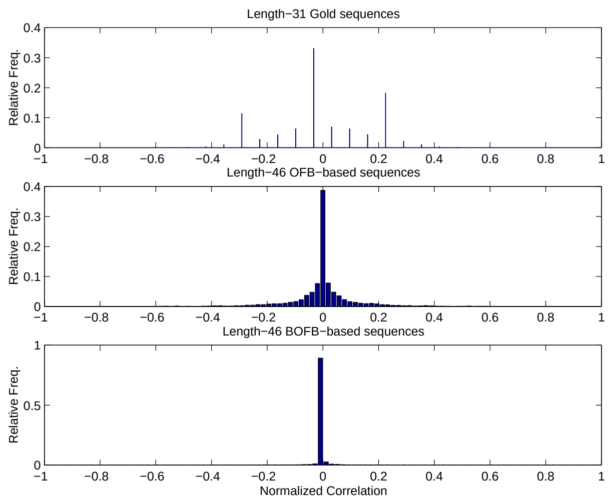

Figure 8: Distributions of correlation values for length-31 Gold sequences and length-46 OFB-based and BOFB-based sequences. $\alpha = 0.5$ has been used in the design.

Gold sequences of similar length, in terms of their correlation properties.

Regarding the practical applicability of these sequences, some comments are in order. First, the codes employed by the transmitter and the receiver are different. This might be of some inconvenience in practice. Nevertheless, the improvement in performance obtained over orthogonal codes might justify using them. Second, it should be pointed out that the FB-based sequences are, in general, multi-valued, as opposed to the binary valued Gold codes [8] or other multi-phase but unit-amplitude sequences (e.g., [7]). Moreover, their lengths are very often larger than their number. For these reasons one might consider the comparisons with Gold codes rather unfair. It should be noted, however, that the PRFB framework provides more freedom on the choice of the length than, e.g., the Gold code construction. Furthermore, it has been shown [29, 31] that allowing the sequence length to exceed the spreading factor has an effect of time averaging (time diversity) that may greatly simplify the equalization and detection tasks. Continuous-valued FB's are being used in Discrete Multitone (DMT) modulation (e.g., [1, 16, 5]). Discrete-valued sequences could be constructed though by appealing to FB's defined on finite fields (e.g., [21]).

Future work should include performance evaluation in realistic MU/MP environments as well as optimization of tree-structured TMUX's whose 2-channel FB's are not necessarily of the same length.

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