A TAP WEIGHT SELECTION METHOD FOR EQUALIZATION OF WIRELESS CHANNELS WITH LARGE MULTIPATH DELAY

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ABSTRACT

In this work we exploit the sparse nature of digital radio channels affected by large multipath delays towards complexity reduction of adaptive equalization algorithms. A sparse channel is defined here as the one having a few nonzero powerful taps separated by many negligible taps. The terrestrial High Definition Television (HDTV) channel model is considered here as a practical example of a sparse wireless channel. A tap-weight selection method is formulated based on the magnitude relationship between the negligible and the powerful tap weights of a linear equalizer. The proposed method eliminates those tap weights that are considered negligible and leave only the most powerful ones. Compared to a conventional adaptive equalizer its possible to achieve at least the same performance using only the most important (powerful) tap weights. Furthermore, the tap-weight selection (TWS) method results in a considerable computational resource saving, which is proportional to the number of negligible taps of the linear equalizer.

1. INTRODUCTION

The classical problem of adaptive equalization has been intensively studied along several years. The use of adaptive and blind equalizers in wireless communication systems is motivated by intersymbol interference (ISI) caused by multipath delay spread in digital radio channels. In particular, equalization of sparse wireless channels, i.e. those with a large multipath delay among its principal impulse response components, has been intensively studied in order to better exploit its structure. Under the assumption of ideal sampling and working at the symbol rate, the length of the equivalent discrete-time CIR can be larger in systems with a high bit-rate, such as the next generation mobile-radio systems and broadcast technologies. In particular, the terrestrial HDTV discrete-time CIR for a symbol-rate of 4.88 Mbaud exhibits a few echoes, the “farthest” one having a time-delay of more than 20 symbol intervals [1]. Such channel can be considered sparse due to its characteristics. The use of linear equalizers in the equalization of sparse channels with a large impulse response may present an elevated computational requirement, as the number of tap inputs is generally large. Some recent works [3] show that the simultaneous use of forward and backward prediction-error transversal filters (FPEF and BPEF) under the constant-modulus (CM) criterion offers a better robustness and a convergence rate faster than that of conventional linear equalizers. In addition to this, few equalizer taps are needed to reduce greatly ISI, leading to an inferior computational complexity. Other equalization strategies can be found in [4], [5]and [6].

However, for sparse wireless channels, all these equalization strategies are not concerned to the relationship between the CIR and the EIR, which we study in this work. A tap-weight selection (TWS) method is proposed here motivated by the presence of negligible tap weights in linear equalizers due to the presence of negligible taps in a sparse CIR. For a sparse channel the optimum Wiener solution can gives us an indication of which tap weights are to be selected. In this work tap-weight selection is done during the adaptation process via LMS algorithm through a periodically observation of the mean square error (MSE) followed by the observation of EIR components and subsequent selection of the most important ones. In this work we develop a method to do the after. Our simulations show it results in an equalizer with few non-zero tap-weights that lead at least to the same performance of conventional equalizers with a computational complexity gain.

The rest of this paper is organized as follows: Section 2 briefly describes the model for a multipath channel as well as sparse channel models used in our computer simulations. Section 3 presents a constrained version of Wiener-Hopf equations for sparse channels. Section 4 introduces the proposed MSE-based method for tap-weight selection. Our simulation results are illustrated in section 5 and section 6 states our conclusions.

2. SPARSE CHANNEL MODELS

The equivalent baseband discrete-time impulse response for a multipath channel corrupted by ISI can be represented as:

$$h(n) = \delta(n) + \sum_{k,k\neq 0}^{L} c(k)\delta(n-k) \qquad \textbf{(1)}$$

where L is the number of ISI components, delayed of k time-samples from the principal component By taking the z-transform of both sides of (1) we get:

$$h(z) = 1 + \sum_{k,k\neq 0}^{L} c(k) z^{-k} \qquad \textbf{(2)}$$

where z^{-k} represents a delay of k time-samples from the principal component. If all values of k are greater than zero we say the channel model is formed only by post-cursors components. For this kind of channels we have a minimum phase channel if $c(k+1) < c(k)$, for all k. In the z-plane such channels have all their zeros located inside the unit circle. On the other hand, if k assumes negative values the channel is classified as non-minimum phase. In this work we deal with both possibilities. For the minimum phase case we use simplified sparse models, as in (3) and (4). Non-minimum phase case is represented by the terrestrial HDTV channel model, which will be detailed later in this section. Here we use the term "sparse" to designate a wireless channel with a few powerful taps separated by many negligible taps. In a simplified model for one delayed multipath, we can start from (2) and vary the multipath delay profile by varying the value of k as in the following:

$$h_1(z) = 1 + cz^{-k} \qquad \textbf{(3)}$$

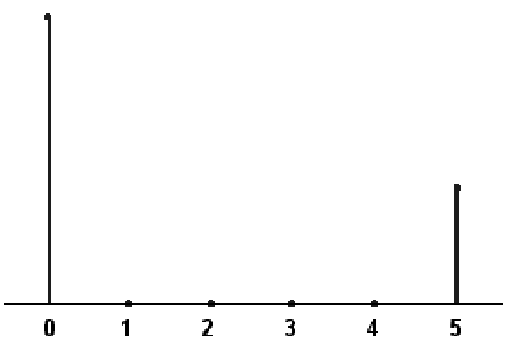

Figure 1: Example of sparse channel with one ISI component. The CIR is obtained from (3) for k =5. We see a multipath delay of five symbol intervals is modeled here.

In figure 1 we see the CIR of a simplified sparse channel for k=5. In another particular case of (2) we have an additional term z^{-k} representing a second ISI component with a negligible power compared to the other two components and situated symmetrically between them. The channel model is now written as:

$$h_2(z) = 1 + c_1 z^{-k_1} + c_2 z^{-k_2} \qquad \textbf{(4)}$$

where 1< k1< k2 and $c_1 << c_2$. Figure 2 illustrates this case.

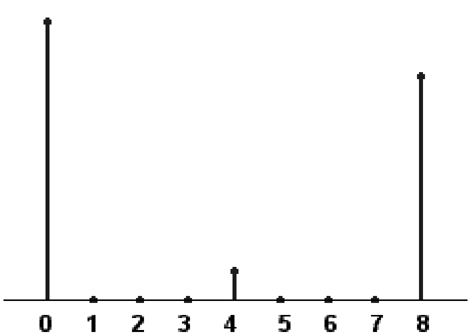

Figure 2: Example of sparse channel with two ISI components. The CIR is obtained from (4) for k_1 =4 and k_2=8. This CIR models a multipath channel with two echoes delayed of four and eight symbol intervals respectively.

A practical example of a sparse channel occurs in digital television broadcasting systems, popularly known as HDTV. The analog CIR for L paths is modeled as follows [1]:

$$h_{HDTV}(t) = g(t) + \sum_{i=1}^{L} c_i e^{j\phi_i} g(t - \tau_i) \qquad \textbf{(5)}$$

where c_i, ϕ_i and τ_i represents respectively the magnitude, phase and time-delay of the i*th* path and *g(t)* is the impulse response of a raised cosine filter. An example of an HDTV channel is obtained using the set of parameters in table 1. Figure 3 exhibits the analog version of its CIR and figure 4 shows the discrete-time version (sampled at the symbol rate) [1].

Multipaths	**Delay (μs)**	**Relative Gain**	**Phase (degrees)**	**Carrier-to-noise ratio (C/N) dB**
1	-1.153	0.1	-24.7	22.9
2	2.203	0.3	151.2	22.9
3	5.046	0.2	-63.8	22.9

Table 1: Multipath parameters (delay, magnitude and phase) of the terrestrial HDTV channel impulse response.

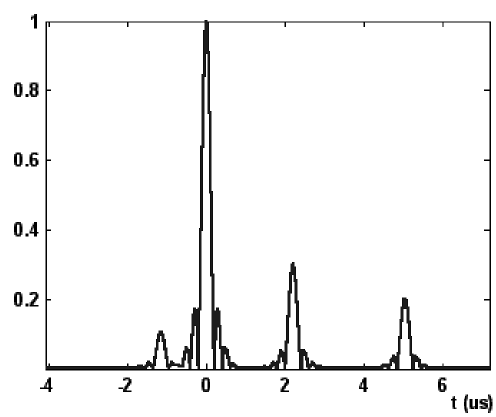

Figure 3: Magnitude of the analog impulse response for the terrestrial HDTV channel model. Observe the multipath delay is very large. The largest one (farthest echo) occurs after 24 symbol intervals.

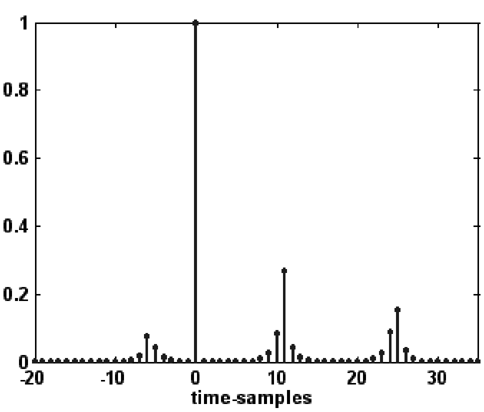

Figure 4: Sampled version of HDTV channel impulse response.

3. CONSTRAINED WIENER-HOPF EQUATIONS FOR SPARSE CHANNELS

The presence of negligible elements in the optimum tap-weight vector of a linear equalizer reflects the sparse profile of channels. Let us suppose a sparse channel for which the CIR and the EIR have the same length. The number of negligible tap weights of a Wiener equalizer can be inferred from the CIR. In a particular case of a sparse channel, illustrated in figure 2, null taps separates the nonzero taps and the spacing among the nonzero taps is the same. In such symmetrical cases the number of null tap weights of the equalizer is exactly the same as those of channel. Also, the spacing among the nonzero tap weights is the same as those of channel. However, in a more general case, where there is not such symmetry, this is not verified. For a successfully trained equalizer, null tap weights never occur as a consequence of the misadjustment of the adaptive algorithm and the noise power.

The motivation of this work is the presence of negligible elements in the optimum Wiener solution, which we wish to eliminate from the adaptive equalization process. Therefore, constrained Wiener-Hopf equations are formulated here, assuming the tap weight vector has some *a priori* null elements. Our constrained optimization problem is based on the method of Lagrange multipliers [2]. Let $f(w) = E[|e(n)|^2]$ be a real-valued function we wish to minimize, where $e(n)$ is the error between the desired response $d(n)$ and the output $y(n)$ of the equalizer. Assuming $\mathbf{w}$ is the M by 1 tap weight vector of the equalizer with K null elements, we wish to find the constrained versions of the correlation matrix of the signal at the equalizer input and the cross-correlation vector between the signal at the equalizer input and desired response $\mathbf{R}_C$ and $\mathbf{p}_C$. This problem is solved by the following system of simultaneous equations:

$$\frac{\partial f}{\partial \mathbf{w}} + \frac{\partial}{\partial \mathbf{w}^*}(\mathrm{Re}[\lambda^* c(\mathbf{w})]) = \mathbf{0} \qquad (6)$$

where $c(\mathbf{w}) = \mathbf{w}^{\mathbf{H}}\mathbf{s}$ is a linear function in $\mathbf{w}$ and $\mathbf{s}$ is a column vector of an M-dimensional orthonormal basis representing our constraint. By solving (6) for λ^* and after some algebra we find:

$$\mathbf{R}_C \mathbf{w} = \mathbf{p}_C \qquad (7)$$

where $\mathbf{R}_C$ has K null rows and $\mathbf{p}_C$ has the same K null elements. Therefore, $\mathbf{R}_C$ has rank M-K and the number of equations of our constrained optimization problem is reduced to M-K. In (7) we can eliminate the row i_k and the column i_k of $\mathbf{R}_C$, where $i_k = i_1,\ldots,i_K$, K<M, are the K null positions in vector $\mathbf{w}$. Such elimination results in a new set of Wiener-Hopf equations, now of dimension M-K:

$$\mathbf{R}'\mathbf{w}' = \mathbf{p}' \qquad (8)$$

The vector $\mathbf{w}$ representing the EIR is determined from $\mathbf{w}'$ by including the K null components on the positions $i_k = i_1,\ldots,i_K$, K<M.

4. MSE-BASED APPROACH FOR TAP-WEIGHT SELECTION

Here we present an MSE-based method for tap-weight selection in the equalization of sparse channels. The objective of the proposed method is to keep the equalizer aware about the power level of the averaged instantaneous squared error, in order to decide correctly the start of the tap-weight selection process. This problem can be formulated in a recursive way by the following set of equations:

$$m(i) = \frac{1}{P}\sum_{n=1}^{P} e_k^{\,2}(n) \qquad (10)$$

where $m(i)$ is a first-averaged instantaneous squared error obtained from averaging N times the last P values of $e_k^2(n)$ at each P iterations of the LMS algorithm, with $i = 1,\ldots, NP$, such that NP is the number of training symbols used. The values of $m(i)$ are used to obtain a second-averaged instantaneous squared error expressed by

$$S(k) = \frac{1}{k}\sum_{i=1}^{k} m(i) \qquad (11)$$

where $S(k)$ is an average of all $m(i)$ until the k-*th* iteration. From (10) and (11) we rewrite $S(k)$ in a recursive fashion:

$$S(k) = \alpha_k m_I(k) + (1-\alpha_k)S(k-1) \qquad (12)$$

where $\alpha_k = 1/k$. The tap-weight selection method must starts when the difference between $S(k)$ and $S(k-1)$ is below a threshold λ_1, sufficiently small to guarantee that the steady state of the LMS algorithm was achieved.
At this point the selection of the most important coefficients starts up. The magnitude of each tap-weight w_i is compared to that of the maximum tap-weight $w_{máx}$ and w_i is considered as a negligible tap-weight if it's below a given percentage of $w_{máx}$ i.e. $w_i < \lambda_2\, w_{máx}$. The value for λ_2 must be carefully chosen. In our simulation results, values for both λ_1 and λ_2 was empirically established for some sparse channel models.

5. SIMULATION RESULTS

Computer simulation results are presented here to demonstrate the performance of the MSE-based criterion for tap-weight selection of linear equalizers. Sparse channel models presented in section 2 are used here. LMS algorithm is used in all cases and comparisons between the classical and the proposed methods are made from the observation of learning curves and impulse responses.

5.1. Wiener Solution for a Simplified Sparse Channel Model: Relationship between CIR and EIR

The two figures below shows the influence between the multipath delays of a simplified channel over the Wiener EIR. Signal-to-Noise Ratio (SNR) is set to 30dB and M =9. EIR components considered. We observe that the number of zero components between the non-zero components of the CIR appears along the horizon of time of the Wiener equalizer. This suggests the CIR may be inferred from the EIR.

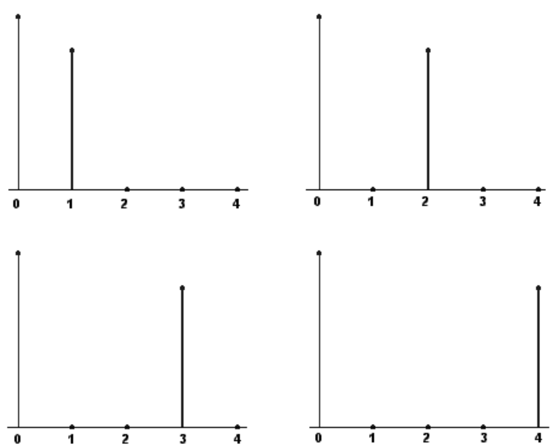

Figure 5: Impulse response behavior of an arbitrary channel as the multipath delay varies.

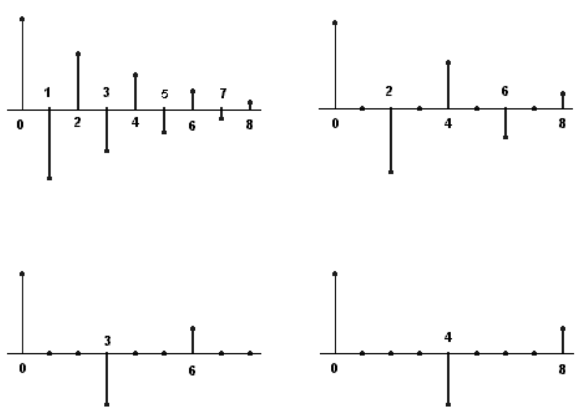

Figure 6:Impulse response behavior of the Wiener equalizer for all the channel situations above, in that sequence.

5.2. MSE Behavior due to the TWS method: a First Approach

Here, as well as in and in the section 5.3, we are concerned to evaluate the impact caused by the selection of the most important tap weights of a linear equalizer for a simplified sparse channel. As we explained in section 4, here we eliminate (force to zero) those tap weights that are considered to be negligible. The main difference here is the supposition that the equalizer has *a priori* knowledge about the number of iterations necessary to the covergence of the LMS algorithm. Therefore (10) and (12) are not used yet. Results for MSE-based tap-weight selection will be shown later in section 5.4.

The channel model described by (3) is used with $c_1 = 0.15$, $k_1 = 4$, $c_2 = 0.8$ and $k_2 = 8$. An M=11 tap-weight linear equalizer is considered here and the LMS step-size parameter is set to 0.045. The SNR is 30dB and the mean square error behavior was evaluated for 500 iterations for binary phase shift keying (BPSK) symbols. After 250 iterations, when the steady state of LMS has already been achieved, the tap-weight selection starts. The threshold λ_2 of the TWS method is set to 0.2. From figure 7 we observe that at the iteration immediately after the first tap-weight selection the MSE shifts 2.5dB downward. The horizontal line indicates the minimum MSE obtained from the classical Wiener solution.

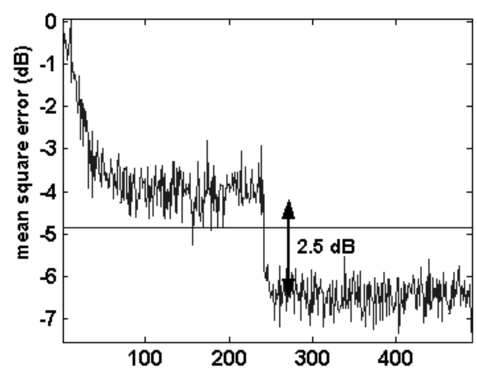

Figure 7: MSE behavior when negligible tap weights are eliminated from the LMS adaptation at iteration number 250.

Figure 8 shows the impulse response of the trained equalizer compared to the classical Wiener solution. We see in the trained case, the tap-weight number four are considered to be negligible and is eliminated.

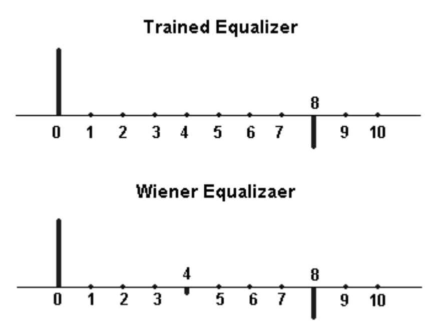

Figure 8: Impulse response of both trained and Wiener equalizers.

5.3. Symbol Error Rate (SER) Performance of the Tap-Weight Selection (TWS) Method

For the same channel model and equalization parameters, the SER is shown in figure 9 according to the variation of the SNR. We observe there is a considerable gain in performance when selection of tap weights is done.

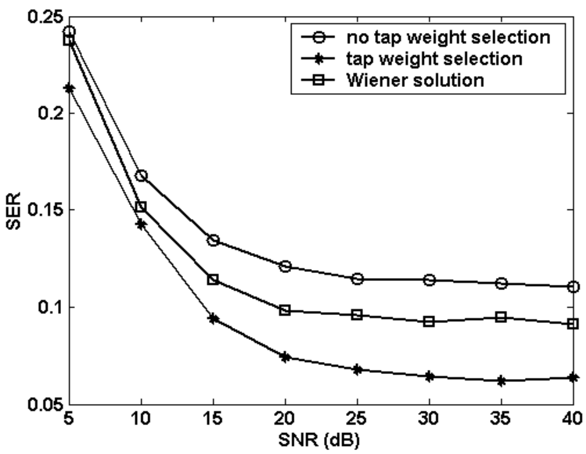

Figure 9: Symbol error rate for an adaptive linear equalizer with selected and non-selected tap weights compared to that of the Wiener solution. We have used here the sparse channel $h_s(z)$.

A key factor for the performance gain of this method is concerned to the definition of the appropriately moment in the training period to start the tap-weight selection. It is acceptable to state the best moment is situated immediately after the convergence of the equalizer in the MSE sense. In other words, after the convergence, the earlier we start the tap-weight selection the smaller is the SER of the training period. Next section is dedicated to the application of the MSE-based tap-weight selection (MSE-TWS) method, which extracts information about the convergence in a way we explained in section 4.

5.4. MSE-Based Tap-Weight Selection (MSE-TWS) Method for Adaptive Equalization of Sparse Channels.

Here we consider an improved way to make the tap-weight selection of linear equalizers, when dealing with sparse channels. The MSE-based criterion is applied here for the adaptive equalization of the terrestrial HDTV channel, described and illustrated in section 1. An M=45 tap-weight linear equalizer is used. The step-size parameter of LMS algorithm is set to 0.025. The SNR is 30dB and the modulation is BPSK. Thresholds λ_1 and λ_2 are set to 0.01 and 0.04, respectively. Figure 10 shows an improved performance when the MSE-TWS method is applied. The minimum MSE after the training period is about 1 dB below that obtained when all tap weights are used. We observe also a faster convergence for the MSE-TWS case. Moreover, from a total of 45 tap weights only 10 of them were used in the adaptation process.

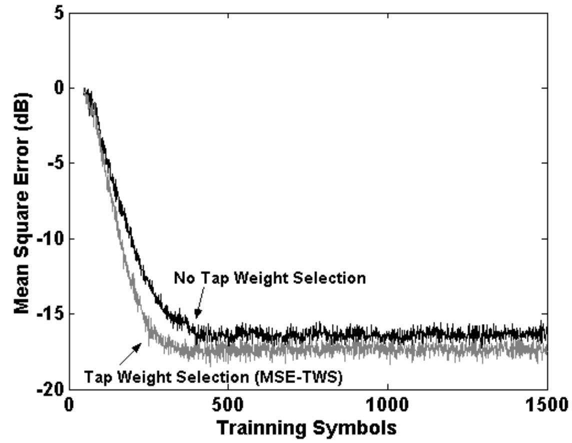

Figure 10: MSE performance of the MSE-TWS method for the terrestrial HDTV channel.

6. CONCLUSIONS AND PERSPECTIVES

We conclude that the proposed MSE-TWS method for the equalization of wireless channels large multipath delay constitutes a practical solution for broadcast technologies, as HDTV. A considerable computational resource saving can be obtained by using only a few tap-weights, as well as an improved performance can be achieved. Furthermore, this technique may be useful for systems with a high bit-rate as the next generation mobile-radio systems, where the equivalent discrete-time CIR can be larger.

To sum up, in the near future, we intend to apply the TWS method in blind equalization criteria. Future studies include an extension of the MSE-TWS method to other adaptive algorithms and other equalization structures. A deeper theoretical analysis may be done in order to establish a more general relationship between the CIR and EIR for channels with large multipath delay.

7. REFERENCES

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