$\{m$-PSK$\}^2$: AN ALTERNATIVE PARTIAL m-PSK OVERLAPPING MODULATION SCHEME

Elvio César Giraudo

Departamento de Engenharia Elétrica - Centro de Tecnologia - Universidade Federal do Ceará - UFC
Caixa Postal 6.001, CEP: 60.455-760, Fortaleza CE - Brasil
Tel: +5585 288-9585, Fax: +5585 288-9574, elvio@dee.ufc.br

ABSTRACT

The aim of this paper is to analyze an alternative modulation technique called $\{m$-PSK$\}^2$ in terms of bit error rate (BER). Varying the overlapping degree in this modulation scheme, it is possible to obtain bandwidth efficiency in a continuous way. The performance of the $\{m$-PSK$\}^2$ is compared to the traditional M-PSK modulation scheme, where m and M are related by $M = 2^i$ and $i = \sqrt{m}, \cdots, 2\sqrt{m} - 1$. Performance curves obtained by the theoretical model and by computer simulation are shown.

1. INTRODUCTION

An $\{m$-PSK$\}^2$ is a modulation technique based on the spectral overlapping of two conventional m-PSK modulations. The $\{m$-PSK$\}^2$ scheme is based on the same principle of an $\{m$-QAM$\}^2$ modulation technique [1][2], but with the m-PSK modulations incorporated.

The purpose of this work is to analyze a modulation technique called $\{m$-PSK$\}^2$ that increases the bit error performance with no sacrifice on the information rate and no expansion of the bandwidth in comparison to a conventional PSK scheme.

Efficient modulation schemes were studied by different authors. Two interesting schemes are shortly discussed here. The former, the Q^2PSK modulation [3], which utilizes two data shaping pulses and two carriers that are pair wise quadrature in phase, to create two more dimensions. The latter, the OFDM method [4], in which multiple user symbols are transmitted in parallel using different overlapping sub carriers with orthogonal signal waveforms.

Here is present the $\{m$-PSK$\}^2$ scheme, an alternative modulation which is based on a partial spectral overlapping of two m-PSK schemes [5], which are not orthogonal. At the receiving end, the two m-PSK signals are separated. Varying the overlapping degree in this modulation scheme, it is possible to obtain fractional bandwidth efficiency.

Monte Carlos simulation using an AWGN channel shows curves of bit error rate (BER) as functions of the energy per bit to noise spectral density ratio (E_b/N_0) for the theoretical and computational models.

This paper is organized as follows. In Section 2 it is described the system model and is analyzed its theoretical performance. In Section 3, there are the computational results. After all, the conclusions are presented in Section 4.

2. THE $\{m$-PSK$\}^2$ SCHEME

The block diagram of an $\{m$-PSK$\}^2$ modulation scheme is given in Fig. 1. It is assumed, with no loss of generality, that a rectangular pulse is transmitted for each constituent m-PSK scheme.

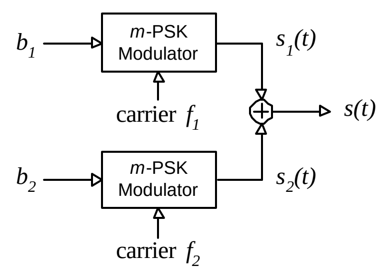

Figure 1. $\{m$-PSK$\}^2$ Modulator.

The input bit stream for each constituent m-PSK modulator is denoted by b_1 and b_2, respectively. For simplification, we are going to consider the same bit rate for both inputs, that is, $r_{b_1} = r_{b_2} = r_b$.

An $\{m$-PSK$\}^2$ modulated wave can be represented by a narrowband signal which is the summation of two m-PSK modulated waves separated in frequency by $\Delta f = f_2 - f_1$ [Hz]:

$$s(t) = A_c cos(2\pi f_1 t + \Theta_1) + A_c cos(2\pi f_2 + \Theta_2) \qquad (1)$$

where A_c, f_i and Q_i are the amplitude, the frequency and the phase of the constituent m-PSK signal $s_i(t)$; $i=1,2$. Each constituent m-PSK modulation occupies a bandwidth equal to r_s, where $r_s = r_b/\log_2 m$ [symbol/s] is the transmitted symbol rate of each constituent m-PSK. Then the total bandwidth of the signal $s(t)$ is $W = r_s\,(1 + \Delta f/r_s)$ [Hz]. Therefore the spectral efficiency R of the $\{m$-PSK$\}^2$ system is given by

$$R = \frac{2r_b}{W} = \frac{2\,log_2 m}{1 + \Delta f/r_s} \quad \text{[bits/s/Hz]} \qquad (2)$$

The narrowband noise over the signal $s(t)$ can be represented in the form

$$n(t) = n_I(t)\cos(2\pi f_0 t) - n_Q(t)\sin(2\pi f_0 t) \tag{3}$$

where $n_I(t)$ and $n_Q(t)$ are the in-phase and quadrature noise components, respectively [5]. Each noise component occupies a bandwidth of W [Hz]. The average power of each component is $N_0\,W$, where N_0 [Watt/Hz] is the two-side noise power spectral density. The frequency f_0 is the mean value of the two carrier frequencies f_1 and f_2. The block diagram of a $\{m\text{-PSK}\}^2$ receiver is given in Fig. 2.

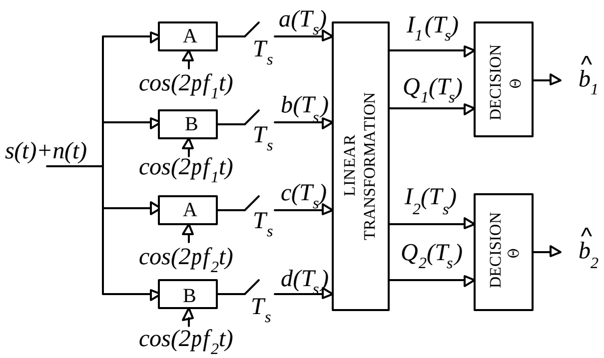

Figure 2. $\{m\text{-PSK}\}^2$ Receiver.

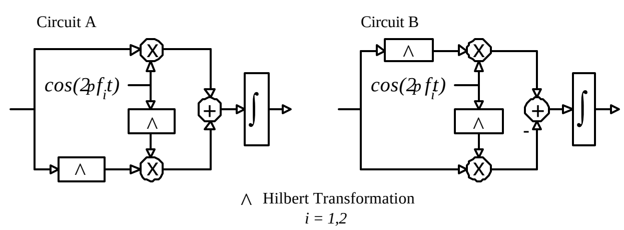

Figure 3. Receiver mixers for the $\{m\text{-PSK}\}^2$ correlators.

The signal $s(t)$ corrupted by noise enters into four correlators. The output of each correlator is then integrated over the sampled period T_s producing the four inputs of the linear transformer, which are written in the following matrix form:

$$\underbrace{\begin{bmatrix} a(T_s) \\ b(T_s) \\ c(T_s) \\ d(T_s) \end{bmatrix}}_{A} = T_s \underbrace{\begin{bmatrix} 1 & 0 & K_1 & K_2 \\ 0 & 1 & -K_2 & K_1 \\ K_1 & -K_2 & 1 & 0 \\ K_2 & K_1 & 0 & 1 \end{bmatrix}}_{M} \underbrace{\begin{bmatrix} A_c\cos\Theta_1 \\ A_c\sin\Theta_1 \\ A_c\cos\Theta_2 \\ A_c\sin\Theta_2 \end{bmatrix}}_{S} + \underbrace{\begin{bmatrix} N_{I1}(T_s) \\ N_{Q1}(T_s) \\ N_{I2}(T_s) \\ N_{Q2}(T_s) \end{bmatrix}}_{N} \tag{4}$$

where

$$N_{I1}(T_s) = \int_0^{T_s} \left[n_I(t)\cos\left(\frac{2\pi\Delta f}{2}t\right) - n_Q(t)\sin\left(\frac{2\pi\Delta f}{2}t\right) \right] dt \tag{5}$$

$$N_{Q1}(T_s) = \int_0^{T_s} \left[n_I(t)\sin\left(\frac{2\pi\Delta f}{2}t\right) + n_Q(t)\cos\left(\frac{2\pi\Delta f}{2}t\right) \right] dt \tag{6}$$

$$N_{I2}(T_s) = \int_0^{T_s} \left[n_I(t)\cos\left(\frac{2\pi\Delta f}{2}t\right) + n_Q(t)\sin\left(\frac{2\pi\Delta f}{2}t\right) \right] dt \tag{7}$$

$$N_{Q2}(T_s) = \int_0^{T_s} \left[-n_I(t)\sin\left(\frac{2\pi\Delta f}{2}t\right) + n_Q(t)\cos\left(\frac{2\pi\Delta f}{2}t\right) \right] dt \tag{8}$$

and

$$K_1 = \frac{\sin(2\pi\Delta f T_s)}{2\pi\Delta f T_s} \;;\; K_2 = \frac{\cos(2\pi\Delta f T_s) - 1}{2\pi\Delta f T_s} \tag{9}$$

or simply $A = T_s\,M\,S + N$. The matrix M is a non-singular matrix, so it has an inverse. The linear transformer then performs a multiplication of the matrix A by $\frac{1}{T_s}M^{-1}$ which produces the output matrix O:

$$\underbrace{\frac{1}{T_s}M^{-1}A}_{O} = S + \underbrace{\frac{1}{T_s}M^{-1}N}_{\tilde{N}} \tag{10}$$

Hence the input of the two decision/decoding devices is given by $O = S + \tilde{N}$ which can be rewritten as:

$$\begin{bmatrix} I_1(T_s) \\ Q_1(T_s) \\ I_2(T_s) \\ Q_2(T_s) \end{bmatrix} = \begin{bmatrix} A_c\cos\Theta_1 \\ A_c\sin\Theta_1 \\ A_c\cos\Theta_2 \\ A_c\sin\Theta_2 \end{bmatrix} + \begin{bmatrix} \tilde{N}_{I1}(T_s) \\ \tilde{N}_{Q1}(T_s) \\ \tilde{N}_{I2}(T_s) \\ \tilde{N}_{Q2}(T_s) \end{bmatrix} \tag{11}$$

where $\tilde{N}$ is the noise matrix with dependent components because of the spectral overlapping.

The first two entries $I_1(T_S)$ and $Q_1(T_S)$ on the left-hand side in (11) are the input of a decision/decoding circuit while $I_2(T_S)$ and $Q_2(T_S)$ are the input of another one.

As similarly studied for $\{m\text{-QAM}\}^2$ systems [1][2], the probability of correct decision Pc, in an $\{m\text{-PSK}\}^2$ scheme, is given by

$$Pc \cong (1 - p_e)^2 \cong 1 - 2p_e \quad ; \text{ for } p_e \ll 1 \tag{12}$$

$$p_e = \operatorname{erf}\left[\frac{A_c}{\sigma\sqrt{2}} \sin\left(\frac{\pi}{m}\right) \right] \tag{13}$$

where p_e is the well known error probability for each constituent m-PSK modulation and σ^2 is the variance of the noise in the decision region, that can be evaluated by

$$\sigma^2 = N\,\delta(\Delta f / r_s) \tag{14}$$

Therefore the probability of symbol error is

$$P_m = 1 - P_C \cong 2p_e \tag{15}$$

On the other hand, the average energy for each m-PSK constellation is $A_c^2 T_s$, the power per symbol channel is A_c^2 and the total power transmitted in an $\{m\text{-PSK}\}^2$ scheme is $2A_c^2$. Considering the following relation

$$\frac{E_b}{N_0} = \frac{S}{N}\frac{W}{r_b} \tag{16}$$

and holding that the noise components in the decision of each constituent m-PSK are weakly correlated for high values of E_b/N_0, after some algebraic manipulation, the bit error rate for the $\{m\text{-PSK}\}^2$ system can be evaluated by

$$BER \cong \frac{1-Pc}{\log_2 m} = 2\,\mathrm{erf}\left[\sqrt{\frac{\log_2 m}{2\left(1+\Delta f/r_s\right) d\left(\Delta f/r_s\right)}\frac{E_b}{N_0}}\,\sin\left(\frac{p}{m}\right)\right] \tag{17}$$

where $d(Df/r_s)$ is a real valued function of Df/r_s. The function $d(Df/r_s)$ depends on the structure of the receiver, i. e., it depends on the correlators, the linear transformation used to separate the information signals at the receiver and the noise components and it can be written as :

$$d(a,b) = \frac{\{1+[1-2r(a,b)](1-k)\}I(a,b)}{k^2} \tag{18}$$

where

$$a = p(r_s + Df)\ ,\ b = pDf\ ,\ k = 1-(k_1^2 + k_2^2) \tag{19}$$

$$r(a,b) = \frac{H(a,b)}{I(a,b)} + \frac{k_1}{k_2}\frac{J(a,b)}{I(a,b)} \tag{20}$$

$$I(a,b) = \sum_{r=0}^{\infty}\frac{(-1)^r}{(2r+1)!(2r+1)(2r+2)}\left\{(1+\frac{b}{a})[(a+b)T_s]^{2r} + (1-\frac{b}{a})[(a-b)T_s]^{2r}\right\} \tag{21}$$

$$H(a,b) = \sum_{r=0}^{\infty}\frac{(-1)^r}{(2r+1)!(2r+1)}\left\{(1+\frac{b}{a})[(a+b)T_s]^{2r} + (1-\frac{b}{a})[(a-b)T_s]^{2r}\right\} \tag{22}$$

$$J(a,b) = \sum_{r=0}^{\infty}\frac{(-1)^r}{(2r+2)!(2r+2)}\left\{(1+\frac{b}{a})[(a+b)T_s]^{2r} - (1-\frac{b}{a})[(a-b)T_s]^{2r}\right\} \tag{23}$$

3. SIMULATION RESULTS

In order to compare the performance between the $\{m\text{-PSK}\}^2$ system with a conventional M-PSK scheme, the symbol period of both systems are related by

$$T_M = T_m\frac{\log_2 M}{2\log_2 m} \tag{24}$$

Fig. 4 shows the spectral efficiency R [bits/s/Hz] versus E_b/N_0 [dB] for the $\{4\text{-PSK}\}^2$ with Df/r_s equal to 1, 7/9, 3/5, 5/11, 1/3, 3/13, 1/7 and 1/15, and bit error rate fixed at 10^{-4}. The spectral efficiency curves of the equivalent M-QAM schemes (M = 16, 32, 64, 128 and 256) are also shown. Notice that, for Df/r_s = 1, the $\{4\text{-PSK}\}^2$ scheme presents no gain (-2.7 dB), in terms of E_b/N_0, over the 16-PSK. For Df/r_s = 3/5, 1/3 and 1/7 the $\{4\text{-PSK}\}^2$ produces gain of 2.2, 5.2 and 5.2 dB over its equivalent 32-, 64- and 128-QAM schemes, respectively. The theoretical and simulated points in Fig. 4 are almost the same which indicates that (17) holds for $\{4\text{-PSK}\}^2$ schemes.

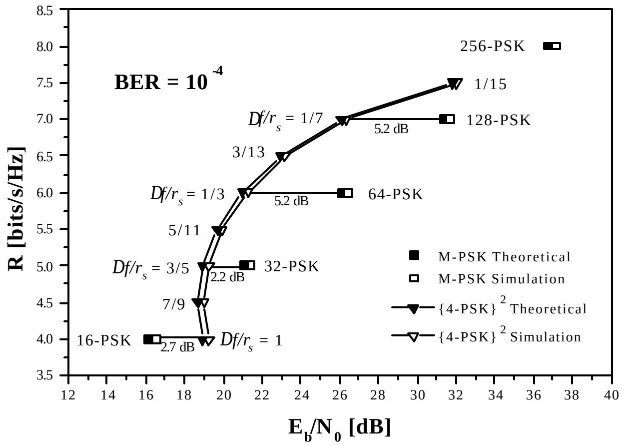

Figure 4 Spectral efficiency for the $\{4\text{-PSK}\}^2$ versus E_b/N_0[dB], for BER=10^{-4}.

For purpose of comparison, the 64-PSK an the $\{4\text{-PSK}\}^2$ with Df/r_s=1/3, present the same spectral efficiency of 6 bits/s/Hz, but the $\{4\text{-PSK}\}^2$ system requires 5.2 dB of E_b/N_0 less than 64-PSK. From the complexity point of view, the $\{4\text{-PSK}\}^2$ needs two 16-PSK constellations instead of one 64-PSK that is more susceptible to noise, and more complicated to implement.

The $\{m\text{-PSK}\}^2$ structure modulation allows us to adjust the bandwidth efficiency in a continuous way varying the degree of overlapping as shown the curve in Fig. 4.

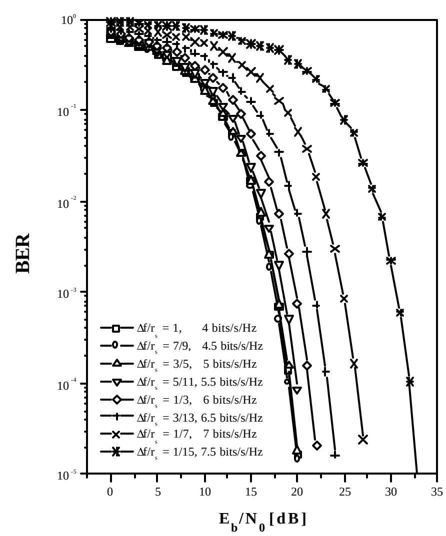

Figure 5 Bit error rate performance for $\{4\text{-PSK}\}^2$.

Fig. 5 shows BER as a function of E_b/N_0 [dB] for the $\{4\text{-PSK}\}^2$ with Df/r_s equal to 1, 7/9, 3/5, 5/11, 1/3, 3/13, 1/7 and 1/15, obtained by computer simulation. These curves show that when the degree of overlapping in the $\{4\text{-PSK}\}^2$ system is increased, the curves shift to the right, pointing that the performance of the system is degraded. Nevertheless the bandwidth efficiency increases.

The BER versus R [bits/s/Hz] in function of E_b/N_0 [dB] as a parameter is shown in Fig. 6 as another way to present the performance of the $\{4\text{-PSK}\}^2$ system. Each curve represents a constant energy per bit to noise spectral density ratio, showing that they are going to get together when the bite rate increases for an apparent and fixed bandwidth. Notice that this fact happens with a decrease of bit error rate, and becomes more apparent for higher E_b/N_0 as shown in Fig.6.

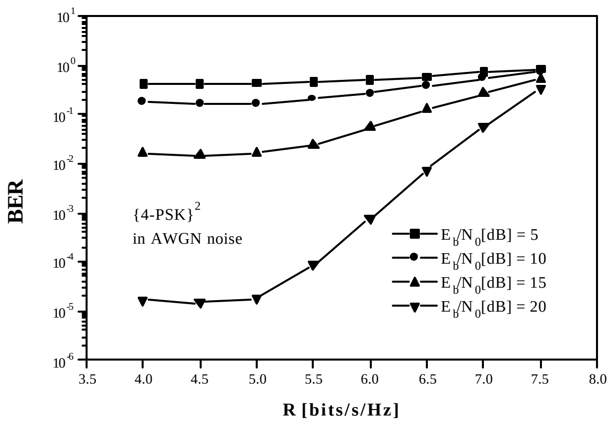

Figure 6 Bit error rate versus spectral efficiency for E_b/N_0 as a parameter.

4. CONCLUSIONS

The $\{m\text{-PSK}\}^2$ scheme is a modulation technique that can present gains, in terms of E_b/N_0 [dB], over a conventional M-PSK system, in view of the same spectral efficiency and bit error rate. This paper has shown that the computer simulations are similar to the theoretical model (17) for $\{4\text{-PSK}\}^2$ scheme. Notice that this modulation scheme can span spectral efficiency from 4 bits/s/Hz for $Df/r_s = 1$ to 7.5 bits/s/Hz for $Df/r_s = 1/15$. Notice also that it is possible to obtain fractional spectral efficiency using $\{m\text{-PSK}\}^2$ schemes. Curves of performance for the $\{4\text{-PSK}\}^2$ scheme were shown in Figs. 4, 5 and 6. Others works on $\{m\text{-QAM}\}^2$ schemes were performed but with coded modulation [6][7].

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