IMPROVING THE PERFORMANCE OF TSVQ OVER NOISY CHANNELS

Weiler A.Finamore, Rafael D. de Azevedo, and Arthur L. A. da Cunha

Center for Telecommunication Studies(CETUC)
Pontifícia Universidade Católica do Rio de Janeiro (PUC-Rio)
Rio de Janeiro RJ, Brazil
weiler@cetuc.puc-rio.br

ABSTRACT

In this work we study the performance of Tree-Structured Vector Quantizers ($TSVQ$) jointly designed with Forward Error Correction Code providing Unequal Error Protection (UEP) for the transmission of still images over noisy channels. Comparisons have shown that this scheme performs better than Channel Optimized Vector Quantizers ($COVQ$) scheme and is less affected by the channel distortion. The design of the $TSVQ$ matched to the noisy channel is less complex than the design of $COVQ$ and it has, in addition, a simpler implementation.

1. INTRODUCTION

To transmit data over a noisy channel requires, in most applications, some kind of error protection. This is the case for instance in the transmission of images captured by remote sensing satellites (RSS) which is the scenario that motivates the present work. Usually the introduction of error control increases the bandwidth and when the latter is limited, data compression is needed to compensate for the additional bit rate introduced by the channel coding. Shannon's well-known separation principle states that one can design and code the source and channel separately with an optimal result provided the compressed and coded block-length are large enough and, the channel is time-invariant. This however is an asymptotic result and, in real-world applications this might not be true since neither is the channel constant with respect to time nor is the block-length large enough for the principle to hold. In this case, joint source/channel coding schemes have proved to be an alternative with significant gains in performance. This immediately leads to the design of joint source-channel quantizers.

Several approaches to vector quantization (VQ) under channel constraint have been proposed in the literature. Source-optimized VQs by efficient index assignment can lead to good results over noisy channels. Algorithms for efficient index assignment are reported in [10, 6, 1]. Source-optimized scheme reported in [9], where the bit-stream of the SPIHT coder is packetized into fixed-length blocks and protected with RCPC codes and error-detection codes, renders astounding performance in SNR yet it has a non-negligible probability of incomplete image decoding.

Another approach is the $COVQ$ proposed in [1, 7], which basically consists in an algorithm that, by taking into account the channel crossover probabilities, produces a codebook optimized for this channel. The resulting quantizer has a better performance when compared to the plain VQ operating over noisy environment since it trades off quantizer accuracy for channel-error resilience. The performance of the $COVQ$ scheme is locally optimal but yet the channel noise introduces an annoying distortion.

The $COVQ$ design can be further improved channel by use of so-called channel noise relaxation [2]. Starting with a channel with a larger error probability, the quantizer can be better designed by relaxing the channel error probability till one gets to the desired channel. This therefore prevents the design from converging to a poor local optimum.

The straightforward solution to the transmission of quantized data over a noisy channel is tandem source-channel coding scheme. Data is quantized with a VQ designed for a noiseless channel [3] and a channel code is used to reduce or even eliminate the effects of the channel noise. This results in a scheme that is not affected by the channel noise but the bits wasted with the code results in large quantization distortion.

Another scheme which jointly design the source and channel codes is reported in [4]. By combining the $COVQ$ with RCPC codes, an iterative design is obtained with a significant gain over $COVQ$. This gain is mainly due to soft decision inherent to the RCPC code and also due to an optimized bit allocation between channel and source codes. A disadvantage of such scheme however is that it does not allow for UEP since there is no order of importance in the index bits.

In [8], an algorithm to design $TSVQ$ matched to the channel was introduced. This is an efficient design technique that although not producing an optimal quantizer has the advan-

Thanks to CNPq (Brazilian Research Council) and INPE (Brazilian Space Agency).

tage of lowering the computational complexity (of the design and the encoding). In addition, the successive refining characteristic of the $TSVQ$ make it amenable to be used with FEC. In this work we study the performance of $TSVQ$ used with UEP over noisy channels which is an intermediate scheme between the $COVQ$ (which does not explicitly uses error correction) and the $VQ + FEC$ tandem scheme. The paper is organized as follows: the formulation of the"Tree-Structured Vector Quantizer" matched to noisy channel is presented in Section 2 with an efficient algorithm to design the quantizer. The problem of transmission of quantized images trough noisy channels is discussed on Section 3. Results are presented on Section 4 with concluding remarks on Section 5.

2. TREE STRUCTURED VECTOR QUANTIZATION OF IMAGES FOR NOISY CHANNELS

The Tree-structured Vector Quantization of an image I can be understood as starting with a vectorization that maps consecutive $L = w \times h$ image cells into a sequence $\{\mathbf{x}_n\} = (\mathbf{x}_1, \ldots, \mathbf{x}_n, \ldots, \mathbf{x}_N)$ of L-dimensional vectors. This sequence is then submitted to a succession of Λ quantizers (we will restrict ourselves to binary codebook quantizers which output an index 0 or 1 if the input vector is closer to centroid 0 or 1). All the indices $\{b_{\lambda,n}\}$ output by the quantizer on layer λ ($\lambda = 1, \ldots, \Lambda$) are sent through a Binary Symmetric Channel (BSC). The codebook used by $TSVQ$ on the first layer is $\mathcal{C}_1 = \{\mathbf{y}_1^{[0]}, \mathbf{y}_1^{[1]}\}$). The codebooks used by $TSVQ$ on layer λ ($\lambda > 1$) are represented by $\mathcal{C}_\lambda^{[\mathbf{b}_{\lambda-1,n}]} = \{\mathbf{y}_\lambda^{[\mathbf{b}_{\lambda-1,n}0]}, \mathbf{y}_\lambda^{[\mathbf{b}_{\lambda-1,n}1]}\}$, where the block which determines this codebook, the length $\lambda - 1$ block of binary indices $\mathbf{b}_{\lambda-1,n} = (b_{1,n} \ldots b_{\lambda-1,n}) \in \{0,1\}^{\lambda-1}$, provides the $(\lambda-1)$-bit approximation of the quantizer input vector. This codebook representation emphasizes that all quantizers used in the successive refinement of $\mathbf{x}_n$, depends not only on the layer λ but also on $\mathbf{b}_{\lambda-1}$, the output of previous quantizers. Let us say that $\mathbf{x}_n$ is emitted by the source; the transmitted indices are $\{\mathbf{b}_{\Lambda,n}\} = \{(b_{1,n}, \ldots, b_{\Lambda,n})\}$ and, at the BSC output, the received blocks are $\{\hat{\mathbf{b}}_{\Lambda,n}\} = \{(\hat{b}_{1,n}, \ldots, \hat{b}_{\Lambda,n})\}$. The reconstructed cell delivered to the user is then the L-dimensional vector $\hat{\mathbf{x}}_n = (\mathbf{y}_\Lambda^{[\hat{b}_{1,n},\ldots,\hat{b}_{\Lambda,n}]})$.

Distortion between the vector $\mathbf{x}_n$ output by the source and the corresponding reconstructed vector $\hat{\mathbf{x}}_n$ delivered at the receiving end will be measured by the squared norm of the difference $d((\mathbf{x})_n, (\hat{\mathbf{x}})_n) = ||(\mathbf{x})_n - (\hat{\mathbf{x}})_n||^2$.

An optimum $TSVQ$ for noiseless channel [8], can be obtained by finding a collection $\mathcal{F}$ of $\sum_{\lambda=0}^{\Lambda-1} 2^\lambda$ codebooks,

$$\mathcal{F} = \{\mathcal{C}_0, \{\mathcal{C}_\lambda^{[\mathbf{b}_\lambda]}\} : (\lambda, \mathbf{b}_\lambda) \in \{1, \ldots, \Lambda\} \times \{0,1\}^\lambda\},$$

that yields an overall minimum average quantization distortion. By using the LBG algorithm [3] one can design a locally optimal $TSVQ$ — but its performance is poor on noisy channel since the channel is not taken into account by the design

Finding an optimum $TSVQ$ for the noisy channel, i.e. finding the collection $\mathcal{F}$ that minimizes the average distortion, is a much harder problem. For a real valued vector source with probability density function $p(\mathbf{x})$, an M-level $TSVQ$ and a channel described by a transition probability $P(j|i)$ between the transmitted vector index i and received index j, the average distortion can be written as

$$D(\mathcal{F}) = \frac{1}{L} \sum_{i=1}^{M} \sum_{j=1}^{M} P(j|i) d(i,j) \tag{1}$$

where $\{\mathcal{S}_1, \ldots, \mathcal{S}_M\}$ is the partition of the L-dimensional space induced by $\mathcal{F}$ and

$$d(i,j) = P(j|i) \int_{\mathcal{S}_i} p(\mathbf{x}) d(\mathbf{x}, \mathbf{c}_j) d\mathbf{x} \tag{2}$$

is the average distortion when a vector $\mathbf{x} \in \mathcal{S}_i$ ends up mapped, by the $TSVQ$ and BSC in the centroid $\mathbf{c}_j$.

An algorithm is presented in [8] for designing $TSVQ$ which takes into account the noisy channel. The procedure is called a CM-$TSVQ$-design since it does not guarantee an optimal encoder-decoder pair. We present an algorithm next which uses a training set instead of a known $p(\mathbf{x})$, developed along the same lines as the CM-$TSVQ$ to design the quantizer.

2.1. Channel-Matched Tree Structured Vector Quantizer algorithm

CM-$TSVQ$ Design Algorithm

Step 1 Let the training set be $\mathcal{S}_0 = \{\mathbf{x}_n\}_{n=1}^N$, and let the evaluated initial centroid be

$$\mathbf{y}_0^{[0]} = \frac{\sum_{n=0}^{N-1} \mathbf{x}_n}{N}. \tag{3}$$

Step 2 $\lambda = 1$
Get the first layer codebook $\mathcal{C}_1$ by creating two new centroids derived from the previous: initially set $\mathbf{y}_1^{[0]} = \mathbf{y}_0^{[0]}$ and $\mathbf{y}_1^{[1]} = \mathbf{y}_0^{[0]} + \varepsilon$, (where ε is a perturbation vector used to *split* the previous centroid in two) and then use the LBG algorithm to get the optimum codebook for the first layer quantizer. The partition $\{\mathcal{S}_1^{[0]}, \mathcal{S}_1^{[1]}\}$ of $\mathcal{S}_0$ induced by $\mathcal{C}_1$ are two new training set associated with the index 0 and 1, respectively.

Step 3 $\lambda = \lambda + 1$.
Find $\mathcal{C}_\lambda^{[\mathbf{b}_{\lambda-1}]} = \{\mathbf{y}_\lambda^{[\mathbf{b}_{\lambda-1}0]}, \mathbf{y}_\lambda^{[\mathbf{b}_{\lambda-1}1]}\}$, the codebook associated to each length $\lambda-1$ distinct block of binary indices

$\mathbf{b}_{\lambda-1}$ by, again, initially setting $\mathbf{y}_\lambda^{[\mathbf{b}_{\lambda-1}0]} = \mathbf{y}_{\lambda-1}^{[\mathbf{b}_{\lambda-1}]}$ and $\mathbf{y}_\lambda^{[\mathbf{b}_{\lambda-1}1]} = \mathbf{y}_{\lambda-1}^{[\mathbf{b}_{\lambda-1}]} + \varepsilon$ and then using the LBG algorithm to get the optimum codebook. The partition induced by this codebook is

$\{\mathcal{S}_\lambda^{[\mathbf{b}_{\lambda-1}0]}, \mathcal{S}_\lambda^{[\mathbf{b}_{\lambda-1}1]}\}$.

The corresponding centroids, for $\mathbf{b}_\lambda \in \{0,1\}^\lambda$ are

$$\mathbf{y}_\lambda^{[\mathbf{b}_\lambda]} = \text{centroid}\left(\mathcal{S}_\lambda^{[\mathbf{b}_\lambda]}\right) \quad (4)$$

$$= \frac{\sum_{\mathbf{x}_n \in \mathcal{S}_\lambda^{[\mathbf{b}_\lambda]}} \mathbf{x} Pr(b_\lambda | b_{\lambda,n})}{\sum_{\mathbf{x}_n \in \mathcal{S}_\lambda^{[\mathbf{b}_\lambda]}} Pr(b_\lambda | b_{\lambda,n})} \quad (5)$$

where $(b_{1,n} \ldots b_{\lambda,n})$ is the index of the λ-bits approximation of $\mathbf{x}_n$.

Step 4 If $\lambda < \Lambda$ go to 2.1.

Step 5 Stop

The design (and coding) complexity of the $CM\text{-}TSVQ$ is much lower than the complexity of $COVQ$ even when a fast implementation of $COVQ$ [5] is used.

3. TRANSMISSION OF QUANTIZED IMAGES TROUGH NOISY CHANNELS

Images VQ compressed and transmitted trough noisy channels are delivered to the user with an overall distortion which is a combination of two terms: the quantization distortion and the channel error distortion. Channel optimized vector quantizers $COVQ$, designed by taking the channel error into account, render a performance better than plain VQ but are still perturbed by an annoying distortion since the transmitted index i (i.e. $(b_{1,n} \ldots b_{\Lambda,n})$) is randomly transformed into an index j (i.e., $(\hat{b}_{1,n} \ldots \hat{b}_{\Lambda,n})$). An way to attenuate this problem is to use Forward Error correct Codes (FEC) — the channel bit error rate is reduced but at the expense of an increased distortion due to quantization. The burden on the rate put by VQ in tandem with FEC can be overcome by using $CM\text{-}TSVQ$ and protecting with FEC only the $TSVQ$ lower layer bits which causes larger damage to the transmission when delivered in error. This is equivalent to providing UEP ($Unequal Error Protection$) to the transmission.

The study of $TSVQ$ with UEP is the subject of the present work. The diagram in Fig. 1 illustrates the general structure of such a $TSVQ$, with an indication that the binary data output by the $TSVQ$ can be *processed* for protection before being sent to the channel. We have investigated two kinds of processing: (1) FEC codes and (2) FEC codes on reverse. The encoding process is done by entering successive blocks of bits of a given layer, say $(b_{\lambda,n+1} \ldots b_{\lambda,n+\ell})$ into the rate $R_\lambda = \frac{\ell}{\nu}$ FEC encoder and transmitting the corresponding output $(u_{\lambda,n+1} \ldots u_{\lambda,n+\nu})$. FEC on reverse means that the TSVQ bit-stream is passed through a FEC decoder which introduces errors whenever the input block does not constitute a legitimate binary codeword. Notice that in this case $\frac{\ell}{\nu} = 1$ meaning no rate increase. For the layers protected with FEC, *processing* is meant to be FEC while for other layers it means “no processing”.

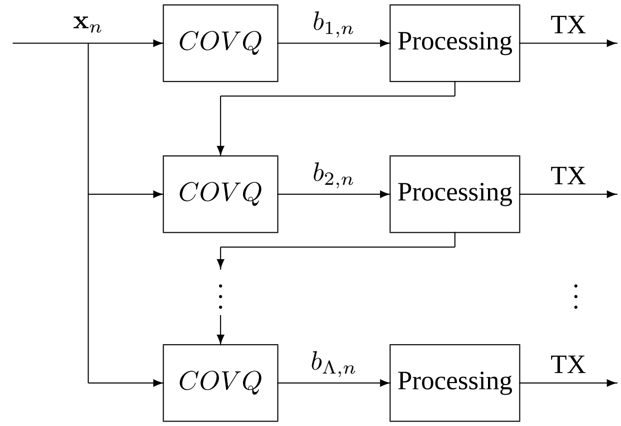

Fig. 1. Building blocks of a TSVQ

Several FEC schemes can be used with the TSVQ. We have chosen to make our investigation by using simple linear block codes. We have performed simulations considering transmission trough a BSC with cross-over probability $\rho_{0|1} = .001$, and used Hamming $(7,4)$ and $(15,11)$ codes. Allocation of code rates R_λ trough the layers were done by trial and error - the lower layers bits were coded with either a rate $\frac{4}{7}$ code or rate $\frac{11}{15}$. One can easily verify that, with these codes, virtually error-free transmission can be attained — the probability of a binary block with two bits in error is about 10^{-5} and 10^{-3} for the Hamming $(7,4)$ and $(15,11)$ respectively. The simulations results are presented next.

4. RESULTS

Extensive simulation results have shown that a better performance, compared to $COVQ$, can be achieved when using the $CM\text{-}TSVQ$ with UEP scheme. Fig. 2 displays the $PSNR \times Rate$ performance of the $CM\text{-}TSVQ$ when compressing the image $Lena$ (512×512). In this same figure the corresponding curves of the $COVQ$, VQ and $VQ + FEC$ are also plotted exhibiting the superior performance of $CM\text{-}TSVQ$. This superior performance can be explained by the use of a form of channel relaxation in the quantizer design. We noted in our simulations that the performance could be improved by designing the first stages of the TSVQ with a larger probability of error (in our experiments we have used $P_e \approx 0.2$).

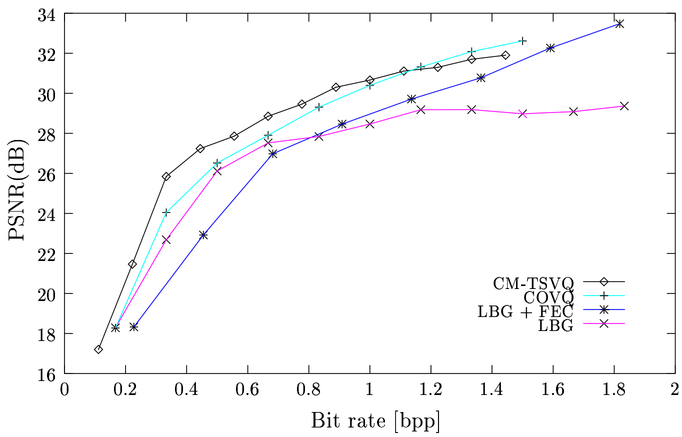

Fig. 2. Comparison between VQ's in a BSC for the Lenna image.

Scheme	Specifications	$PSNR$ (dB)
$COVQ$	3×2, 9 bits per cell	32.54
Scheme 1. $H^{1\ldots3}0^*$	$CM\text{-}TSVQ$ ($P_e = 0.2$), 12-bit codebook	33.27
Scheme 2. $H^{1+2}H0^*$	$CM\text{-}TSVQ$ ($P_e = 0.2$), 12-bit codebook	33.37
Scheme 3. $H^{1+2}H^{3+4}0^*$	$CM\text{-}TSVQ$ ($P_e = 0.2$) 12-bit codebook	33.58
Scheme 4. $H^{1+2,3+4,5+6,7}0^*$	$CM\text{-}TSVQ$ ($P_e = 0.2$) 12-bit codebook	33.29
Scheme 5. $H_D^{1\ldots4}0^*$	TSVQ 13-bit codebook	31.35

Table 1. Performance over a BSC at 1.5bpp, $P_e = 10^{-3}$, for the 512×512 Lena image.

It must be pointed out that although $CM\text{-}TSVQ$ has a slightly better $PSNR$ performance, it still exhibits typical COVQ artifacts (salt-and-pepper like noise). The use of UEP allows us to overcome this problem. The protection of the $TSVQ$ lower layer bit stream with FEC reduces the impact of the channel noise. Tables 1 and 2 list the $PSNR$ results obtained with some of the simulated schemes. On Table 1, the schemes are described by the following notation: scheme $H^{1\ldots3}0^*$ works with each of the first three layers protected with an H(15,11) FEC code; scheme $H^{1+2}H0^*$ $H^{1+2}H^{3+4}0^*$ works with the bit-stream from the first and second layers interleaved and protected with an H(15,11), plus an H(15,11) on the third layer; scheme $H^{1+2,3+4,5+6,7}0^*$ works with an interleaved H(15,11) on layers 1+2, 3+4 and 5+6 and an H(15,11) code on the seventh layer; finally the scheme $H_D^{1\ldots4}0^*$ works with an H(15,11) code on reverse on the first and second layers. $P_e = 0.2$ is the probability of error for the first $TSVQ$ stage. The schemes on Table 2 are similarly described.

For natural images, the $CM\text{-}TSVQ$ plus FEC has a near 1dB improvement in $PSNR$ over $COVQ$. For RSS images such improvement was not observed. In both cases however, the decoded images are subjectively better as illustrated in Fig. 3 and Fig. 4 (salt-and-pepper like noise is drastically reduced). It should be pointed out that $CM\text{-}TSVQ$ has a lower computational complexity encoder.
Another important characteristic of the $CM\text{-}TSVQ + FEC$ is its robustness to channel mismatch. $CM\text{-}TSVQ$ has a lower rate of decrease in $PSNR$ when the channel cross-over probability is increased than does $COVQ$ — Table 3 presents the values of $PSNR$ for three channels transition probabilities (the quantizers were designed for the $P_e = .001$ channel). As can be seen, the decrease in $PSNR$ is lower than $COVQ$ indicating that the proposed scheme would perform better in a fading, time-varying channel. Our results also show that the $LBG + FEC$ is a wasteful scheme and performs poorly, in general, as compared to both the $CM\text{-}TSVQ$ and $COVQ$.

5. CONCLUDING REMARKS

We have shown in this work that the performance of $CM\text{-}TSVQ$ can be significantly improved with UEP. Performance better than $COVQ$ was observed. Subjective evaluation of the performance also points the $CM\text{-}TSVQ$ as a better scheme.

We are currently investigating the problem of code rate allocation among the layers of the quantizer as well as the use of coding schemes other than linear block codes, e.g., RCPC codes with soft decision.

Other forms of *processing* are also being investigated.

6. REFERENCES

[1] N. Farvardin. A study of vector quantization for noisy channels. *IEEE Trans. Information Theory*, 36:799–809, July 1990.

[2] Shrinivas Gadkari and Kenneth Rose. Robust vector quantizer design by channel relaxation. *IEEE Trans. Communication*, 47(8):1113–1116, August 1999.

[3] A. Gersho and R. M. Gray. *Vector Quantizaztion and Signal Compression*. Kluwer, Norwell, MA, 1991.

[4] A. J. Goldsmith and M. Effros. Joint design of fixed-rate source codes and multiresolution channel codes. *IEEE Transactions on Communications*, 46(10):1301–1312, October 1998.

Scheme	Specifications	$PSNR(dB)$
$COVQ$	3×2 9 bits per cell	41.18
Scheme 1. R: $H^{1 \ldots 5}0^*$ G: $H^{1,2}0^*$ B: $H^{1,2}0^*$	$CM\text{-}TSVQ$ ($P_e = 0.18$), 13-bit codebook 12-bit codebook 8-bit codebook	40.99
Scheme 2. R: $H^{1 \ldots 4}0^*$ G: $H^{1 \ldots 3}0^*$ B: $H^{1,2}0^*$	$CM\text{-}TSVQ$ ($P_e = 0.18$), 13-bit codebook 12-bit codebook 8-bit codebook	40.91
Scheme 3. R: $H^{1 \ldots 5}0^*$ G: $H^{1 \ldots 3}0^*$ B: $H0^*$	$CM\text{-}TSVQ$ ($P_e = 0.18$), 13-bit codebook 12-bit codebook 8-bit codebook	40.99
Scheme 4. R: $H^{1 \ldots 3}0^*$ G: $H^{1,2}0^*$ B: $H0^*$	$CM\text{-}TSVQ$ ($P_e = 0.18$), 13-bit codebook 12-bit codebook 8-bit codebook	40.93

Table 2. Performance over a BSC at 1.5bpp, $P_e = 10^{-3}$, for a 512×512 RSS image.

	Lena			RSS		
P_e	.001	.005	.01	.001	.001	.01
$COVQ$	32.54	29.12	26.71	40.98	37.84	35.55
Proposed	33.58	31.44	28.62	40.84	39.22	37.78

Table 3. $PSNR(dB)$ results under channel mismatch

[5] J. Han and H. Kim. Efficient algorithm for channel-optmised vector quantization. *Electronic Letters*, 35(176):1305–1306, August 1999.

[6] N. Jayant J. De Marca. An algorithm for assigning binary indices to the codevectors of a multi-dimension quantizer. *in Proc. Int. Conf. Communications*, pages 1128–1132, 1987.

[7] V. Vaishampayan N. Farvardin. On the performance and complexity of channel-optimized vector quantizers. *IEEE Trans. Information Theory*, 37(1):155–160, January 1991.

[8] N. Farvadin N. Phamdo and T. Moriya. A unified approach to tree-structyred and multistage vector quantization for noisy channels. *IEEE Trans.on Information Theory*, 39(3):835–850, May 1993.

[9] P. G. Sherwood and K. Zeger. Progressive image coding on noisy channels. *Proceedings of the Data Compression Conference*, March 1997.

[10] K. Zeger and A. Gersho. Pseudo-gray coding. *IEEE Trans. Commun.*, 38:2147–2158, 1990.

Fig. 3. Performance over a BSC $P_e = 10^{-3}$, clockwise: (a) cut from 512×512 Lena, original; (b) $COVQ$; (c) Table 1, scheme 1; (d) Table 1, scheme 3.

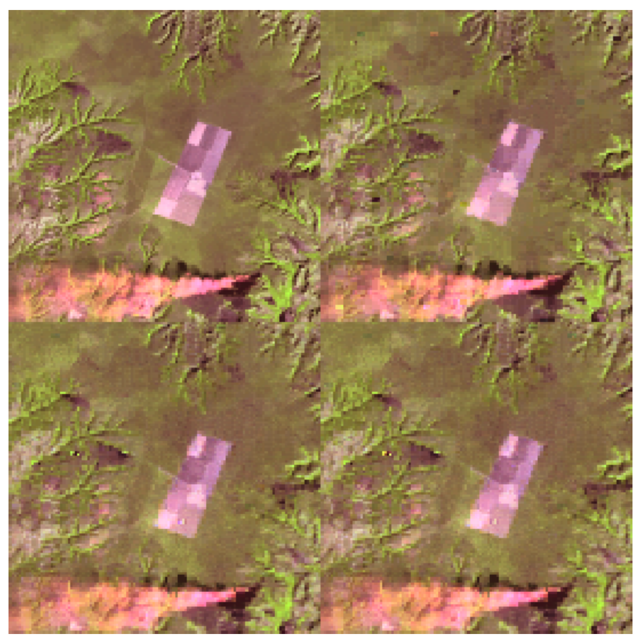

Fig. 4. Performance over a BSC $P_e = 10^{-3}$, clockwise: (a) cut from a typical RSS image; (b) COVQ (c) Table 2, scheme 1; (d) Table 2, scheme 3.