ANALYSIS OF MILLIMETER WAVE RESONATOR

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ABSTRACT

The direct and efficient Transverse Transmission Line (TTL) method is used to analyze the unilateral fin-line resonator. In this method the Maxwell's equations in Fourier transformed domain, are firstly used to obtain the electromagnetic fields in terms of the transversal electromagnetic fields. An inhomogeneous equation system is obtained, in which the current densities in the conductor fins are related to the electric fields in the slot. The TTL method in conjunction with the Galerkin's procedure and Parseval's relation are applied to obtaining a homogeneous equation system with two variables. The determinant of this system gives the complex resonance frequency.

I. INTRODUCTION

A slot resonator in unilateral fin line is a circuit element that is extensively used as a building block in the design of fin-line filters. It is also useful in other millimeter-wave applications, such as oscillators and mixers. The resonator of unilateral fin line consists of a rectangular waveguide cavity with three dielectric regions, being the second region a substrate placed in the most central part and covered by a conductive sheet with a rectangular slot of width w and length l [1-6]. The cavity has length L of, at least, 15 times the length l to reduce the effects of its terminal walls on the fields in the slot resonator. In this resonator type, lowest order resonance occurs when the electrical length of the resonator becomes equal one half wavelength of the fin line. In the Fig. 1 a representation in 3D of the slot resonator in fin line is shown.

In the direct and concise TTL method, the Maxwell's equations in Fourier transformed domain, are firstly used to obtain the electromagnetic fields in terms of the transversal electromagnetic fields. In the unilateral fin line rectangular slot resonator are considered “x” and “z” electromagnetic fields quantities in terms of “y” direction fields.

An inhomogeneous equation system is obtained, in which the current densities in the conductor fins are related to the electric fields in the slot. The Transverse Transmission Line (TTL) method in conjunction with the Galerkin’s procedure and Parseval’s relation are applied to obtaining a homogeneous equation system with two variables. The determinant of this system is the complex resonance frequency.

The full wave method of the TTL gives precise and concise equations and the results bring good advantages. Computational programs are developed in Fortran Power Station and MATLAB 5.0. The results for the unilateral fin line resonator are presented for different parameters including substrate thickness, length and width of the slot resonator, in 2 and 3D.

Fig. 1. Three-dimensional view of the rectangular slot resonator in unilateral fin line.
II. THEORY

The unilateral fin line resonator analysed in this work, is limited in its length and the equations are obtained in the spectral domain in "x" and "z" directions as functions of the "y" direction fields. Therefore the field equations are applied for double Fourier transformed defined as:

\[
\tilde{f}(\alpha_n, \beta_k) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y,z) \cdot e^{j\alpha_n x} \cdot e^{j\beta_k z} \, dx \, dz
\]  

(1)

Where \(\alpha_n\) is the spectral variable in the "x" direction and \(\beta_k\) the spectral variable in the "z" direction.

Using the relation among the current densities in the metal fins and the fields in the slot interface of the conductive ribbon, the following equations are obtained:

\[
\begin{align*}
Y_{xx} \tilde{E}_{xt} + Y_{xz} \tilde{E}_{zt} &= \tilde{J}_{zt} \\
Y_{zx} \tilde{E}_{xt} + Y_{zz} \tilde{E}_{zt} &= \tilde{J}_{xt}
\end{align*}
\]  

(2)

(3)

Where the equations above can be represented in matrix form:

\[
\begin{bmatrix}
Y_{xx} & Y_{xz} \\
Y_{zx} & Y_{zz}
\end{bmatrix}
\begin{bmatrix}
\tilde{E}_{xt} \\
\tilde{E}_{zt}
\end{bmatrix} = 
\begin{bmatrix}
\tilde{J}_{zt} \\
\tilde{J}_{xt}
\end{bmatrix}
\]  

(4)

The "Y" matrix is the green admittance functions.

The electric field distributions \(E_x(\alpha_n, \beta)\) and \(E_y(\alpha_n, \beta)\) in the slot can be expanded in terms of known base functions \(\hat{e}_{xm}(\alpha_n, \beta)\) and \(\hat{e}_{zm}(\alpha_n, \beta)\), respectively.

The component of the field in the z direction in a structure of unilateral finline is despicable being able to, therefore, to be ignored without damage of the results.

With the use of just a component of the electric field \((E_x)\), was chosen the base function that is expressed in the space domain for:

\[
\begin{align*}
f_x(x,z) &= f_x(x) \cdot f_y(z) \\
f_x(x) &= \frac{1}{\sqrt{\frac{W}{2} - x^2}} \\
f_y(z) &= \cos\left(\frac{\pi z}{1}\right)
\end{align*}
\]  

(5)

(6)

(7)

that in the spectral domain are transformed in:

\[
\tilde{f}_x(\alpha_n, \beta_k) = \frac{2\pi^2 \cdot \cos\left(\frac{\beta_k}{2}\right)}{\pi^2 - (\beta_k)^2} \cdot J_0\left(\alpha_n \frac{w}{2}\right)
\]  

(8)

where \(J_0\) is the function of Bessel of first species and order zero.

Using the Gallerkin’s method, a particular case of the moment method, and Parseval’s relation the current densities on the conducting fins are eliminated, because the Fourier transformed of the current densities on the conducting fins are related to the Fourier transform of the electric fields \(E_x\) and \(E_y\) in the slot region,

\[
\begin{bmatrix}
K_{xx} & K_{xz} \\
K_{zx} & K_{zz}
\end{bmatrix}
\begin{bmatrix}
a_x \\
a_z
\end{bmatrix} = 
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\]  

(9)

One complex non-trivial solution of (9) is obtained, which characteristic equation of the determinant is made equal to zero. The characteristic equation is general and can be applied to slot resonator of arbitrary shape. The solution of the characteristic equation has as result the complex resonance frequency.

III. RESULTS

Computational programs are developed in Fortran Power Station and MATLAB 5.0, and the numerical results in 2 and 3D, for the unilateral fin line resonator are presented for different parameters including substrate thickness, length and width of the slot resonator. The programs are iterative and easy to use.

The Fig. 2 illustrates the variation of the resonance frequency as function of the region 1 thickness substrate. The resonator has, \(w=1.778\) mm, \(L=3.6\) mm, \(\varepsilon_{r2}=2.22\), \(da=7.112\) mm, \(db=3.556\) mm (WR-28), and \(g=0.127\) mm.

In the Fig. 3, is shown that the resonance frequency decreases as the length of the slot resonant increases. It is noticed although this relationship stays unaffected same when it is used larger values of thickness and of height of the dielectric substrate. The resonator has, \(da=7.112\) mm, \(db=3.556\) mm (WR-28), and \(g=0.127\) mm.

In the Fig. 4, is shown that the resonance frequency decreases as the length of the slot resonant increases. It is noticed although this relationship stays unaffected same when it is used larger values of thickness and of height of the dielectric substrate. The resonator has, \(da=7.112\) mm, \(db=3.556\) mm (WR-28), and \(g=0.127\) mm.

The Fig. 4 shows 3-D results of the real resonance frequency as function of the normalized width waveguide and of the dielectric substrate thickness. The frequency
range is between 20.0 and 30.0 GHz. There is a considerable variation when the normalized width of the waveguide increases. The resonator has, $a=7.112$ mm, $b=3.556$ mm (WR-28), $\varepsilon_r=12.0$, $s=2.8$ mm and $l=3.6$ mm.

IV. CONCLUSIONS

The unilateral fin line rectangular resonator was analyzed using the concise full wave TTL Method. Results for the complex resonance frequency confirm the exactness of the TTL method applied. Graphics that show the variation of the resonance frequency with relation to the normalized width, and thickness of dielectric substrate and length of the fin line rectangular resonator were also presented in 2-D and 3-D. The Transverse Transmission Line - TTL is an efficient and accurate method applied to the analysis and design of fin line rectangular resonator. This is a very versatile method that can be used without or with loss substrate, as semiconductor, in various planar structures. This work received financial support from CNPq.

V. REFERENCES