

# PERFORMANCE ANALYSIS OF EIGENSTRUCTURE-BASED TECHNIQUES FOR ARRAY PROCESSING IN WIRELESS COMMUNICATIONS (\*)

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## ABSTRACT

**In this contribution we make use of eigenstructure-based techniques in order to evaluate the performance of some spatial adaptive algorithms. These techniques are based on the concepts of reduced-rank modeling and sub-spaces weighting. We study both MMSE-SS (Minimum Mean Squared Error – Signal Sub-Space) and MSINR-EC (Maximum Signal-to-Interference-plus-Noise Ratio – Eigencanceler) techniques. The Weighted Sub-Space (WSS) algorithm is also applied, for the case where the number of co-channel interferers is not known. These algorithms can achieve improved performance relative to traditional methods, such as the Direct Matrix Inversion algorithm (DMI), both MSINR and MMSE criteria-based, and optimum Maximal Ratio Combining. Furthermore, the WSS approach demonstrates to be robust also in temporal processing applications as adaptive equalization.**

## 1. INTRODUCTION

It is well known that fading is a limiting factor for communication systems in wireless environments. Furthermore, in mobile radio, we have to deal with Co-Channel Interference (CCI). These limitations, if left unchecked, degrade bit error rate (BER) performance leading to a poor transmission quality.

Along years of study, much work has been done in order to increase capacity of wireless communication systems. Classical solutions include adaptive temporal equalization for frequency selective fading channels and Adaptive Antenna Array (AAA) diversity for flat fading channels [1]. Intersymbol and co-channel interferences can be reduced by those means. For the case of absence of CCI, the Maximal Ratio Combiner (MRC) is the optimal solution in the sense of maximizing the Signal-to-Noise Ratio (SNR) [2]. Conversely, if CCI is present, MRC is no longer optimum in maximizing the Signal-to-Interference-plus-Noise Ratio (SINR), and for this case we will concentrate on AAA processing by means of two equivalent criteria. The first is the maximization of the SINR (MSINR) and the second is the Minimum Mean-Squared Error (MMSE). The eigenstructure-based techniques used in this work may be divided into two different approaches, one of them based on Reduced-Rank (RR) modeling and the other based on sub-space weighting. RR-based algorithms are the MSINR-EC [3,4] and MMSE-SS [5] and they work with the noise and signal sub-spaces respectively. One algorithm based on sub-space weighting is the WSS. For interference reduction in the presence of CCI, improved BER performance can be achieved by these techniques over traditional ones, such as the DMI (both MSINR and MMSE criteria-based) and MRC algorithms.

We point out that one of the limitations of RR techniques is the need of a priori knowledge about the number of CCI sources present. In addition to this, the RR algorithms are not efficient in a frequency selective fading environment. This is because both MSINR-EC and MMSE-SS are rank-selection techniques. Such limitations disappear in sub-space weighting methods, as the WSS algorithm. WSS shows to be robust not only in the AAA spatial processing but also in temporal processing applications, as adaptive equalization.

This paper is organized as follows: section 2 briefly describes the signal model. Section 3 derives the expressions of the MMSE and

MSINR criteria for eigenstructure-based algorithms. Simulation results are presented next, in section 4. Section 5 introduces sub-space techniques for temporal equalization, with some preliminary results and in section 6 we draw some conclusions.

## 2. SIGNAL MODEL

Let us assume a mobile communication system that employs an  $N$  element linear Adaptive Antenna Array (AAA) at the base station. We represent the signal received at the array as  $\mathbf{x}[n] = [x_0[n], x_1[n], \dots, x_{N-1}[n]]^T$ , where the superscript T denotes transpose and each entry  $x_k[n]$ ,  $k = 0, 1, \dots, N-1$ , represents the signal received at the antenna element  $k$  after coherent demodulation, matched filtering and sampling at  $t = nT$ . The environment is assumed to be a flat Rayleigh fading channel in the presence of white Gaussian noise and co-channel interference. The signal vector is then represented as:

$$\begin{aligned} \mathbf{x}[n] &= \mathbf{u}[n] + \mathbf{i}[n] + \mathbf{v}[n] \\ &= A_s \mathbf{h}_s a[n] + \sum_{j=1}^L A_j \mathbf{h}_j z_j[n] + \mathbf{v}[n], \end{aligned} \quad (1)$$

where  $\mathbf{u}[n]$ ,  $\mathbf{i}[n]$  and  $\mathbf{v}[n]$  are the vector components relative to desired user, interference and noise signals, respectively.  $A_s$  and  $A_j$  are the desired user and interference signal amplitudes, respectively. The complex gain for the desired user signal is included in the vector  $\mathbf{h}_s$ , while for interferers signals the gains are represented by the vectors  $\mathbf{h}_j$ , where  $j = 1, \dots, L$ , and  $L$  is the number of CCI sources present. It is assumed that spacing between antenna elements is larger than the coherence distance in such a way that fading is independent between any two antennas. Therefore, these spatial channel vectors are complex-valued, zero-mean and multivariate Gaussian distributed with uncorrelated real and imaginary parts, and unity variance. Furthermore, they are mutually independent and assumed to be stationary over a time lag corresponding to a prescribed number of symbol periods. The sequence  $a[n]$  represents the uncorrelated data symbols for the user of interest, and it assumes the values  $\{+1, -1\}$  with equal probabilities (BPSK modulation is employed). The ambient noise, represented by the vector  $\mathbf{v}[n]$ , is complex-valued, stationary, zero-mean, white Gaussian distributed with variance  $\sigma_v^2$ .

We further consider an asynchronous sampling model for the CCI symbol. The term  $z_j[n]$  in eq. (1) represents the convolution between the mutually independent CCI data symbols,  $b_j[m]$ , and the overall equivalent impulse response,  $g(t)$ , for the transmitter, channel and receiver together. This term can be written as:

$$z_j[n] = \sum_{m=-\infty}^{+\infty} b_j[m] g(nT - mT - \tau_j), \quad (2)$$

where  $g(t)$  has a raised-cosine pulse shape with excess bandwidth  $\beta$ , and the uncorrelated CCI data symbols  $b_j[n]$  are mutually independent (also independent of  $a[n]$ ) and may assume values  $\{+1, -1\}$  with equal probability. The lack of synchronism among users is modeled by the random variable  $\tau_j$ , which represents the timing phase for the  $j$ -th interferer, and it is assumed to have a uniform

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distribution over the interval  $[0, T]$ , where  $T$  is the symbol period. This allows the CCI signals to be sampled in time instants different from those of the user of interest.

### 3. EIGENSTRUCTURE-BASED ALGORITHMS

The output of the AAA optimum combiner is given by  $y[n] = \mathbf{w}^H \mathbf{x}[n]$ , where the optimum weight vector  $\mathbf{w}$ , in the minimum mean square error sense, is expressed as  $\mathbf{w} = \mathbf{R}^{-1} \mathbf{r}$ ,  $\mathbf{R}$  representing the covariance matrix of the received signal vector and  $\mathbf{r}$  the cross-correlation vector between the desired signal and the received vector,  $\mathbf{r} = E\{a^*[n] \mathbf{x}[n] \mathbf{h}_s\} = A_s \mathbf{h}_s$ , which is estimated by:

$$\hat{\mathbf{r}} = \frac{1}{M} \sum_{n=1}^M a^*[n] \mathbf{x}[n], \quad (3)$$

where the asterisk represents complex conjugation. We have used  $M$  to denote the number of time samples of the training sequence, i.e., the window size used in the calculation.

Since the covariance matrix is Hermitian, it is possible to diagonalize the matrix  $\mathbf{R}$  by using a *Unitary Similarity Transformation*. The resulting diagonalized matrix is expressed as follows [6]:

$$\mathbf{\Lambda} = \mathbf{Q}^H \mathbf{R} \mathbf{Q}, \quad (4)$$

where the  $N$ -by- $N$  matrix  $\mathbf{Q}$  has as its columns the orthonormal set of eigenvectors  $\mathbf{q}_1, \mathbf{q}_2, \dots, \mathbf{q}_N$  of the matrix  $\mathbf{R}$ , and  $\mathbf{\Lambda}$  is a diagonal matrix which has the associated eigenvalues  $\lambda_1, \lambda_2, \dots, \lambda_N$  for the elements of its main diagonal.

Owing to the orthonormal nature of the eigenvectors, we find that  $\mathbf{Q}^H \mathbf{Q} = \mathbf{I}$ , where  $\mathbf{I}$  is the  $N \times N$  identity matrix. From this and eq. (4) we rewrite matrix  $\mathbf{R}$  as follows [6]:

$$\mathbf{R} = \mathbf{Q} \mathbf{\Lambda} \mathbf{Q}^H = \sum_{k=1}^N \lambda_k \mathbf{q}_k \mathbf{q}_k^H \quad (5)$$

It is important to recall that, as  $\mathbf{R}$  is hermitian so is  $\mathbf{R}^{-1}$ , and their eigenvectors are the same. For this reason,  $\mathbf{R}^{-1}$  may be obtained from eq. (5), by only changing  $\lambda_k$  by  $1/\lambda_k$ .

From eq. (1) and the assumption of mutual independence among the user signal, interference and noise, it is clear that we can rewrite  $\mathbf{R}$  as a sum of matrices,  $\mathbf{R} = \mathbf{R}_u + \mathbf{R}_i + \mathbf{R}_v$ , where  $\mathbf{R}_u$ ,  $\mathbf{R}_i$  and  $\mathbf{R}_v$  are respectively the covariance matrices of the components,  $\mathbf{u}[n]$ ,  $\mathbf{i}[n]$  and  $\mathbf{v}[n]$ , of the signal  $\mathbf{x}[n]$ .

From this consideration, and under the assumption that the desired user signal,  $\mathbf{u}[n]$ , and the noise,  $\mathbf{v}[n]$ , are white signals, it is straightforward to note that each  $\lambda_k$  can be decomposed in three components,  $\lambda_k^{(u)}$ ,  $\lambda_k^{(i)}$  and  $\lambda_k^{(v)}$ , associated to each of the signal components,  $\mathbf{u}[n]$ ,  $\mathbf{i}[n]$  and  $\mathbf{v}[n]$ , respectively. From this and eq. (4), we can rewrite the matrix of eigenvalues associated to the eigenvectors of the matrix in eq. (5), as:

$$\mathbf{\Lambda} = \mathbf{\Lambda}_u + \mathbf{\Lambda}_i + \mathbf{\Lambda}_v, \quad (6a)$$

or equivalently,

$$\lambda_k = \lambda_k^{(u)} + \lambda_k^{(i)} + \lambda_k^{(v)}, \quad k = 1, \dots, N. \quad (6b)$$

To make this statement, we consider the Karhunen-Loève expansion [6]. We will analyze three different spatial adaptive algorithms, which make use of the eigenstructure of the covariance matrix, and are supposed to be a better solution to the optimum combiner in different link situations. Each of these methods is based on a different covariance matrix, and therefore each matrix has different compositions on the space of representation.

The selection of the space of representation is based on an energy criterion, taking into account eq. (6b) and the fact we have organized the eigenvectors of the covariance matrix in the descending order of their associated eigenvalues, such that,  $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_p \geq \dots \geq \lambda_N$ , where  $p$  is the value that approximates the signal sub-space dimension. This means that nearly the whole signal sub-space can be defined by the  $p$  largest eigenvalues,  $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_p$ ,  $p < N$ . On the other hand, we assume the noise sub-space is composed by the remaining eigenvalues, i.e.,  $\lambda_{p+1} \geq \dots \geq \lambda_N$ .

In order to take conclusions regarding their efficiency and applicability we evaluate the BER performance for each of these algorithms and compare them to traditional methods. Two of the following algorithms (MMSE-SS and MSINR-EC) consider the *a priori* knowledge about the number of CCI sources present, while the third one (WSS) estimates this value by weighting the space relative to the interference covariance matrix.

#### - MMSE-SS:

The MMSE cost function is intended to minimize the mean-squared error (MSE) between the array output and the desired signal. Let us assume that for the MMSE cost function the signal sub-space is composed by the desired signal plus CCI components. We may express the MMSE covariance matrix as:

$$\begin{aligned} \hat{\mathbf{R}}_{\text{MMSE}} &= \frac{1}{M} \sum_{n=1}^M \mathbf{x}[n] \mathbf{x}^H[n] \\ &= \hat{\mathbf{R}}_u + \hat{\mathbf{R}}_i + \hat{\mathbf{R}}_v \end{aligned} \quad (7)$$

where,

$$\left. \begin{aligned} \hat{\mathbf{R}}_u &= \frac{1}{M} \sum_{n=1}^M (\mathbf{u}[n] \mathbf{u}^H[n]) \\ \hat{\mathbf{R}}_i &= \frac{1}{M} \sum_{n=1}^M (\mathbf{i}[n] \mathbf{i}^H[n]) \\ \hat{\mathbf{R}}_v &= \frac{1}{M} \sum_{n=1}^M (\mathbf{v}[n] \mathbf{v}^H[n]) \end{aligned} \right\} \quad (8)$$

However, we can estimate this covariance matrix by a reduced-rank approximation, which makes use only of the eigenvectors associated to the largest  $p$  eigenvalues [5]. Therefore, based on eq. (5), the RR approach to the MMSE estimate can be written simply as:

$$\hat{\mathbf{R}}_{\text{MMSE-SS}} = \sum_{k=1}^p \lambda_k \mathbf{q}_k \mathbf{q}_k^H \quad (9)$$

Here, it is supposed  $p = L+1$ . This procedure leads to the following sub-space approximation of the weight vector [5,6]:

$$\mathbf{w}_{\text{MMSE-SS}} = \hat{\mathbf{R}}_{\text{MMSE-SS}}^{-1} \hat{\mathbf{r}} = \left( \sum_{k=1}^p \frac{1}{\lambda_k} \mathbf{q}_k \mathbf{q}_k^H \right) \hat{\mathbf{r}}, \quad (10)$$

Note that each selected eigenvector is weighted by its respective eigenvalue, which may be expressed as in (6b).

#### - MSINR-EC:

The MSINR cost function attempts to maximize the Signal-to-Interference-plus-Noise Ratio (SINR) at the array output. In this case, we assume the signal sub-space is composed only by the CCI components, and we can write the covariance matrix estimate as follows:

$$\hat{\mathbf{R}}_{\text{MSINR}} = \frac{1}{M} \sum_{n=1}^M (\mathbf{x}[n] - \mathbf{h}_s a[n])(\mathbf{x}[n] - \mathbf{h}_s a[n])^H \quad (11)$$

The sub-space approach for estimation of the covariance matrix is based on the application of a reduced-rank method, known as the *Eigencanceller (EC)* [3,4], which performs interference cancellation by making use of the noise sub-space, which is said to be orthogonal to the interference sub-space. In estimating the Eigencanceller covariance matrix, we employ only the eigenvectors associated to the  $N-p$  smallest eigenvalues of the matrix  $\hat{\mathbf{R}}_{\text{MSINR}}$ . As a result we obtain the estimation of the inverse covariance matrix, as follows:

$$\hat{\mathbf{R}}_{\text{MSINR-EC}}^{-1} = \frac{1}{\sigma_v^2} \sum_{k=p+1}^N \mathbf{q}_k \mathbf{q}_k^H, \quad (12)$$

where  $p = L$ . This procedure leads to the following MSINR-EC weight vector [6]:

$$\mathbf{w}_{\text{MSINR-EC}} = \hat{\mathbf{R}}_{\text{MSINR-EC}}^{-1} \hat{\mathbf{r}} = \left( \frac{1}{\sigma_v^2} \sum_{k=p+1}^N \mathbf{q}_k \mathbf{q}_k^H \right) \hat{\mathbf{r}} \quad (13)$$

Once the computation of the covariance matrix of this algorithm does not take into account the desired user signal sub-space, we can state that  $\lambda_k^{(u)} = 0, \forall k$ . Therefore, eq. (6a) can be rewritten as:

$$\mathbf{\Lambda}_{\text{MSINR-EC}} = \mathbf{\Lambda}_i + \mathbf{\Lambda}_v \quad (14a)$$

Which leads to:

$$\lambda_k^{(\text{MSINR-EC})} = \lambda_k^{(i)} + \lambda_k^{(v)}, k = 1, \dots, N. \quad (14b)$$

#### - WSS Technique

It is still possible to deal with the problem of the AAA optimum combiner considering the more practical assumption that the number of interferers is not known in advance and must, therefore, be estimated by any means. To accomplish this task the WSS method works with the eigenvalues of the signal sub-space, in the absence of noise. These eigenvalues are weighted by a non-linear function, which is chosen to emphasize the eigenvalues associated to large interferers, leaving them alone, and force to zero those associated to weak interferers, if exist.

In modeling WSS covariance matrix, we assume the signal sub-space is composed only by the CCI components. Thus, we can express the covariance matrix from eq. (11), just by subtracting the noise power. This leads to:

$$\hat{\mathbf{R}} = \hat{\mathbf{R}}_{\text{MSINR}} - \sigma_v^2 \mathbf{I} \quad (15)$$

From this and eq. (6b), we see  $\lambda_k^{(v)} = 0$ . Therefore,  $\lambda_k = \lambda_k^{(i)}$ ,  $k = 1, \dots, N$ , are the estimated eigenvalues representing the interference in absence of noise. The non-linear function used to weight the sub-spaces is composed by the addition of two  $\text{TAN}^{-1}(\bullet)$  functions, each one properly scaled and shifted, in such a way to

provide the behavior discussed above. This function is used to estimate the interference plus noise matrix, which can be written as [6]:

$$\hat{\mathbf{R}}_{\text{wss}} = \sum_{k=1}^N f(\lambda_k) \lambda_k \mathbf{q}_k \mathbf{q}_k^H + \sigma_v^2 \mathbf{I}, \quad (16)$$

where  $f(\lambda_k)$  is the non-linear function we have just described.

At last, it is possible to make a quantitative analysis of the eigenvalues composition, by the following observation:

$$\lambda_k \equiv \begin{cases} \lambda_k^{(u)} + \lambda_k^{(i)}, & \text{for MMSE-SS only} \\ \lambda_k^{(i)}, & \text{for MSINR-EC only} \end{cases} \left. \vphantom{\lambda_k} \right\} \text{for } k \in [1, p] \quad (17)$$

$$\lambda_k \equiv \begin{cases} \lambda_k^{(v)}, & \text{for both MMSE-SS} \\ & \text{and MSINR-EC} \end{cases} \left. \vphantom{\lambda_k} \right\} \text{for } k \in [p+1, N]$$

## 4. SIMULATION RESULTS

In this section we present performance results relative to the BER of the eigenstructure-based algorithms, for the signal model discussed before. We make some conclusions by varying critical parameters as the number of CCI sources, the signal-to-interference ratio (SIR) per interferer and number of antenna elements. Simulation results were based on a slot format composed of 162 symbols, 14 of which are for training in the beginning of the slot [7]. The remaining samples were used to evaluate the BER. All The results are averages of 480.000 Monte Carlo runs. In this work we have considered that all interferers signals have the same power.

### 4.1. Reduced Rank Algorithms for MMSE and MSINR criteria.

For these simulation results we employ  $N = 4$  antenna elements. The simulation results were obtained for a SNR range of  $-6$  to  $+6$  dB per antenna element. Two cases were studied, one for  $L = 1$  and the other for  $L = 3$  CCI sources. Two scenarios are examined: the first one represents a less severe influence of the interferers (SIR = 7dB), while in the other CCI power is stronger (SIR = 0dB).

#### - SIR = 7 dB:

In this situation, for  $L = 1$ , figure 1 shows that the MMSE-SS outperforms its classical version (MMSE-DMI). However, comparing to figure 2, we see that the MSINR-EC does not have any gain in performance relative to the MSINR-DMI, thereby showing the strong sensitivity of the MSINR-EC to the degrees of freedom of the array, i.e., MSINR-EC severely degrades its performance when we decrease the difference between the number of antenna elements ( $N$ ) and the number of signals involved ( $L + 1$ ).

In the cases of figures 1 and 2, the SIR is relatively high, such that the BER of the MRC remains still at a low level. However, as the value of the SNR per antenna increases, we note the performance of MRC is not so much improved as those of the other algorithms.

For  $L = 3$ , we observe, in figure 2, that all the algorithms, except the MSINR-EC, present nearly the same performance, compared to the case in figure 1. Also clear in figure 2 is the fact that MMSE-SS and MMSE-DMI performances are exactly the same, as expected, since for this case ( $L = 3$ ) there is no rank reduction ( $p = N$ ). Here the contribution of the interferer power,  $\lambda_4^{(i)}$ , to the eigenvalue,  $\lambda_4^{(\text{MSINR-EC})}$ , associated to noise sub-space is larger than in the case  $L = 1$ .

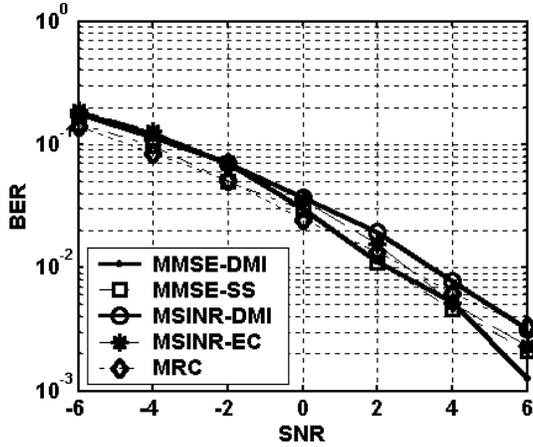


Figure 1: BER performance of the MMSE and MSINR criteria based algorithms, for  $N = 4$  antenna elements,  $L = 1$  CCI source and  $SIR = 7\text{dB}$ .

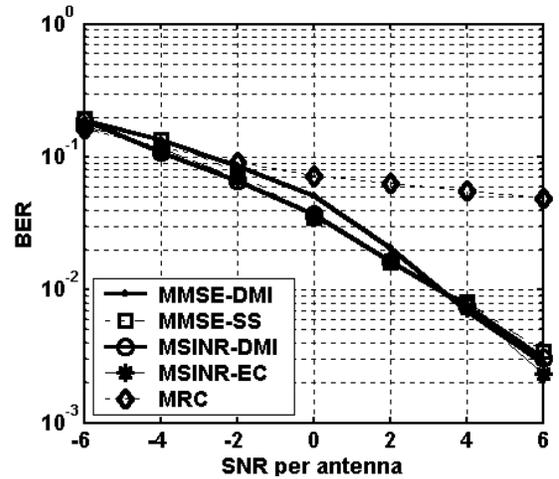


Figure 3: BER performance of the MMSE and MSINR criteria based algorithms, for  $N = 4$  antenna elements,  $L = 1$  CCI source and  $SIR = 0\text{dB}$ .

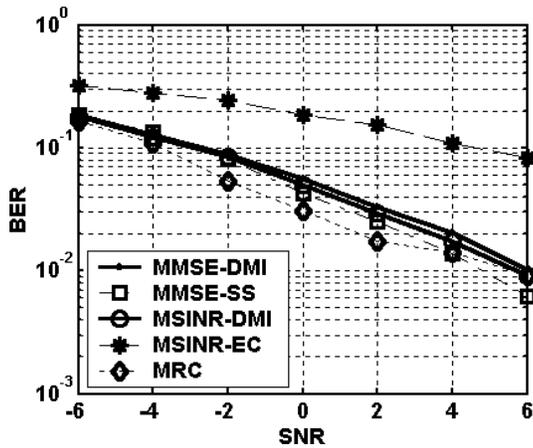


Figure 2: BER performance of the MMSE and MSINR criteria based algorithms, for  $N = 4$  antenna elements,  $L = 3$  CCI sources and  $SIR = 7\text{dB}$ .

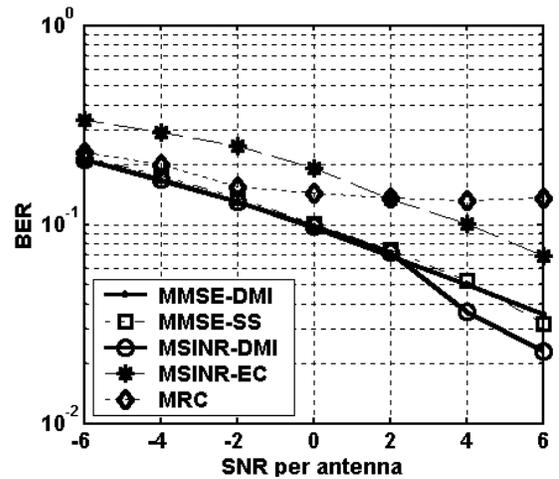


Figure 4: BER performance of the MMSE and MSINR criteria based algorithms, for  $N = 4$  antenna elements,  $L = 3$  CCI sources and  $SIR = 0\text{dB}$ .

-  $SIR = 0\text{dB}$ :

Now we compare the performance of the same algorithms for a scenario presenting higher co-channel interference. Comparisons between figures 1 and 3 show MRC is severely affected by a stronger CCI, as expected, while MSINR-EC and MMSE-SS performances remain almost unchanged.

For the case  $L=3$ ,  $SIR=0\text{dB}$  the same conclusion can be taken regarding the MRC, MMSE-SS and MSINR-EC performances, as it is clear from comparison between figures 2 and 4. This comparison also illustrates the property of the Eigencanceler to be robust in a situation of increased interference power. However, MMSE-SS performance was superior to MSINR-EC even after some degradation due to enhanced CCI power.

#### 4.2. Eigenstructure-Based Algorithms Performance with Increasing Number of CCI Sources

The results presented in this section are concerned to the robustness of the eigenstructure-based techniques relative to the quantity of interferers present. Here, we have used an 8-antenna elements array, to allow a minimum degree of freedom, even when up to 6 CCI components are present.

As we can see in figure 5, MSINR-EC is the most sensitive to the increasing number of interference sources. On the other hand, WSS presents the lowest degradation in performance, which show this technique is suitable to be used in a scenario with several weak interferers.

We also verified WSS performance in a higher interference scenario ( $SIR = 0\text{dB}$ ) in comparison to the RR algorithms. Figure 6 shows that in this case, WSS still works better than RR methods. We also observe that MSINR-EC and MMSE-SS exhibit nearly the same performance degradation. However, when the number of interferers is larger than 4 (i.e., less than 3 degrees of freedom), the degradation of MSINR-EC becomes stronger. This result reinforces the sensitivity of MSINR-EC relative to number of CCI sources present (see figures 1 and 2).

In the case of a weaker interference ( $SIR=7\text{dB}$ ) we can state the superiority of WSS and MMSE-SS compared to MSINR-EC is due to the eigenstructure of covariance matrix that makes use of the signal sub-space. When we deal with a high number of interferers, compared to  $N$ , their energy spread out towards the  $N$ -dimension of

the covariance matrix and the noise sub-space approach is no more a good alternative.

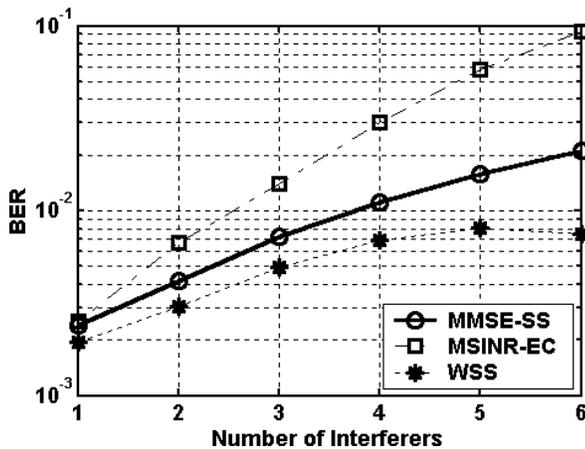


Figure 5: BER performance of the MMSE-SS, MSINR-EC and WSS algorithms, for  $N = 8$  antenna elements,  $SNR = 0dB$  and  $SIR = 7dB$ .

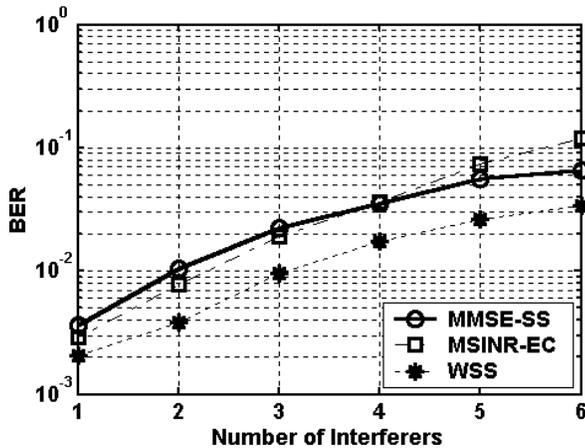


Figure 6: BER performance of the MMSE-SS, MSINR-EC and WSS algorithms, for  $N = 8$  antenna elements,  $SNR = 0dB$  and  $SIR = 0dB$ .

If the power levels of CCI and noise are close, as occurs in the case of figure 6, the powers of interferers and noise are found mixed along the  $N$ -dimension space. Thus, the correct eigenvector selection in both signal and noise sub-space approaches are no more guaranteed, as there is no more separation criterion between signal and noise sub-spaces. In other words, the contributions of both signal and noise power to the eigenvalues are so close that it is no longer possible to distinguish if the energy represented by a given eigenvalue is concerned, in its major part, with the interferer signal or with the noise. This fact is more detailed in the next section.

### 4.3. Eigenvalue analysis for MMSE-SS, MSINR-DMI and WSS algorithms.

In this section we are concerned to the analysis of eigenvalue distribution along the  $N$ -dimension of the covariance matrix. This kind of observation allows us to relate the effect of both SNR and SIR levels on the composition of these eigenvalues for each of reduced rank and sub-space algorithms.

The eigenvalue distribution shown in figure 7 is obtained for  $SIR=0dB$  and  $SNR=0dB$ , in the presence of 1 CCI component. Using

an  $N=4$  element antenna array, for MSINR-EC most of the interference power is concentrated in the larger eigenvalue, indicating the use of a reduced rank model for the covariance matrix. The 3 smallest eigenvalues represent the noise power (i.e. their magnitudes equals noise variance).

For WSS, there is not noise contribution and the eigenvalues represent only the interference power. That is why this power is not as concentrated as in the MSINR-EC case. Since MMSE-SS covariance matrix contains also the desired user signal contribution, we observe there is an offset in the eigenvalue distribution curve.

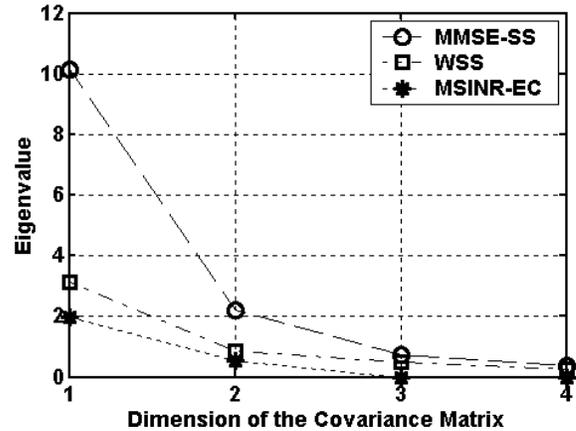


Figure 7: Eigenvalue distribution for  $SIR = 0dB$ ,  $SNR = 0dB$ ,  $L = 1$  CCI source, with  $N = 4$  antenna elements.

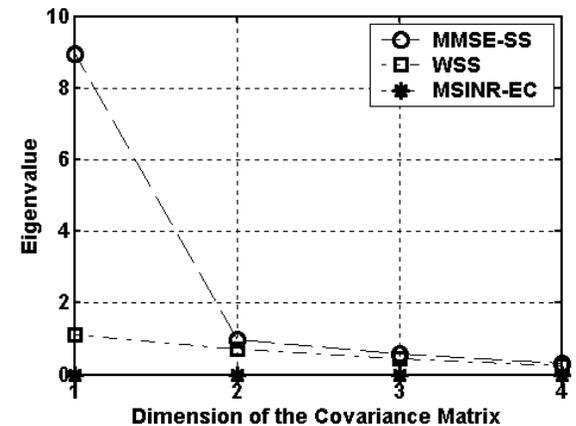


Figure 8: Eigenvalue distribution for  $SIR = 15dB$ ,  $SNR = 0dB$ ,  $L = 1$  CCI source, with  $N = 4$  antenna elements.

A very illustrative situation is shown in figure 8. In this simulation we have used a  $SIR=15dB$  and  $SNR=0dB$  scenario. Thus, in this case, the interference is very weak, while the noise is kept strong. Therefore, since WSS covariance matrix includes only the interferer sub-space, its eigenvalues are close to zero, as expected. It is well known MSINR-EC covariance matrix includes interference plus noise energy. This fact is clear from the figure 8. Once the CCI present is very small the eigenvalues are all close to the noise variance. At last, we observe the main difference of MMSE-SS approach relative to the MSINR-EC is the inclusion of the desired user signal power in the magnitude of the first eigenvalue.

## 5. WSS TECHNIQUE FOR TEMPORAL EQUALIZATION

In this section we present a preliminary result of the WSS algorithm applied to the adaptive linear equalization problem. To illustrate frequency selective fading, an ISI channel model was chosen. Figure 9 shows its discrete impulse response, where  $T$  is the symbol period.

The channel model is composed by two ISI components that span one symbol period on each side of the desired signal component. We use a 9 tap linear adaptive equalizer. The SNR is set to 25 dB and 8-PSK modulation is employed. After the tap weight acquisition, we obtain the output constellation for  $6 \times 162 = 972$  transmitted symbols. The global impulse response (channel + equalizer) is also observed.

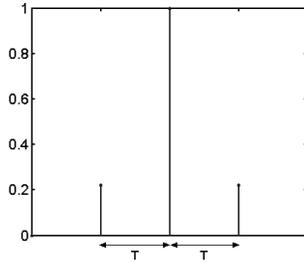


Figure 9: ISI channel impulse response

According to figure 10 we can state MMSE-WSS works better than MSINR-DMI suggesting the use of sub-space methods also for temporal processing. We can see this in another way if we observe figure 11. The residual ISI after equalization is larger if no sub-space weighting is used

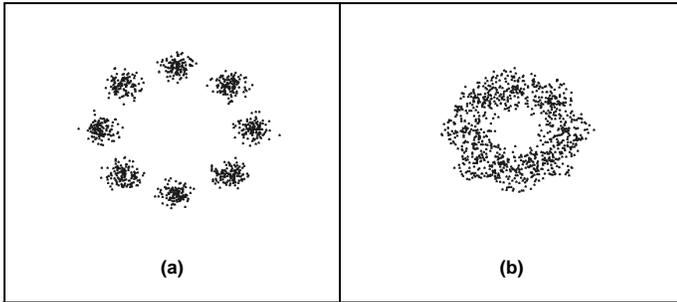


Figure 10: Output Constellations for both (a) MMSE-WSS and (b) MSINR-DMI

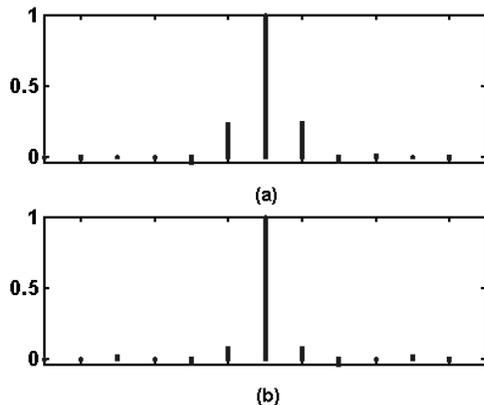


Figure 11: Global impulse response for (a) MSINR-DMI and (b) MMSE-WSS.

## 6. SUMMARY AND CONCLUSIONS

It has been shown that algorithms based on sub-space and reduced rank techniques can improve BER performance of an antenna diversity array. For flat Rayleigh fading in the presence of co-channel interferers, these techniques, especially MMSE-SS and WSS, outperformed conventional methods. MSINR-EC was shown to be very sensitive to the number of interferers, but robust to the CCI power enhancement. MMSE-SS employs the desired user signal sub-space, and therefore presented improved performance even in the presence of several CCI components. However, in a more severe scenario of interference power, it showed a considerable degradation, although its BER performance still remained better than that of MSINR-EC. On the other hand, WSS algorithm performed the most stable. In this sub-space method, eigenvectors (representing interference sub-space) are never completely discarded, but properly weighted. All eigenstructure-based algorithms studied in this work were superior in performance compared to MRC for a stronger CCI scenario, especially for high SNR. These conclusions suggest their employment in future mobile communication systems where high capacity is desired under high levels of co-channel interference.

The work focused in the analysis of the eigenstructure of the covariance matrices employed by each method, and some considerations were made about the eigenvalues distribution obtained by the respective matrices. This distribution reflects the concentration of the power of the signal components over the sub-spaces. We can say that each eigenvector points out the direction where is concentrated the power related to the associated eigenvalues.

Finally, it was presented an application of the algorithms studied in the problem of temporal adaptive linear equalization, and preliminary results have confirmed the superiority of the WSS algorithm even in this case. Future results expected involve the application of other eigenstructure-based techniques to the adaptive equalization problem and possible extension to space-time processing.

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