\{m-PSK\}^2: AN ALTERNATIVE PARTIAL m-PSK OVERLAPPING MODULATION SCHEME

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ABSTRACT

The aim of this paper is to analyze an alternative modulation technique called \{m-PSK\}^2 in terms of bit error rate (BER). Varying the overlapping degree in this modulation scheme, it is possible to obtain bandwidth efficiency in a continuous way. The performance of the \{m-PSK\}^2 is compared to the traditional M-PSK modulation scheme, where \(m\) and \(M\) are related by \(M = 2^m\) and \(i = \sqrt{m \cdots 2 \sqrt{m - 1}}\). Performance curves obtained by the theoretical model and by computer simulation are shown.

1. INTRODUCTION

An \{m-PSK\}^2 is a modulation technique based on the spectral overlapping of two conventional m-PSK modulations. The \{m-PSK\}^2 scheme is based on the same principle of an \{m-QAM\}^2 modulation technique [1][2], but with the m-PSK modulations incorporated.

The purpose of this work is to analyze a modulation technique called \{m-PSK\}^2 that increases the bit error performance with no sacrifice on the information rate and no expansion of the bandwidth in comparison to a conventional PSK scheme.

Efficient modulation schemes were studied by different authors. Two interesting schemes are shortly discussed here. The former, the QPSK modulation [3], which utilizes two data shaping pulses and two carriers that are pair wise quadrature in phase, to create two more dimensions. The latter, the OFDM method [4], in which multiple user symbols are transmitted in parallel using different overlapping sub carriers with orthogonal signal waveforms.

Here is present the \{m-PSK\}^2 scheme, an alternative modulation which is based on a partial spectral overlapping of two m-PSK schemes [5], which are not orthogonal. At the receiving end, the two m-PSK signals are separated. Varying the overlapping degree in this modulation scheme, it is possible to obtain fractional bandwidth efficiency.

Monte Carlos simulation using an AWGN channel shows curves of bit error rate (BER) as functions of the energy per bit to noise spectral density ratio \((E_b/N_0)\) for the theoretical and computational models.

This paper is organized as follows. In Section 2 it is described the system model and is analyzed its theoretical performance. In Section 3, there are the computational results. After all, the conclusions are presented in Section 4.

2. THE \{m-PSK\}^2 SCHEME

The block diagram of an \{m-PSK\}^2 modulation scheme is given in Fig. 1. It is assumed, with no loss of generality, that a rectangular pulse is transmitted for each constituent m-PSK scheme.

\[
b_1 \xrightarrow{m-PSK \text{ Modulator}} s_1(t) \\
b_2 \xrightarrow{m-PSK \text{ Modulator}} s_2(t)
\]

\[s(t) = A_s \cos(2\pi f_1 t + \Theta_1) + A_s \cos(2\pi f_2 + \Theta_2)\] (1)

where \(A_s, f, \Theta\) are the amplitude, the frequency and the phase of the constituent m-PSK signal \(s(t); i = 1, 2\). Each constituent m-PSK modulation occupies a bandwidth equal to \(r_s\), where \(r_s = r_0 \log m\) [symbol/s] is the transmitted symbol rate of each constituent m-PSK. Then the total bandwidth of the signal \(s(t)\) is \(W = r_s (1 + f_2/r_s)\) [Hz]. Therefore the spectral efficiency \(R\) of the \{m-PSK\}^2 system is given by

\[R = \frac{2r_s}{W} = 2 \log m \frac{f_1}{1 + f_2/r_s} \text{ [bits/s/Hz]}\] (2)
The narrowband noise over the signal $s(t)$ can be represented in the form

$$n(t) = n_1(t) \cos(2\pi f_1 t) - n_Q(t) \sin(2\pi f_1 t)$$  \hspace{1cm} (3)$$

where $n_1(t)$ and $n_Q(t)$ are the in-phase and quadrature noise components, respectively [5]. Each noise component occupies a bandwidth of $W$ [Hz]. The average power of each component is $N_0 W$, where $N_0$ [Watt/Hz] is the two-side noise power spectral density. The frequency $f_0$ is the mean value of the two carrier frequencies $f_1$ and $f_2$. The block diagram of a $\{m$-PSK$\}^2$ receiver is given in Fig. 2.

The signal $s(t)$ corrupted by noise enters into four correlators. The output of each correlator is then integrated over the sampled period $T_s$ producing the four inputs of the linear transformer, which are written in the following matrix form:

$$\begin{bmatrix}
d(T_s) \\
b(T_s) \\
c(T_s) \\
d(T_s)
\end{bmatrix} = T_s \begin{bmatrix} 1 & 0 & K_1 & K_2 \\
0 & 1 & -K_2 & K_1 \\
K_1 & -K_2 & 1 & 0 \\
K_2 & K_1 & 0 & 1
\end{bmatrix} \begin{bmatrix} A_1 \cos\Theta_1 \\
A_1 \sin\Theta_1 \\
A_2 \cos\Theta_2 \\
A_2 \sin\Theta_2
\end{bmatrix}$$  \hspace{1cm} (4)$$

where

$$N_{I_1}(T_s) = \int_0^{T_s} [n_1(t) \cos(\frac{2\pi f_1 t}{2}) - n_Q(t) \sin(\frac{2\pi f_1 t}{2})] dt$$  \hspace{1cm} (5)$$

$$N_{Q_1}(T_s) = \int_0^{T_s} [n_1(t) \sin(\frac{2\pi f_1 t}{2}) + n_Q(t) \cos(\frac{2\pi f_1 t}{2})] dt$$  \hspace{1cm} (6)$$

and

$$K_i = \frac{\sin(2\pi f_1 T_s)}{2\pi f_1 T_s}; \hspace{1cm} K_2 = \frac{\cos(2\pi f_1 T_s) - 1}{2\pi f_1 T_s}$$  \hspace{1cm} (9)$$

or simply $A = T_s M + N$. The matrix $M$ is a non-singular matrix, so it has an inverse. The linear transformer then performs a multiplication of the matrix $A$ by $\frac{1}{T_s} M^{-1}$ which produces the output matrix $O$:

$$\frac{1}{T_s} M^{-1} A = S + \frac{1}{T_s} M^{-1} N$$  \hspace{1cm} (10)$$

Hence the input of the two decision/decoding devices is given by $O = S + \tilde{N}$ which can be rewritten as:

$$\begin{bmatrix} I_1(T_s) \\
Q_1(T_s) \\
I_2(T_s) \\
Q_2(T_s)
\end{bmatrix} = \begin{bmatrix} A_1 \cos\Theta_1 + \tilde{N}_1(T_s) \\
A_1 \sin\Theta_1 + \tilde{N}_2(T_s) \\
A_2 \cos\Theta_2 + \tilde{N}_3(T_s) \\
A_2 \sin\Theta_2 + \tilde{N}_4(T_s)
\end{bmatrix}$$  \hspace{1cm} (11)$$

where $\tilde{N}$ is the noise matrix with dependent components because of the spectral overlapping.

The first two entries $I_1(T_s)$ and $Q_1(T_s)$ on the left-hand side in (11) are the input of a decision/decoding circuit while $I_2(T_s)$ and $Q_2(T_s)$ are the input of another one.

As similarly studied for $\{m$-QAM$\}^2$ systems [1][2], the probability of correct decision $P_c$, in an $\{m$-PSK$\}^2$ scheme, is given by

$$P_c = (1 - p_r)^2 \equiv 1 - 2 p_r; \hspace{1cm} \text{for} \hspace{0.5cm} p_r \ll 1$$  \hspace{1cm} (12)$$

$$p_r = \text{erf}\left[\frac{A}{\sigma \sqrt{2}} \sin\left(\frac{\pi}{m}\right)\right]$$  \hspace{1cm} (13)$$

where $p_r$ is the well known error probability for each constituent $m$-PSK modulation and $\sigma^2$ is the variance of the noise in the decision region, that can be evaluated by

$$\sigma^2 = N \delta(\Delta f/s_r)$$  \hspace{1cm} (14)$$

Therefore the probability of symbol error is

$$P_m = 1 - P_c = 2p_r$$  \hspace{1cm} (15)$$

On the other hand, the average energy for each $m$-PSK constellation is $A_m^2 T_s$, the power per symbol channel is $A_m^2$ and...
the total power transmitted in an $[m\text{-PSK}]^2$ scheme is $2A^2$.

Considering the following relation

$$E_b = \frac{S W}{N_0}$$

and holding that the noise components in the decision of each constituent $m\text{-PSK}$ are weakly correlated for high values of $E_b/N_0$, after some algebraic manipulation, the bit error rate for the $[m\text{-PSK}]^2$ system can be evaluated by

$$BER \equiv \frac{1-P_c}{\log_2 m} = 2 \exp \left[ \frac{-2r_{(a,b)} \log_2 m - E_b \sin \left( \frac{\pi}{m} \right)}{2(1+D/r)} \delta(D/r) \right]$$

where $\delta(D/r)$ is a real valued function of $D/r$. The function $\delta(D/r)$ depends on the structure of the receiver, i.e., it depends on the correlators, the linear transformation used to separate the information signals at the receiver and the noise components and it can be written as:

$$\delta(a,b) = \left\{ \begin{array}{ll}
\left[ 1 + \frac{1}{2} r(a,b) \right] (1-k) H(a,k) \\
\end{array} \right. \quad \text{for } a=b
$$

where

$$a = \pi(r, r) \cdot b = \pi(a, r) \cdot k = 1 - (k_1^2 + k_2^2)$$

$$r(a,b) = H(a,b) I(a,b)$$

and

$$H(a,b) = \sum_{\ell=0}^{\infty} \frac{(-1)^{\ell+1}}{(2\ell+1)(2\ell+1)(2\ell+2)^2} \left[ \left( 1+\frac{b}{a} \right) \left( a+b \right)^{\ell} + \left( 1-\frac{b}{a} \right) \left( a-b \right)^{\ell} \right]$$

$$I(a,b) = \sum_{\ell=0}^{\infty} \frac{(-1)^{\ell}}{(2\ell+1)(2\ell+1)(2\ell+2)^2} \left[ \left( 1+\frac{b}{a} \right) \left( a+b \right)^{\ell} + \left( 1-\frac{b}{a} \right) \left( a-b \right)^{\ell} \right]$$

3. SIMULATION RESULTS

In order to compare the performance between the $[m\text{-PSK}]^2$ system with a conventional M-PSK scheme, the symbol period of both systems are related by

$$T_s = \frac{\log_2 M}{2 \log_2 m}$$

Fig. 4 shows the spectral efficiency $R$ [bits/s/Hz] versus $E_b/N_0$ [dB] for the $[4\text{-PSK}]^2$ with $\Delta f/r_1$ equal to 1, 7/9, 3/5, 5/11, 1/3, 1/7 and 1/15, and bit error rate fixed at $10^{-4}$. The spectral efficiency curves of the equivalent M-QAM schemes ($M = 16$, 32, 64, 128 and 256) are also shown. Notice that, for $\Delta f/r_1 = 1$, the $[4\text{-PSK}]^2$ scheme presents no gain (-2.7 dB), in terms of $E_b/N_0$, over the 16-PSK. For $\Delta f/r_1 = 3/5$, 1/3 and 1/7 the $[4\text{-PSK}]^2$ produces gain of 2.2, 5.2 and 5.2 dB over its equivalent 32-, 64- and 128-QAM schemes, respectively. The theoretical and simulated points in Fig. 4 are almost the same which indicates that (17) holds for $[4\text{-PSK}]^2$ schemes.
system is degraded. Nevertheless the bandwidth efficiency increases.

The BER versus $R$ [bits/s/Hz] in function of $E_b/N_0$ [dB] as a parameter is shown in Fig. 6 as another way to present the performance of the $\{4\text{-PSK}\}^2$ system. Each curve represents a constant energy per bit to noise spectral density ratio, showing that they are going to get together when the bite rate increases for an apparent and fixed bandwidth. Notice that this fact happens with a decrease of bit error rate, and becomes more apparent for higher $E_b/N_0$ as shown in Fig.6.

![Figure 6](image-url)  
*Figure 6* Bit error rate versus spectral efficiency for $E_b/N_0$ as a parameter.

### 4. CONCLUSIONS

The $\{m\text{-PSK}\}^2$ scheme is a modulation technique that can present gains, in terms of $E_b/N_0$ [dB], over a conventional $M$-PSK system, in view of the same spectral efficiency and bit error rate. This paper has shown that the computer simulations are similar to the theoretical model (17) for $\{4\text{-PSK}\}^2$ scheme. Notice that this modulation scheme can span spectral efficiency from 4 bits/s/Hz for $\Delta f/\Delta f_r = 1$ to 7.5 bits/s/Hz for $\Delta f/\Delta f_r = 1/15$. Notice also that it is possible to obtain fractional spectral efficiency using $\{m\text{-PSK}\}^2$ schemes. Curves of performance for the $\{4\text{-PSK}\}^2$ scheme were shown in Figs. 4, 5 and 6. Others works on $\{m\text{-QAM}\}^2$ schemes were performed but with coded modulation [6][7].

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### REFERENCES


