A Fast Algorithm for Joint Downlink Beamforming and Power Control in UMTS Systems

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Abstract— **Downlink Beamforming and Power Control are techniques used to improve the capacity or the quality of service in wireless systems. In this paper we propose a fast iterative algorithm for joint downlink beamforming and power control. The performance of this algorithm is compared with decoupled strategies through simulations. Results show that the fast algorithm outperforms the decoupled strategies and has a low computational complexity.**

I. Introduction

THE 3G mobile systems based on CDMA (Code Division Multiple Access) such as the UMTS (Universal Mobile Telecommunications System) are very sensitive to the quality of power control. Indeed, all mobiles transmit and receive at the same time interfering each other. Thus, obtaining an acceptable SNR (Signal to Noise Ratio) for each one of them requires a fine tuning of their allocated power.

These systems also envisage the use of adaptive antennas on the base station to improve either their capacity and quality of service. Therefore, the crucial problem of the power control becomes more complex since it must be treated jointly with that related to the receive or transmit beamforming for each user.

In uplink, the receive beamforming is automatically adapted to the received powers. However, even in uplink, the power control instructions send to the mobiles must be jointly set with the beamforming solution. In downlink, the need for jointly determining the powers and the beamforming for all users seems more intuitive. In this paper, we initially propose to study the downlink problem. However, as it will be seen in the sequel, the resolution of this problem is related to the resolution of the uplink one.

The rest of the paper is organized as follows. Section II introduces some background on downlink beamforming. The problem of joint downlink beamforming and power control is stated in Section III and developed in Section IV. Section V presents an algorithm that solves this problem and some discussions are stated in Section VI. In Section VII we propose a novel and faster algorithm. Section VIII presents some simulations that illustrate and compare the performance of the algorithms and, finally, some conclusions are drawn in Section IX.

II. Downlink

Let us consider the transmission towards each mobile by means of a purely spatial antenna array. The useful energy per bit received by each mobile i is expressed as:

$$E_{b_i} = SF_i p_i \mathbf{w}_i^H \mathbf{R}_i \mathbf{w}_i \tag{1}$$

where SF_i is the ith user spreading factor, p_i is the ith user transmit power and $\mathbf{w}_i$ is the normalized transmit antenna weight vector for the ith user $\left(\mathbf{w}_i^H \mathbf{w}_i = 1\right)$. $\mathbf{R}_i$ is the Downlink Channel Covariance Matrix (DCCM) of the ith user, which can be expressed in function of the downlink channel as:

$$\mathbf{R}_i = \int_0^\infty \mathbf{h}_i(t)\mathbf{h}_i^H(t)dt \tag{2}$$

where $\mathbf{h}_i(t)$ is the overall (transmit and receive filters included) impulse response of the multi-sensor channel.

The interference power received by mobile i due to transmission towards the other mobiles is given by

$$P\mathrm{I}_i = \sum_{k\neq i} p_k \mathbf{w}_k^H \mathbf{R}_i \mathbf{w}_k \tag{3}$$

Finally, the ratios $\frac{E_b}{N_0}$ before decoding are given by

$$\left(\frac{E_b}{N_0}\right)_i = \frac{SF_i p_i \mathbf{w}_i^H \mathbf{R}_i \mathbf{w}_i}{\sum\limits_{k\neq i} p_k \mathbf{w}_k^H \mathbf{R}_i \mathbf{w}_k + \sigma^2} \quad \forall i \tag{4}$$

where $\sigma 2$ accounts for thermal noise and extra-cellular interference at each mobile receiver and is assumed to be constant and identical for all mobiles.

This expression attests that the ratios $\frac{E_b}{N_0}$ at each mobile depend on all of transmit powers and beamforming weights assigned for each user. The optimization of these ratios must be done for all mobiles and implies on the jointly determination of the transmit power and the transmit weight vector for each one of them.

As this optimization is done at the base station, it supposes the knowledge of the DCCM for all users. These matrices can only be truly estimated at the mobile, so feedback from mobile is necessary [1], [2], [3]. However, in TDD (Time Division Duplex) systems, uplink and downlink share the same frequency, so the uplink and downlink channels are the same. Thus, the DCCM can be directly obtained estimating the Uplink Channel Covariance Matrix (UCCM). This approach can be extended to FDD (Frequency Division Duplex), where the DCCM can be obtained by frequency transposition of the UCCM [4], [5], [3]. In this article, the DCCM are assumed to be known.

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III. Problem Statement

When the number of mobiles is small, there are several solutions that reach the $\frac{E_b}{N_0}$ required by the QoS (Quality of Service) assigned for each user. Let us suppose that the beamforming weights are precalculated, e.g. using a matched spatial filter [6]. It can be possible to determine a set of transmit powers in order to reach the $\frac{E_b}{N_0}$ targets. The difference between such a solution and the optimal one is that the total radiated power is greater, increasing the extra-cellular interference and decreasing system capacity. It is thus convenient to add a supplementary statement:

$$\sum_i p_i = \min \tag{5}$$

On the other hand, when the number of mobiles is large, there may not be a solution that reachs the targets. Therefore, it is interesting to search a solution that maximizes and equals the ratios $\frac{E_b}{N_0}$:

$$\left(\frac{E_b}{N_0}\right)_i = cC_i \tag{6}$$

where C_i is the target value necessary to ensure the QoS for user i and c is a proportionality coefficient lower than 1 common for all users.

Moreover, the solution that reaches the target values may lead to excessive transmit powers, exceeding the maximum power available at the base station. Thus, it is advisable to search a solution in the sense of (6) and respects the maximum power available:

$$\sum_i p_i \leq P_{\max} \tag{7}$$

Finally, the problem can be stated as:

$$\left\{\begin{array}{ll} \sum\limits_i p_i \mathbf{w}_i^H \mathbf{w}_i = \min \leq P_{\max} & \\ \forall i \quad \frac{SF_i p_i \mathbf{w}_i^H \mathbf{R}_i \mathbf{w}_i}{\sum\limits_{k\neq i} p_k \mathbf{w}_k^H \mathbf{R}_i \mathbf{w}_k + \sigma^2} = cC_i & \end{array}\right. \tag{8}$$

IV. Development

This optimization under constraints can be solved using the Lagrange multipliers. For clarity reasons, the constraints can be rewritten as:

$$\forall i \quad \frac{p_i \mathbf{w}_i^H \frac{\mathbf{R}_i}{\gamma_i} \mathbf{w}_i}{\sum\limits_{k\neq i} p_k \mathbf{w}_k^H \mathbf{R}_i \mathbf{w}_k + \sigma^2} = c \tag{9}$$

where $\gamma_i = \frac{C_i}{SF_i}$ is the chip level target for the ith user. This constraints can also be written as:

$$p_i \mathbf{w}_i^H \frac{\mathbf{R}_i}{\gamma_i} \mathbf{w}_i - c\left(\sum_{k\neq i} p_k \mathbf{w}_k^H \mathbf{R}_i \mathbf{w}_k + \sigma^2\right) = 0 \tag{10}$$

Replacing the original form of the constraints in equation (8) with the one of equation (10) and using the Lagrange multipliers, lead us to the Lagrange cost function:

$$J = \sum_i p_i \mathbf{w}_i^H \mathbf{w}_i - \sum_i \chi_i \left(p_i \mathbf{w}_i^H \frac{\mathbf{R}_i}{\gamma_i} \mathbf{w}_i - c\left(\sum_{k\neq i} p_k \mathbf{w}_k^H \mathbf{R}_i \mathbf{w}_k + \sigma^2\right)\right) \tag{11}$$

where χ_i are the Lagrange multipliers.

A. Optimal transmit weights

The Lagrange cost function is a quadratic function of the transmit weights $\mathbf{w}_i$. Thus, its optimum can be found zeroing the gradient with respect to all $\mathbf{w}_i$'s.

Considering the user u, the partial derivative of the Lagrange cost function with respect to $\mathbf{w}_u^H$ is:

$$\frac{\partial J}{\partial \mathbf{w}_u^H} = 2p_u \mathbf{w}_u - \chi_u 2p_u \frac{\mathbf{R}_u}{\gamma_u} \mathbf{w}_u + c\sum_{i\neq u} \chi_i 2p_u \mathbf{R}_i \mathbf{w}_u = 0 \tag{12}$$

Remarking that the above equation does not depend on p_u, the optimum condition is:

$$\forall u \quad \mathbf{w}_u - \chi_u \frac{\mathbf{R}_u}{\gamma_u} \mathbf{w}_u + c\sum_{i\neq u} \chi_i \mathbf{R}_i \mathbf{w}_u = 0 \tag{13}$$

To emphasize the physical meaning of the Lagrange multipliers in the sequel, it is useful to pose $\alpha_i = c\sigma 2\chi_i$. It follows that:

$$\forall u \quad \frac{\mathbf{R}_u}{\gamma_u} \mathbf{w}_u - \frac{c}{\alpha_u}\left(\sum_{i\neq u} \alpha_i \mathbf{R}_i + \sigma^2 \mathbf{I}\right) \mathbf{w}_u = 0 \tag{14}$$

where $\mathbf{I}$ is the identity matrix of order M and M is the number of antenna elements in the array.

The above equation is essential since it shows that $\mathbf{w}_u$ is eigenvector of the generalized eigen-decomposition of $\left(\frac{\mathbf{R}_u}{\gamma_u}, \mathbf{R}_{u_T}\right)$, where $\mathbf{R}_{u_T} = \sum\limits_{i\neq u} \alpha_i \mathbf{R}_i + \sigma^2 \mathbf{I}$.

Equation (14) seems to show that the determination of the transmit weight vectors does not depend on the transmit powers. However, as it will be shown in the sequel, the transmit weight vectors depend on the Lagrange multipliers which, by their turn, depend on the transmit powers.

B. Uplink-Downlink Equivalence

Left-multiplying equation (14) for user i by $\mathbf{w}_i^H$, it follows that:

$$\frac{\alpha_i \mathbf{w}_i^H \mathbf{R}_i \mathbf{w}_i}{\mathbf{w}_i^H \left(\sum\limits_{k\neq i} \alpha_k \mathbf{R}_k\right) \mathbf{w}_i + \sigma^2} = c\gamma_i \tag{15}$$

The above equation is the uplink $\frac{E_b}{N_0}$ calculated with the DCCM. The α_i's are then equivalent to the mobile transmit powers. And the $\mathbf{w}_i$'s are the receive weight vectors used in the base station. Equations (9) and (15) highlight the equivalence between the uplink and the downlink problems:

$$\frac{\alpha_i \mathbf{w}_i^H \frac{\mathbf{R}_i}{\gamma_i} \mathbf{w}_i}{\mathbf{w}_i^H \left(\sum\limits_{k\neq i} \alpha_k \mathbf{R}_k\right) \mathbf{w}_i + \sigma^2} = \frac{p_i \mathbf{w}_i^H \frac{\mathbf{R}_i}{\gamma_i} \mathbf{w}_i}{\sum\limits_{k\neq i} p_k \mathbf{w}_k^H \mathbf{R}_i \mathbf{w}_k + \sigma^2} \tag{16}$$

The above equation can be rewritten as:

$$\alpha_i \sum_{k\neq i} p_k \mathbf{w}_k^H \mathbf{R}_i \mathbf{w}_k + \sigma^2 \alpha_i = p_i \sum_{k\neq i} \alpha_k \mathbf{w}_i^H \mathbf{R}_k \mathbf{w}_i + \sigma^2 p_i \tag{17}$$

Summing the equations of all users, il follows that:

$$\sum_i \alpha_i \sum_{k\neq i} p_k \mathbf{w}_k^H \mathbf{R}_i \mathbf{w}_k + \sigma^2 \sum_i \alpha_i = \sum_i p_i \sum_{k\neq i} \alpha_k \mathbf{w}_i^H \mathbf{R}_k \mathbf{w}_i + \sigma^2 \sum_i p_i \quad (18)$$

Adding the term $\alpha_i p_i \mathbf{w}_u^H \mathbf{R}_u \mathbf{w}_u$ to both sides, it can be easily shown the relation between the uplink and downlink powers:

$$\sum_i p_i = \sum_i \alpha_i \quad (19)$$

Equation (15) also shows that in order to maximize the $\frac{E_b}{N_0}$ of each mobile or, equivalently, to reach the targets with a minimum total transmit power, the transmit weight vectors $\mathbf{w}_i$ must be the eigenvectors associated with the maximum eigenvalues of equation (14).

C. Uplink Power Control

It is worth to notice that the determination of the transmit powers is based on the knowledge of the uplink powers α_i. At given $\mathbf{w}_i$'s, these values can be find respecting the constraints of equation (15):

$$\forall i \quad \alpha_i \left(\mathbf{w}_i^H \frac{\mathbf{R}_i}{\gamma_i} \mathbf{w}_i \right) - c \sum_{k\neq i} \alpha_k \left(\mathbf{w}_i^H \mathbf{R}_k \mathbf{w}_i \right) = c\sigma^2 \quad (20)$$

Defining the matrix $\mathbf{D}$, whose diagonal elements are:

$$d_{i,i} = \mathbf{w}_i^H \frac{\mathbf{R}_i}{\gamma_i} \mathbf{w}_i\,, \quad (21)$$

the matrix $\mathbf{M}$, whose generic elements i, k are:

$$m_{i,k} = \begin{cases} \mathbf{w}_i^H \frac{\mathbf{R}_k}{\gamma_i} \mathbf{w}_i & i \neq k \\ 0 & i = k \end{cases} \quad (22)$$

and the vector :

$$\boldsymbol{\alpha} = \begin{bmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_U \end{bmatrix}^T, \quad (23)$$

the linear system described by equation (20) can be rewritten as:

$$\mathbf{D}\boldsymbol{\alpha} - c\mathbf{M}\boldsymbol{\alpha} = c\sigma^2 \mathbf{1} \quad (24)$$

where $\mathbf{1}$ is a column vector whose elements are ones and U is the number of mobile users.

D. Downlink Power Control

As for the case of uplink power control, the downlink powers p_i can be find respecting the constraints of equation (9):

$$\forall i \quad p_i \left(\mathbf{w}_i^H \frac{\mathbf{R}_i}{\gamma_i} \mathbf{w}_i \right) - c \sum_{k\neq i} p_k \left(\mathbf{w}_k^H \mathbf{R}_i \mathbf{w}_k \right) = c\sigma^2 \quad (25)$$

Using the same definition for matrices $\mathbf{D}$ and $\mathbf{M}$, it follows that:

$$\mathbf{D}\mathbf{p} - c\mathbf{M}^T \mathbf{p} = c\sigma^2 \mathbf{1} \quad (26)$$

where $\mathbf{p} = \begin{bmatrix} p_1 & p_2 & \cdots & p_U \end{bmatrix}^T$.

E. Existence of a solution

To ensure that the solution is valid, both uplink powers (24) and dowlink powers (26) must be positive.

We will show that c must be lower than the maximum eigenvalue of $\left(\mathbf{D}, \mathbf{M}^T\right)$ in order that equations (24) and (26) provide a valid solution. Considering the solution provided by equation (26) :

$$\mathbf{p} = c\sigma^2 \left(\mathbf{D} - c\mathbf{M}^T \right)^{-1} \mathbf{1}\,, \quad (27)$$

a sufficient condition for the vector $\mathbf{p}$ to have positive elements is that the matrix $\left(\mathbf{D} - c\mathbf{M}^T\right)^{-1}$ is formed, by its turn, by positive elements. This matrix can be written as:

$$\mathbf{D}^{-1} \left(\mathbf{I} - c\mathbf{D}^{-1}\mathbf{M}^T \right)^{-1} \quad (28)$$

Provided that the matrices $\mathbf{D}$ and $\mathbf{M}$ have positive elements, it suffices that all the elements of the matrix $\left(\mathbf{I} - c\mathbf{D}^{-1}\mathbf{M}^T\right)^{-1}$ are positive for the above condition to be respected. If the maximum eigenvalue of $c\mathbf{D}^{-1}\mathbf{M}^T$ is lower than 1, this matrix can be expanded as:

$$\left(\mathbf{I} - c\mathbf{D}^{-1}\mathbf{M}^T \right)^{-1} = \mathbf{I} + \sum_{m=1}^{\infty} \left(c\mathbf{D}^{-1}\mathbf{M}^T \right)^m \quad (29)$$

The above expression shows on the one hand that the matrix $\left(\mathbf{I} - c\mathbf{D}^{-1}\mathbf{M}^T\right)$ is invertible and on the other hand that all the elements of its inverse are positive because those of $\mathbf{D}$ and $\mathbf{M}$ are positive.

Ultimately, it suffices to choose c lower than the maximum eigenvalue of $\mathbf{D}^{-1}\mathbf{M}^T$ or, equivalently, lower than the maximum eigenvalue $c_{\max}$ of $\left(\mathbf{D}, \mathbf{M}^T\right)$. Provided the similarity between equations (24) and (26), the value adopted to obtain the downlink powers also guarantees positive values for the uplink powers.

V. DBPC Algorithm

Provided that it is necessary to know all the α_i's in order to obtain the downlink weights $\mathbf{w}_i$'s and, in a reciprocal way, it is necessary to know the $\mathbf{w}_i$'s to compute the uplink powers, we propose an iterative algorithm which finds the optimum solution:

1. Initialization: $\boldsymbol{\alpha} = \mathbf{0}$
2. Update the downlink weights: $\mathbf{w}_i$ is the eigenvector associated with the maximum eigenvalue of $\left(\frac{\mathbf{R}_i}{\gamma_i}, \sum_{k\neq i} \alpha_k \mathbf{R}_k + \sigma 2\mathbf{I} \right)$
3. Compute the matrices $\mathbf{D}$ and $\mathbf{M}$, as given by equations (21) and (22)
4. Compute $c_{\max}$, maximum eigenvalue of $(\mathbf{D}, \mathbf{M})$
   (a) If $c_{\max} \geq 1$, then $c = 1$
   (b) If $c_{\max} < 1$, then $c = c_{\max} - \varepsilon$
5. Update the uplink power vector: $\boldsymbol{\alpha} = (\mathbf{D} - c\mathbf{M})^{-1} c\mathbf{1}$
6. Stop test on the variation of the total transmit power. Back to step 2.

After convergence, the downlink powers are computed using equation (27). This algorithm is called DBPC (Downlink Beamforming and Power Control) algorithm in the following. The algorithm convergence is shown in Appendix A.

VI. DISCUSSIONS

The algorithm previously obtained is roughly similar to that proposed in [7] and used in [8] to carry out the optimal allocation of user terminals to base stations. It converges very quickly but the computational complexity associated with each iteration is very high, being proportional to the cube of the number of users. Besides, to our best knowledge, it is not possible to easily modify this algorithm in order to respect the maximum power.

On the other hand, in FDD systems, where the DCCM are obtained from the UCCM by frequency transposition [5], it is imperative to estimate the matrices of covariance over a great number of frames in order to obtain reliable information on the physical paths, since the channel phases are different from uplink to downlink.

At each new frame, the uplink channels are estimated and used to update the estimated DCCMs:

$$\mathbf{R}_i(n) = \lambda \mathbf{R}_i(n-1) + \sum_{l=0}^{L-1} \mathbf{T}\mathbf{h}_i^{UL}(l)\mathbf{h}_i^{UL^H}(l)\mathbf{T}^H \qquad (30)$$

where λ is the forgetting factor, $\mathbf{T}$ is the frequency transposition matrix, L is the channel impulse response length and $\mathbf{h}_i^{UL}(l)$ is the overall uplink impulse response of the multi-sensor channel at instant l.

After updating the estimated DCCM, these matrices are only slightly disturbed and the optimal downlink weights differ little from the previous ones. However, to determine the new downlink weights, it is necessary to achieve an iteration of the DBPC algorithm and this iteration has a very high computational complexity.

Thus, a fast algorithm, which has a slower convergence but a much lower computational complexity, is proposed in the following.

VII. FAST-DBPC ALGORITHM

The computational complexity of the previous algorithm is mainly due to the resolution of the linear system (26), which is necessary in order to determine the downlink powers.

Observing that the maximum eigenvalue λ_u of the eigendecomposition of $\left(\frac{\mathbf{R}_u}{\gamma_u}, \mathbf{R}_{u_T}\right)$ is proportional to $\frac{c}{\alpha_u}$, we propose to take $\frac{c}{\lambda_u}$ as the new value of α_u. Thus, the α_i's can be directly and quickly obtained from equation (14).

Hence, the computational complexity to update the downlink power vector $\boldsymbol{\alpha}$ becomes negligible. From now on, the eigendecompositions necessary to compute the downlink weights are responsible for the computational complexity. They can however be computed using the power method in order to achieve a lower computational complexity.

The proposed Fast-DBPC algorithm steps are as follows:

1. Initialization: $\boldsymbol{\alpha} = \mathbf{0}$ and $\mathbf{w}_i = \mathbf{0}$
2. Compute $\mathbf{R}_T = \sum_k \alpha_k \mathbf{R}_k$ and its inverse $\mathbf{R}_T^{-1}$
3. Update the downlink weights using the power method: $\mathbf{v}_i = \mathbf{R}_T^{-1}\mathbf{R}_i\mathbf{w}_i$; $\lambda_{i_T} = \sqrt{\mathbf{v}_i^H \mathbf{v}_i}$; $\mathbf{w}_i = \frac{1}{\lambda_{i_T}}\mathbf{v}_i$
4. Compute the eigenvalues: $\lambda_i = \frac{\lambda_{i_T}}{(1-\alpha_i\lambda_{i_T})\gamma_i}$
5. Update the uplink powers: $\alpha_i = \frac{c}{\lambda_i}$ with $c = \max\left(1, \frac{\sum \frac{1}{\lambda_i}}{P_{\max}}\right)$ in order to respect the maximum power constraint
6. Stop test on the variation of the total transmit power or on the variation of the target c if the maximum power is reached. Back to step 2.

TABLE I
UMTS SERVICES

Service	SF	C [dB]
Voice	64	7
Data 64kbps	16	6.5
Data 144kbps	8	6
Data 384kbp	4	5

This algorithm has computational complexity of $O(N3) + 3UN2$ per iteration, where N is the number of antenna elements and U is the number of mobile users. This computational complexity, which is proportional to the number of users, is to be compared with that of the DBPC algorithm which is of $UO(N3) + U^2N2 + O(U3)$. One can easily see that, if from one frame to another the matrices of covariance are only slightly disturbed so that only one iteration is enough to converge, the fast algorithm has a much lower computational complexity, being more suitable for practical implementations. However, even in situations where the optimal downlink weights are obtained without preliminary initialization, simulations show that the total computational complexity of the fast algorithm is at least twice lower when the number of mobiles is greater than about 10 (see Table II.

Finally, contrary to the DBPC algorithm, the proposed fast algorithm respects the maximum power constraint present in any system.

VIII. SIMULATIONS RESULTS

In order to evaluate the performance of the algorithms, an 8-element linear antenna array is used at the base station to serve a 120° sector of a single cell in a UMTS/WCDMA system. The inter-element distance is $\frac{\lambda_c}{2}$, where λ_c is the carrier wavelength. The cell radius is 1 km and the path loss is proportional to $r^{-3.6}$, where r is the distance between the mobile and the base station. The noise power at the mobile ($\sigma 2$) is -100.2 dBW. The maximum transmit power of the base station is 40 W. The multi-sensor channel model of [9] is used. The angle of arrival for each user is a uniform random variable in $[-60°, +60°]$ and the distance between the mobile and the base station is also a uniform random variable in $[50, 1000]$m. The angular spread $\Delta\theta$ is modelled as a gaussian random variable with mean equals 30° and standard deviation equals 2.64°.

Firstly, the DBPC algorithm is compared with decoupled beamforming and power control for different services provided by UMTS. The characteristics of these services are show in Table I. Two algorithms for downlink beamforming are used, namely the SICR (Summed Inverse Carrier-to-interference Ratio) and the MSF (Matched Spatial Filter), proposed in [4] and [6] respectively. Figure 1 shows a Monte Carlo simulation for 100 trials, where SICR+PC denotes the SICR beamforming and power control; and MSF+PC denotes the MSF beamforming and power control. The DBPC algorithm outperforms the other two in all cases. Besides, as the number of users becomes large, spatial noise becomes *spatial-white-noise* like and the MSF+PC algorithm leads to a capacity greater than the SICR+PC one.

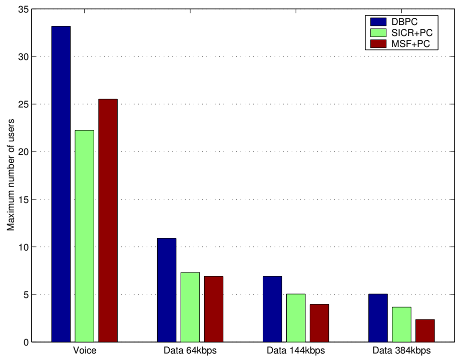

Fig. 1. Maximum number of users for different services of the UMTS system.

The capacity increase is also related to the number of users (which is related to the provided services), as shown in Figure 2.

Figure 3 shows the evolution of the maximum number of users according to the required $\frac{E_b}{N_o}$, for SF=4. As expected for a few number of users, the MSF+PC has the worst performance since the spatial noise is concentrated in some particular directions and the algorithm does not include nulling. On the other hand, since the SICR+PC is a beamforming algorithm with nulling, it outperforms the MSF+PC one. However, the SICR+PC is not yet optimal, the optimal solution being achieved by the DBPC algorithm.

Figures 4 and 5 show the convergence of the DBPC and the Fast-DBPC algorithms. The dotted line corresponds to the steady-state transmit power and the dashed line stands for the iteration number where convergence is reached. By convergence, in the following, we mean that the total transmit power is reached at 99% of the steady-state value and $c_{\max}$ is also reached at 99%. At this point, the solution is close enough to the optimal one.

A comparison of these two algorithms is made regarding this convergence condition. The mean number of iterations to converge taking into account a Monte Carlo simulation for 1000 trials is shown in Table II, where U is the number of users, N_{DBPC} is the mean number of iterations for the DBPC algorithm and $N_{\text{F-DBPC}}$ is the mean number of iterations for the Fast-DBPC algorithm. The theoretical $N_{\text{F-DBPC}}$ to N_{DBPC} ratio at the same computational complexity is denoted by $\mathcal{T}\left\{\frac{N_{\text{F-DBPC}}}{N_{\text{DBPC}}}\right\}$.

For $U > 8$, the Table II shows that the Fast-DBPC algorithm has a lower computational complexity than the DBPC one, while

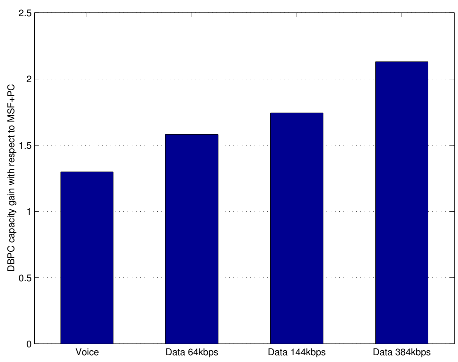

Fig. 2. DBPC capacity gain with respect to MSF+PC.

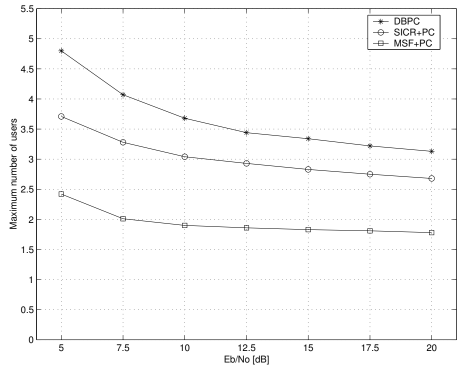

Fig. 3. Maximum number of users in function of the required $\frac{E_b}{N_o}$, for SF=4.

both reach the optimal solution. The Fast-DBPC computational complexity in this case is half the DBPC one.

TABLE II

CONVERGE SPEED COMPARISON

U	N_{DBPC}	$N_{\text{F-DBPC}}$	$\frac{N_{\text{F-DBPC}}}{N_{\text{DBPC}}}$	$\mathcal{T}\left\{\frac{N_{\text{F-DBPC}}}{N_{\text{DBPC}}}\right\}$
4	1.077	5.475	5.0836	2.6036
8	1.558	6.907	4.4332	4.4380
16	2.045	8.493	4.1531	8.1983
20	2.220	11.967	5.3905	10.2547

IX. CONCLUSION

In this paper the joint downlink beamforming and power control problem is stated and an iterative algorithm which leads to the optimum solution is derived. A fast algorithm with a lower computational complexity is proposed. Moreover, the fast algorithm respects the maximum power constraint and is more suitable for situations where the covariance matrices are recursively estimated.

Simulations have shown that the proposed algorithm outperforms other decoupled downlink beamforming and power control strategies and enables capacity increase in UMTS systems. However, the same methodology can be applied to any wireless network, such as FDMA and TDMA, also increasing the capacity of these systems.

APPENDIX A - ALGORITHM CONVERGENCE

Each iteration of the DBPC algorithm is composed of 2 stages. Firstly, the weights $\mathbf{w}_i$ are updated. Secondly, the uplink power vector $\boldsymbol{\alpha}$ is updated. A sufficient condition for the algorithm to converge is that on each step the solution tends towards the optimal solution.

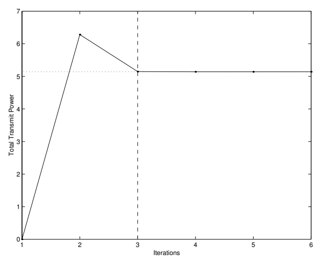

Fig. 4. Converge of the DBPC Algorithm

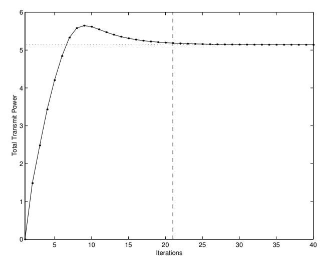

Fig. 5. Converge of the Fast-DBPC Algorithm

Let us consider one iteration of the algorithm. The entities $\mathbf{w}_i^-$ and $\boldsymbol{\alpha}^-$ obtained in the previous iteration are such that the constraints (9) are verified:

$$\frac{\alpha_i^- \mathbf{w}_i^{-H} \frac{\mathbf{R}_i}{\gamma_i} \mathbf{w}_i^-}{\mathbf{w}_i^{-H} \left(\sum_{k \neq i} \alpha_k^- \mathbf{R}_k \right) \mathbf{w}_i^- + \sigma^2} = c^- \quad \forall i \tag{31}$$

The optimal solution however is not reached yet because either the targets are not reached ($c^- < 1$), or the total transmit power is not minimal.

A. Downlink weights update

As the α_i^-'s are been computed using the $\mathbf{w}_i^-$'s in order to respect the constraints, the downlink weights are not any more eigenvectors of equation (14). Thus, the downlink weights are updated using equation (14) and the vector $\boldsymbol{\alpha}^-$. It follows that:

$$\frac{\alpha_i^- \mathbf{w}_i^{+H} \frac{\mathbf{R}_i}{\gamma_i} \mathbf{w}_i^+}{\mathbf{w}_i^{+H} \left(\sum_{k \neq i} \alpha_k^- \mathbf{R}_k + \sigma^2 \right) \mathbf{w}_i^+} = \lambda_{i,\max} > c^- \quad \forall i \tag{32}$$

The $\lambda_{i,\max}$ are the maximum eigenvalues associated with the eigenvectors $\mathbf{w}_i^+$ and are all higher than c^-. Thus, the new downlink weights approaches the optimal solution.

B. Uplink powers update

The gain obtained at the preceding step ($\lambda_{i,\max} > c^-$) is exploited at this stage either to make grow the value c^- in order to approach the target 1, or to minimize the total transmit power when the targets are already reached.

At the end of the computation of the downlink weights $\mathbf{w}_i^+$, the equality (32) can be expressed in matricial form as:

$$\mathbf{D}^+ \boldsymbol{\alpha}^- - \boldsymbol{\Lambda} \mathbf{M}^+ \boldsymbol{\alpha}^- = \boldsymbol{\Lambda} \mathbf{1} \tag{33}$$

where $\boldsymbol{\Lambda}$ is a diagonal matrix composed of the $\lambda_{i,\max}$'s and the matrices $\mathbf{D}^+$ and $\mathbf{M}^+$ are computed using $\mathbf{w}_i^+$. The α_i's are updated in order to respect the constraints:

$$\mathbf{D}^+ \boldsymbol{\alpha}^+ - c^+ \mathbf{M}^+ \boldsymbol{\alpha}^+ = c^+ \mathbf{1} \tag{34}$$

Let us initially treat the case where the targets are not reached yet. c^+ is then chosen slightly lower than $c_{\max}$, maximum eigenvalue of $(\mathbf{D}^+, \mathbf{M}^+)$. Let us demonstrate that c^+ is greater than c^-. The Equation (33) can be rewritten as:

$$\mathbf{D}^+ \boldsymbol{\alpha}^- - \beta \mathbf{M}^+ \boldsymbol{\alpha}^- = \boldsymbol{\Lambda} \mathbf{1} + (\boldsymbol{\Lambda} - \beta \mathbf{I}) \mathbf{M}^+ \boldsymbol{\alpha}^- \tag{35}$$

where $\beta = \min \lambda_{i,\max}$.

Using the fact that the diagonal elements of $\boldsymbol{\Lambda}$ are strictly positive, the following inequality holds:

$$\mathbf{D}^+ \boldsymbol{\alpha}^- - \beta \mathbf{M}^+ \boldsymbol{\alpha}^- > \mathbf{0} \tag{36}$$

Left-multiplying the above equation by $\boldsymbol{\alpha}^{-T}$, it follows that:

$$\frac{\boldsymbol{\alpha}^{-T} \mathbf{D}^+ \boldsymbol{\alpha}^-}{\boldsymbol{\alpha}^{-T} \mathbf{M}^+ \boldsymbol{\alpha}^-} > \beta \tag{37}$$

The eigenvector $\mathbf{v}$ of $(\mathbf{D}^+, \mathbf{M}^+)$, associated with the maximum eigenvalue $c_{\max}$, makes this ratio maximum:

$$\frac{\mathbf{v}^T \mathbf{D}^+ \mathbf{v}}{\mathbf{v}^T \mathbf{M}^+ \mathbf{v}} = c_{\max} > \beta \tag{38}$$

Ultimately:

$$c^+ = c_{\max} > \beta > c^- \tag{39}$$

Let us treat now the case where the targets are reached. In this case, the value of c^+ is fixed at 1. And the vector $\boldsymbol{\alpha}^+$ must satisfy the following equation:

$$\mathbf{D}^+ \boldsymbol{\alpha}^+ - \mathbf{M}^+ \boldsymbol{\alpha}^+ = \mathbf{1} \tag{40}$$

Let us demonstrate that the total transmit power decreases; in other words, that:

$$\sum_i \alpha_i^+ < \sum_i \alpha_i^- \tag{41}$$

In order to do so, one should left-multiply equation (33) by $\boldsymbol{\Lambda}^-1$:

$$\boldsymbol{\Lambda}^{-1} \mathbf{D}^+ \alpha^- - \mathbf{M}^+ \alpha^- = \mathbf{1} \tag{42}$$

Defining $\boldsymbol{\alpha}^+ = \boldsymbol{\alpha}^- + \Delta \boldsymbol{\alpha}$, the equality of (40) and (42)

$$\mathbf{D}^+ \alpha^- + \mathbf{D}^+ \Delta\alpha - \mathbf{M}^+ \alpha^- - \mathbf{M}^+ \Delta\alpha = \boldsymbol{\Lambda}^{-1} \mathbf{D}^+ \alpha^- - \mathbf{M}^+ \alpha^- \tag{43}$$

shows that all the elements of $\Delta\boldsymbol{\alpha}$ are negative:

$$\Delta \boldsymbol{\alpha} = - \underbrace{\left(\mathbf{D}^+ - \mathbf{M}^+\right)^{-1}}_{I} \underbrace{\left(\mathbf{I} - \boldsymbol{\Lambda}^{-1}\right)}_{II} \underbrace{\mathbf{D}^+ \boldsymbol{\alpha}^-}_{III} \tag{44}$$

Indeed, vector III has all its elements strictly positive because all the elements of $\boldsymbol{\alpha}^-$ and $\mathbf{D}^+$ are, by their turn, strictly positive. The matrix II also has all its elements strictly positive because $\lambda_{i,\max} \geq 1, \forall i$. Lastly, matrix I has the same property, shown by equation (29).

Ultimately:

$$\sum_i p_i^+ = \sum_i \alpha_i^+ < \sum_i \alpha_i^- = \sum_i p_i^- \tag{45}$$

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