International Telecommunications Symposium – ITS2002, Natal, Brazil

Orthogonal Pulse Shape Modulation for Impulse Radio

João A. Ney da Silva and Marcello L. R. de Campos
COPPE/Universidade Federal do Rio de Janeiro, RJ, Brazil

Abstract—**In this paper we propose and test a new modulation strategy to be used in impulse-radio ultra-wide band (IR-UWB) modulation. The use of short-duration pulses to convey information has recently emerged as a powerful alternative to future-generation spread-spectrum wireless systems. The proposed method has shown significant advantages over existing modulation schemes for IR-UWB modulation, specially when a high throughput is desired. The technique is based on the design of pulses using Hermite functions, such that an orthogonal M-ary modulation scheme is obtained. The paper presents detailed information of the design procedure and simulations showing preliminary results for AWGN channels.**

I. Introduction

In recent years there has been a growing interest in Ultra-Wide Bandwidth communications (**UWB**) by means of Impulse Radio (**IR**). This technique has emerged as a possible alternative to future wireless communications systems. IR/UWB systems differ from current systems basically by the lack of a continuous sinusoidal carrier. In this kind of systems, transmitted signals are impulsive, i.e., they are extremely short in time, and therefore have a highly spread spectrum. This high frequency content, with no DC component, allows its carrierless transmission. The technology to generate these pulses, with duration in the order of 1ns, is already available, and have been used for years in radar systems.

The initial interest in this area came from military applications, in which the communications security is enhanced due to the low probabilities of detection/interception afforded by the short pulse durations and its spread spectrum with low power density. The technique also looks promising for civilian applications, due to the low power required, the multiuser capacity, the availability of the technology to generate and receive the pulses with little complexity and to its high capability to mitigate the effects of multipath by using more than one copy of the pulse to construct the received signal (specially in indoor/short distance wireless environments).

As there is no current regulation for allocating bandwidths as wide as 1GHz, IR/UWB systems must be treated as interference by other systems. This affects IR/UWB systems, for its signals need to compete with a variety of interfering signals and yet not interfere with other existing systems. These conditions lead to the same requirements of the military systems aiming to minimize the detection/interception probability, i.e., the spectral content must be kept to the minimum power necessary to guarantee the desired communication quality.

Being composed of extremely short pulses precisely located in time inside a relatively long time frame, while general noise and interfering signals occur randomly and continuously along this frame, UWB signals have a great advantage over noise. As it is shown in this paper, this fact along with efficient modulation and detection schemes put IR/UWB communications in a privileged condition, being able to operate below ambient noise, with signal-to-noise ratios (SNR) as low as $-20dB$. Indeed IR/UWB proves to be a strong candidate for future communication systems, including future generations of cellular systems.

In this work we propose a new form of modulation for IR/UWB systems based on orthogonal pulses. The addition of orthogonality into IR/UWB communications opens new opportunities for evolution of these systems. The orthogonal pulse waveforms can be used either as a form to expand multiple access (or multi-user) capacity, a form to expand IR/UWB systems, or as an alternative form of data modulation, which we develop and test in this paper, showing its high efficiency and its good own features.

The proposed Pulse Shape Modulation (**PSM**) scheme is compared with the well known Pulse Position Modulation (**PPM**) [1], which is reviewed in the next section. The PSM scheme is introduced in Section III. Section IV describes the methodology for obtaining four orthogonal pulses, suitable to compose a quaternary orthogonal PSM. Section V presents some simulations and in Section VI the conclusions are presented.

II. Pulse Position Modulation

Pulse Position Modulation [1] consists in representing each symbol by a corresponding delay in the transmitted pulse. The transmitted signal is then expressed by:

$$x_P(t) = \Sigma_j w(t - jT_f - \tau_j) \tag{1}$$

where τ_j is the delay of the j-th pulse corresponding to the symbol represented. In a binary modulation we have typically $\tau_j = 0$ representing bit 0 and $\tau_j = \delta$ representing bit 1, where δ is the time delay adopted in the scheme. The pulse waveform is represented by $w(.)$. T_f is the frame time (from the system viewpoint) or pulse repetition time (from the user viewpoint). In [1] the authors suggested a totally temporal modulation-based system, in which the binary data are modulated by PPM and multiple access is achieved with a pseudo-random time hopping technique. The resulting signal is expressed by:

$$s^{(k)}(t) = \Sigma_j w(t - jT_f - c_j^{(k)} T_c - \delta.d^{(k)}_{\lfloor j/N_S \rfloor}) \tag{2}$$

where k indicates the k-th user, $c_j^{(k)}$ represents the pulse delay pattern, T_c is the basic time delay unit and $d^{(k)}_{\lfloor j/N_S \rfloor}$ represents the data stream. N_S is the number of pulses being modulated by one single symbol. Notice that $\delta.d^{(k)}_{\lfloor j/N_S \rfloor}$ is equivalent to τ_j. This system is depicted in figure 1 for user $k = 1$.

João A. Ney da Silva is with the Instituto Militar de Engenharia and with COPPE/Universidade Federal do Rio de Janeiro, Caixa Postal 68504, RJ, Brazil, 21945-970. E-mail: abdalla@lps.ufrj.br.

Marcello L. R. de Campos is with COPPE/Universidade Federal do Rio de Janeiro, Caixa Postal 68504, RJ, Brazil, 21945-970. E-mail: campos@lps.ufrj.br.

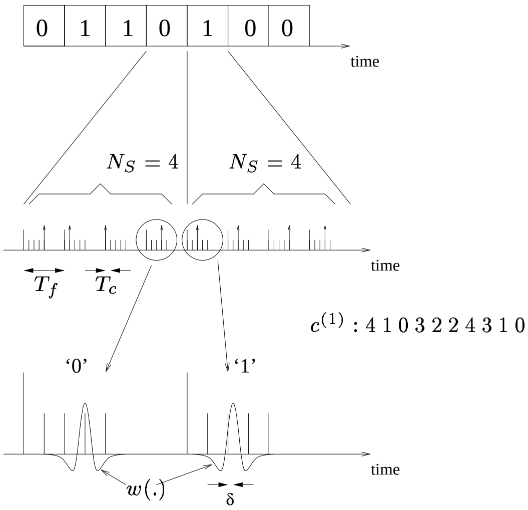

Fig. 1. A totally temporal modulation-based scheme, with uniform pulse train spacing and pseudo-random time-hopping for multiple access and PPM for data modulation.

In the PPM scheme, time modulation leads to a variable time between pulses, coded by pseudo-random sequences which gives a random aspect to the signal, both in time and frequency. The resulting good features are the code division channelization avoiding catastrophic collisions with a power distribution more uniform than that achieved with a purely deterministic TDMA scheme. It also decorrelates the in-band interfering signals by the use of the coded time hops. Detection can be accomplished by a simple receiver, in which a matched filter or a signal correlator is used to estimate the pulse arrival time, possibly using many copies of the same pulse created by multipath propagation.

The pulse waveform, $w(t)$, can be any pulse with a short effective duration. Two waveforms commonly used are the Gaussian and Rayleigh pulses which are given, respectively, by the following expressions:

$$w_G(t,\sigma) = \frac{1-(t/\sigma)^2}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-t^2}{2\sigma^2}\right) \quad (3)$$

$$w_R(t,\sigma) = (t/\sigma^2) \exp\left(\frac{-t^2}{2\sigma^2}\right) \quad (4)$$

where σ is a time scale factor.

Conroy [2] first investigated these two pulses highlighting the fact that they are respectively even and odd time functions, what makes them orthogonal in time. It was also noted that the power spectrum densities of the two pulses are similar, allowing to use both pulses simultaneously, sharing the same bandwidth, hence doubling the total capacity of the system. This could be achieved using ordinary PPM for data modulation having half of the users using one kind of pulses while the other half use the other kind. Another possibility is to send simultaneously two bits from a given data stream, where each bit is carried by one of the two kinds of pulses. The Gaussian and Rayleigh pulses are plotted in figure 2.

In PPM, orthogonal signals can be obtained by making δ in Eq. (2) greater or equal to the width of the pulse waveform, but a time slot of twice the pulse width would be required for each bit, reducing the total capacity of the system.

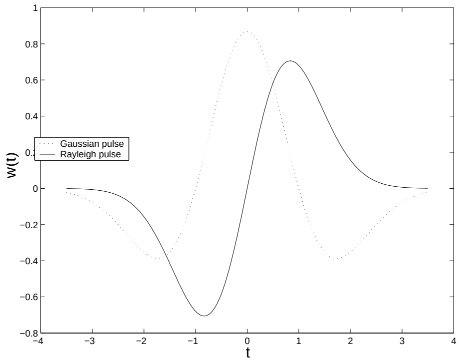

Fig. 2. Gaussian and Rayleigh Pulses.

The scheme initially suggested for PPM [1] performs reception through correlation of the received signal with a special template waveform composed by the sum of the two possible transmitted pulses, being one of them inverted and delayed:

$$v(t) = w_G(t) - w_G(t-\delta) \quad (5)$$

where δ was previously chosen to minimize the cross-correlation between $w_G(t)$ and $w_G(t-\delta)$. This scheme allows the use of a single correlator whose output gives positive values when applied to in-phase pulses and negative values to pulses delayed by δ. This scheme neglects the possible signal inversion due to reflections in the propagation path or to the relative positions of transmitter and receiver. So, it depends upon an additional scheme to determine the polarity of the received signal, whose efficiency and complexity may affect the whole system.

III. Pulse Shape Modulation

The new form of IR/UWB modulation proposed in this work, Pulse Shape Modulation (PSM), is based on the orthogonality of pulse waveforms. The basic idea is to represent bits 1 and 0 by two orthogonal pulses, that can be, for example, the Gaussian and Rayleigh pulses given in Eqs (3) and (4), respectively.

Initially, the motivation for this form of modulation in IR/UWB systems is related to the possibility of requiring less time precision than PPM and being more immune to multipath. It is desirable to have a detection design independent of the received signal polarity, avoiding some of the requirements posed by PPM. The binary case described above is an example of orthogonal PSM. Generic M-ary modulation can be achieved by combining M distinct orthogonal pulses, in which case we have an M-ary orthogonal PSM, which will be explored in the following sections.

A. Orthogonal Functions

From the solution of the Sturm-Liouville partial differential equations some sets of orthogonal functions are specified over defined intervals [3], [4], e.g., Legendre, Bessel and Hermite functions among other polynomial-based functions. These are infinite sets with the property that any square-integrable (or finite-energy) function can be projected (or expanded) in any of the sets, i.e., any finite-energy function can be expressed as a linear combination of the functions of any of these orthogonal sets. In other words we can say that any of these sets form a basis for the space $\mathbf{L_2}$ of the square-integrable functions.

We can take finite subspaces from $\mathbf{L_2}$ by selecting only a finite number of functions from an orthogonal set. Therefore, the N-dimensional Hermite space can be defined as the space spanned by the N first Hermite functions [5], $\mathcal{H}_N = \mathsf{span}\{\psi_n(t)\}_{n=0}^{N-1}$. Figure 3 shows some of the first Hermite functions, given by the expression

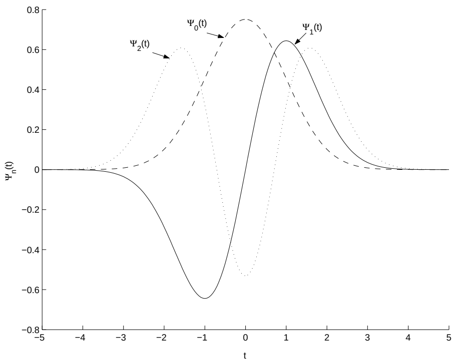

Fig. 3. First Three Hermite Functions.

$$\psi_n(t) = \frac{H_n(t)e^{-t^2/2}}{\sqrt{2^n n! \sqrt{\pi}}} \tag{6}$$

where $H_n(t)$ are the Hermite polynomials, recursively obtained by the formulas:

$$H_0(t) = 1 \tag{7}$$

$$H_1(t) = 2t \tag{8}$$

$$H_{n+1}(t) = 2tH_n(t) - 2nH_{n-1}(t) \quad (n \geq 1). \tag{9}$$

The expressions for the first Hermite functions are given below:

$$\psi_0(t) = \frac{1}{\sqrt[4]{\pi}}\, e^{-t^2/2} \tag{10}$$

$$\psi_1(t) = \frac{2t}{\sqrt{2\sqrt{\pi}}}\, e^{-t^2/2} \tag{11}$$

$$\psi_2(t) = \frac{2t^2 - 1}{\sqrt{2\sqrt{\pi}}}\, e^{-t^2/2} \tag{12}$$

$$\psi_3(t) = \frac{2t^3 - 3t}{\sqrt{3\sqrt{\pi}}}\, e^{-t^2/2} \tag{13}$$

$$\psi_4(t) = \frac{4t^4 - 12t^2 + 3}{2\sqrt{6\sqrt{\pi}}}\, e^{-t^2/2} \tag{14}$$

The Fourier Transform of the Hermite functions is presented next:

$$\Psi_{n+1}(\omega) = \sqrt{\frac{2}{n+1}} j\omega \Psi_n(\omega) + \sqrt{\frac{n}{n+1}} \Psi_{n-1}(\omega) \tag{15}$$

An interesting fact, proved with the help of time-frequency analysis theory [5], is that the Hermite spaces are maximally concentrated both in time and in frequency. This allows the shortest effective duration waveforms without having a frequency content higher than necessary. Besides avoiding further difficulties for the signal generation, this guarantees optimal use of the communications resources measured by the duration-bandwidth product. Hence, Hermite spaces constitute promising signal spaces for the desired orthogonal functions.

B. Hermite Pulses

We will define Nth-order Hermite pulses as any pulses belonging to the N-dimensional Hermite space. Therefore, we formally define a Hermite pulse as one expressed by:

$$w_{H_N}(t) = q_{N-1}(t) \exp\left(\frac{-t^2}{2}\right) \tag{16}$$

where $q_{N-1}(t)$ is a polynomial in the variable t with degree $N-1$. It can be easily seen that the Gaussian and Rayleigh pulses are particular cases of Hermite pulses. We now define the vector composed by the N first Hermite functions as:

$$\vec{\psi}_N(t) = \left[\; \psi_0(t)\ \psi_1(t)\ \ldots\ \psi_{N-1}(t) \;\right] \tag{17}$$

Therefore, any Nth-order Hermite pulse can be expressed as:

$$w_{H_N}(t) = \vec{\psi}_N(t)\mathbf{w_{H_N}} \tag{18}$$

where $\mathbf{w_{H_N}}$ is a real column vector that relates the considered pulse to the N first orthonormal Hermite functions, and it will be called *projection vector*. This means that each signal of the Hermite space $\mathcal{H}_N$ is being expressed through its orthonormal basis. In this context, the Gaussian and Rayleigh pulses with $\sigma = 1$ can be expressed by:

$$\mathbf{w_{G_{H_3}}} = \frac{1}{2\sqrt[4]{4\pi}} \begin{bmatrix} 1 \\ 0 \\ -\sqrt{2} \end{bmatrix} \text{ and } \mathbf{w_{R_{H_3}}} = \frac{2}{\sqrt[4]{4\pi}} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \tag{19}$$

The orthogonality of these two pulses becomes obvious by the orthogonality of their projection vectors, since they represent projections on an orthogonal basis. Now we wish to find a set of M Nth-order Hermite pulses, $M \leq N$, orthogonal to each other, having no DC component and occupying the most similar possible frequency bands. This set of pulses can be represented by a matrix $N \times M$ formed by the projection vectors $\mathbf{w_{H_N}}$ from each pulse:

$$\mathbf{A} = \left[\; \mathbf{w_{H_N,1}}\ \ \mathbf{w_{H_N,2}}\ \ \ldots\ \ \mathbf{w_{H_N,M}} \;\right] \tag{20}$$

where the second index designates each particular pulse.

Notice that the requirement of absence of DC component eliminates the trivial solution of using exactly the Hermite functions as the set of desired pulses. Moreover, it poses a linear restriction to the vector components $\mathbf{w_{H_N}}$ and therefore to the lines of matrix $\mathbf{A}$, lowering its rank and leading to:

$$M \leq N - 1 \tag{21}$$

C. A Set of Three Orthogonal Pulses

In this section we will describe the general procedure to get a set of orthogonal pulses taking as an example a set of three pulses with the desired characteristics. The basic methodology is introduced, defining the figures of merit that quantify the quality of a candidate set.

According to (21), in order to get three orthogonal pulses, the space dimension must be $N \geq 4$. So we will search for three 4th-order orthogonal Hermite pulses. The projection vector of each pulse becomes $\mathbf{w_{H_4}} = [w_0\ w_1\ w_2\ w_3]^T$. Hence,

$$w_{H_4}(t) = w_0\psi_0(t) + w_1\psi_1(t) + w_2\psi_2(t) + w_3\psi_3(t) \tag{22}$$

The requirement of zero DC component leads to $w_2 = -\sqrt{2}\, w_0$. As a consequence, we have:

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ -\sqrt{2}a_{11} & -\sqrt{2}a_{12} & -\sqrt{2}a_{13} \\ a_{41} & a_{42} & a_{43} \end{bmatrix} \quad (23)$$

For this matrix to be orthogonal, it must satisfy $\mathbf{A}^T\mathbf{A} = \mathbf{I_3}$, which leads to an undetermined system with 6 equations and 9 variables. Among the infinite possible solutions we will select one which leads to three pulses of similar bandwidths. This can be obtained using equation (15) to calculate *mean central frequency* and *frequency spreading*, whose definitions are analogous to the statistical definitions of mean value and variance and usually referred to as mean frequency and bandwidth in the time-frequency literature. A search for the set of pulses that minimizes a determined functional based on these measures can be made. For such, we define the energy spectral density of a generic pulse as $E_w(\omega)$, and then we can define the central frequency of a pulse as [6]:

$$\omega_0 = \frac{\int_0^\infty \omega |E_w(\omega)|^2 d\omega}{\int_0^\infty |E_w(\omega)|^2 d\omega} = 2\int_0^\infty \omega |E_w(\omega)|^2 d\omega \quad (24)$$

since the signal is normalized. This results in:

$$\omega_0 = \frac{8}{\sqrt{\pi}} \left[{w_0}^2 + \frac{{w_1}^2}{4} + \frac{3{w_3}^2}{8} - \frac{\sqrt{6}}{12} w_1 w_3 \right] \quad (25)$$

Next we define the squared mean frequency spreading as:

$$\sigma_\omega^2 = \frac{\int_0^\infty (\omega - \omega_0)^2 |E_w(\omega)|^2 d\omega}{\int_0^\infty |E_w(\omega)|^2 d\omega} \quad (26)$$

which, for the considered pulse, results in:

$$\sigma_\omega^2 = \frac{15}{2}{w_0}^2 + \frac{3}{2}{w_1}^2 + \frac{7}{2}{w_3}^2 - \sqrt{6} w_1 w_3 - {\omega_0}^2 \quad (27)$$

Using the definitions above, we can define the edge-frequencies, ω_1 and ω_2, of the spectrum as those enclosing a given portion of the total energy:

$$\omega_1 = \omega_0 - \alpha\sigma_\omega \text{ and } \omega_2 = \omega_0 + \alpha\sigma_\omega \quad (28)$$

It was empirically found that for $\alpha = 1.3$ the width $\omega_2 - \omega 1$ corresponds approximately to the classical $-3dB$ bandwidth. Finally the function to be minimized is defined as:

$$G = (\omega_{1,\mathbf{1}} - \omega_{1,\mathbf{2}})^2 + (\omega_{2,\mathbf{1}} - \omega_{2,\mathbf{2}})^2 + (\omega_{1,\mathbf{1}} - \omega_{1,\mathbf{3}})^2 + (\omega_{2,\mathbf{1}} - \omega_{2,\mathbf{3}})^2 + (\omega_{1,\mathbf{2}} - \omega_{1,\mathbf{3}})^2 + (\omega_{2,\mathbf{2}} - \omega_{2,\mathbf{3}})^2 \quad (29)$$

where the second index corresponds to the considered pulse.

The pulses found are shown in figure 4, and correspond to the following vectors:

$$\mathbf{w_{1_{H_4}}} = \left[0 \;\; -3/\sqrt{10} \;\; 0 \;\; 1/\sqrt{10}\right]^T \quad (30)$$

$$\mathbf{w_{2_{H_4}}} = \left[-1/\sqrt{6} \;\; 1/\sqrt{3} \;\; -1/(2\sqrt{5}) \;\; -3/2\sqrt{5}\right]^T \quad (31)$$

$$\mathbf{w_{3_{H_4}}} = \left[1/\sqrt{6} \;\; -1/\sqrt{3} \;\; -1/2\sqrt{5} \;\; -3/(2\sqrt{5})\right]^T \quad (32)$$

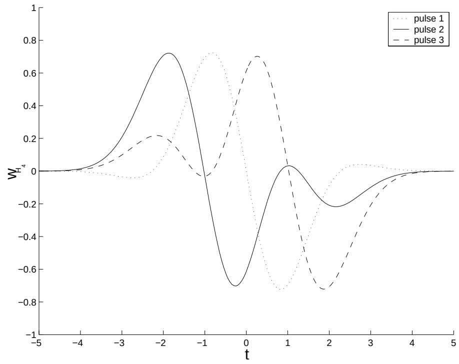

Fig. 4. Three optimal orthogonal pulses.

The three pulses are given by:

$$w_{1_{H_4}}(t) = \frac{-2\sqrt{2}t^3 + 3(2\sqrt{3} + \sqrt{2})t}{2\sqrt{15\sqrt{\pi}}}\, e^{-t^2/2} \quad (33)$$

$$w_{2_{H_4}}(t) = \frac{-6\sqrt{2}t^3 + 4\sqrt{5}t^2 + (9\sqrt{2} - 2\sqrt{3})t - 4\sqrt{5}}{2\sqrt{30\sqrt{\pi}}}\, e^{-t^2/2} \quad (34)$$

$$w_{3_{H_4}}(t) = \frac{-6\sqrt{2}t^3 - 4\sqrt{5}t^2 + (9\sqrt{2} - 2\sqrt{3})t + 4\sqrt{5}}{2\sqrt{30\sqrt{\pi}}}\, e^{-t^2/2} \quad (35)$$

The spectra of these pulses can be seen in figure 5. Note that the second and third pulses have identical spectra. Observing closely their time representations, it can be seen that they are inverted versions (both in time and amplitude) of each other, i.e., $w_{3_{H_4}}(t) = -w_{2_{H_4}}(-t)$, what explains this fact.

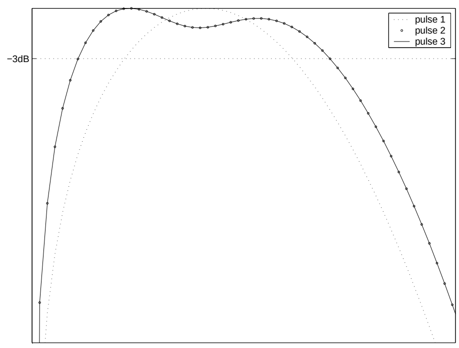

Fig. 5. Frequency spectra of the three proposed orthogonal pulses.

IV. A Set of Four Orthogonal Pulses

For the purpose of data modulation it is desirable to have a number of $M = 2^K$ waveforms to represent a set of K bits, constituting an M-ary modulation. In this section we will search for a set of four orthogonal pulses so that a quaternary orthogonal modulation can be implemented. In order to obtain four orthogonal pulses, we already know from (21) that we shall have $N \geq 5$. In this case, the projection vector is $\mathbf{w_{H_5}} = [w_0\ w_1\ w_2\ w_3\ w_4]^T$. And similarly to section III-C, the requirement of zero DC component yields $w_2 = -\sqrt{2}w_0 - \frac{\sqrt{3}}{2}w_4$.

Now we have a 4×4 matrix $\mathbf{A}$, which leads to a system with 10 equations and 16 variables. If we followed the same procedure used for the set of three pulses, we would have to choose the value of six variables in each iteration, instead of three as before. This would render the dimension of the search space prohibitive. In addition, the equations involved become increasingly more complex. As a consequence, an alternative procedure is proposed.

We are looking for a set of orthogonal vectors, therefore we can use well-known methods to generate orthogonal squared matrices to obtain the candidate sets. We begin by decomposing matrix $\mathbf{A}$ in a (reduced) square matrix, $\mathbf{A_R}$, and a line combination matrix , $\mathbf{C}$:

$$\mathbf{A} = \mathbf{C}\,\mathbf{A_R} \tag{36}$$

The orthogonal matrix equation can be rewritten as:

$$\mathbf{A^T A} = \mathbf{I_4} \quad \Rightarrow \quad \mathbf{A_R^T C^T C A_R} = \mathbf{I_4} \tag{37}$$

where $\mathbf{C^T C}$ can be decomposed in its eigenvalues and eigenvectors as

$$\mathbf{C^T C} \;=\; \mathbf{V^T \Lambda V} \tag{38}$$

Defining the matrix $\tilde{\mathbf{A}}$ as:

$$\tilde{\mathbf{A}} = \mathbf{\Lambda^{1/2} V A_R} \tag{39}$$

it is easy to see that $\tilde{\mathbf{A}}^\mathbf{T}\tilde{\mathbf{A}} = \mathbf{I_4}$. So, we now have a square orthogonal matrix. We can then apply known methods to generate orthogonal matrices for obtaining, iteratively, the matrix $\tilde{\mathbf{A}}$, and determining next the corresponding matrix $\mathbf{A_R}$ (and $\mathbf{A}$) by the equation:

$$\mathbf{A_R} = \mathbf{V^{-1}}\left(\mathbf{\Lambda^{1/2}}\right)^{\mathbf{-1}}\tilde{\mathbf{A}} \tag{40}$$

We can scan the space of possible solutions in an ordered manner looking for the solution that minimizes the functional similarly to what was done in the three-pulse case. The functional to be minimized is essentially the same of equation (29):

$$\begin{aligned} G = {} & (\omega_{1,\mathbf{1}} - \omega_{1,\mathbf{2}})^2 + (\omega_{2,\mathbf{1}} - \omega_{2,\mathbf{2}})^2 + (\omega_{1,\mathbf{1}} - \omega_{1,\mathbf{3}})^2 + \\ & (\omega_{2,\mathbf{1}} - \omega_{2,\mathbf{3}})^2 + (\omega_{1,\mathbf{1}} - \omega_{1,\mathbf{4}})^2 + (\omega_{2,\mathbf{1}} - \omega_{2,\mathbf{4}})^2 + \\ & (\omega_{1,\mathbf{2}} - \omega_{1,\mathbf{3}})^2 + (\omega_{2,\mathbf{2}} - \omega_{2,\mathbf{3}})^2 + (\omega_{1,\mathbf{2}} - \omega_{1,\mathbf{4}})^2 + \\ & (\omega_{2,\mathbf{2}} - \omega_{2,\mathbf{4}})^2 + (\omega_{1,\mathbf{3}} - \omega_{1,\mathbf{4}})^2 + (\omega_{2,\mathbf{3}} - \omega_{2,\mathbf{4}})^2 \end{aligned} \tag{41}$$

where ω_1 and ω_2 are given by (28), and ω_0 and σ_ω^2 are calculated using (24) and (26).

The search procedure begins by the construction of an orthogonal 4×4 matrix, $\tilde{\mathbf{A}}$, making the product of an upper by a lower orthonormal Hessenberg matrix, both obtained by the product of 3 rotations each, given by the parameters $\theta_i, i = 1, ..., 6$. Next, (40) is used to determine $\tilde{\mathbf{A}}$ and the parameters w_0 to w_4 from each pulse. With these parameters in hands, we calculate the value of the functional under minimization using (41).

The best set of 4 pulses found is shown in figure 6, and corresponds to the following vectors:

$$\begin{aligned} \mathbf{w_{1_{H_5}}}^T &= \left[-\frac{4}{\sqrt{165}}\;\frac{\sqrt{4-2\sqrt{2}}}{4}\;\frac{\sqrt{11}}{\sqrt{30}}\;\frac{\sqrt{4+2\sqrt{2}}}{4}\;-\frac{2}{\sqrt{110}}\right] \\ \mathbf{w_{2_{H_5}}}^T &= \left[-\frac{4}{\sqrt{165}}\;-\frac{\sqrt{4-2\sqrt{2}}}{4}\;\frac{\sqrt{11}}{\sqrt{30}}\;-\frac{\sqrt{4+2\sqrt{2}}}{4}\;-\frac{2}{\sqrt{110}}\right] \\ \mathbf{w_{3_{H_5}}}^T &= \left[\frac{\sqrt{3}}{\sqrt{22}}\;-\frac{\sqrt{4+2\sqrt{2}}}{4}\;0\;\frac{\sqrt{4-2\sqrt{2}}}{4}\;-\frac{2}{\sqrt{11}}\right] \\ \mathbf{w_{4_{H_5}}}^T &= \left[\frac{\sqrt{3}}{\sqrt{22}}\;\frac{\sqrt{4+2\sqrt{2}}}{4}\;0\;-\frac{\sqrt{4-2\sqrt{2}}}{4}\;-\frac{2}{\sqrt{11}}\right] \end{aligned} \tag{42}$$

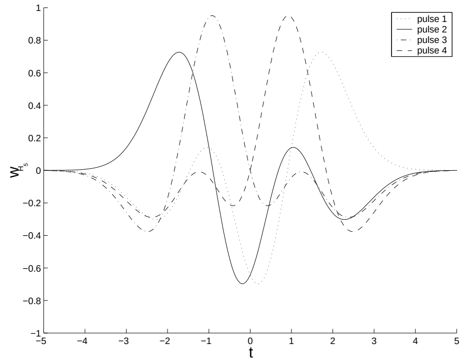

Fig. 6. Four Orthogonal Hermite Pulses.

The spectra of the pulses found can be seen in figure 7. Notice once more that two pairs of pulses have identical spectra. In figure 6, notice again that each pulse corresponds to the inverted version of another one, more precisely, $w_{2_{H_5}}(t) = w_{1_{H_5}}(-t)$ and $w_{4_{H_5}}(t) = w_{3_{H_5}}(-t)$, what explains the identical frequency contents.

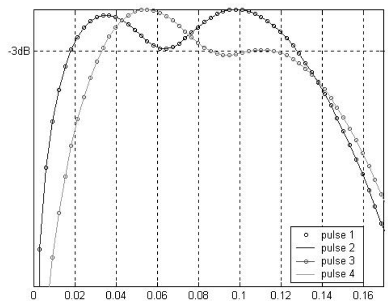

Fig. 7. Frequency Spectra of the Four Orthogonal Pulses.

V. SIMULATIONS

In order to evaluate several modes of PSM as candidates for data-modulation methods for IR/UWB systems, simulations have been performed using routines developed in MATLAB© to compute the bit error rate (BER) for some values of signal-to-noise ratio (SNR), considering an AWGN channel. For comparison, we computed the performance of the following modulation schemes:

1. *Binary PPM schemes:*

(a) **BPPM1** - *PPM with one correlator*: the original PPM scheme described is section II, proposed in [1].

(b) **BPPM2** - *PPM with two correlators*: a PPM modified to prevent problems with signal inversion doing separate correlations with each of the two possible delay values and deciding for the greatest absolute value.

2. *Binary PSM schemes:*

(a) **BPSM2** - *Binary PSM with two correlators*: uses the two orthogonal Gaussian and Rayleigh pulses to represent the bits 0 and 1 as described in section III. The received signal is passed through two independent correlators, each designed for one of the two standard pulses. Just like scheme (1b), this scheme is immune to signal inversion problems.

(b) **BPSM1** - *Binary PSM with one correlator*: same as above but using only one correlator, with a detection scheme similar to the one used by scheme (1a).

3. *Quaternary PSM schemes:*

(a) **QOPSM4** - *Quaternary Orthogonal PSM with four correlators*: uses the four pulses designed in section IV, given by Eq. (42). In the detection, it decides for the greatest absolute value among the four correlations made with each standard waveform. It is also immune to signal inversion problems.

(b) **QOPSM2** - *Quaternary Orthogonal PSM with two correlators*: same as above but using only two correlators, with a detection scheme similar to the one used by scheme (1a).

(c) **QNPSM** - *Quaternary Non-orthogonal PSM*: uses four waveforms that are linear combinations of the basic Gaussian and Rayleigh pulses. The detection uses two correlators that give the weights that each basic pulse have in the received waveform, thus deciding for the closest symbol. It makes no assumption about the polarity of the received waveform and it is immune to signal inversion problems.

The simulated environment consists in a system with a single user with fixed duty cycle given by the ratio of the effective duration of the pulses of $0.7ns$ by a time frame of $T_f = 100ns$. In the PPM schemes the effective duration is increased by $\delta = \delta_{opt} = 0.156ns$. The effect of transmission through an additive Gaussian channel is obtained by simple addition of Gaussian white noise with the power fit to the desired SNR. The results are presented in figure 8.

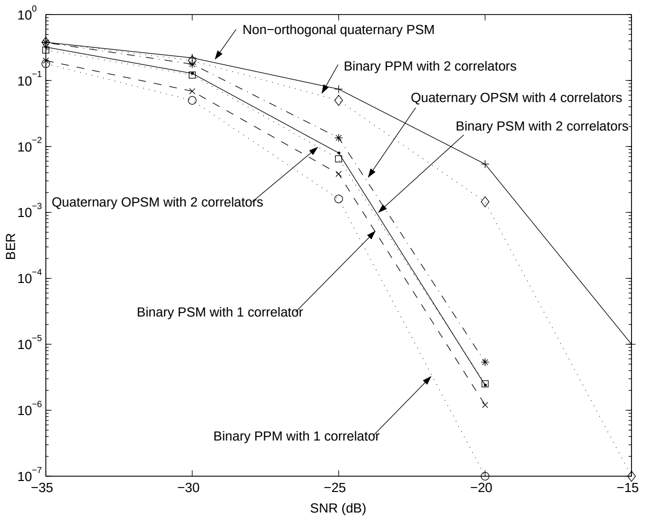

Fig. 8. Performance of various modulation schemes in presence of additive Gaussian noise.

Among all the tested binary schemes, the original PPM (1a-BPPM1) reached the best results. However it is remarkable the loss of quality of PPM when using two correlators (1b-BPPM2). With this scheme it is necessary an increase of about $5dB$ in the SNR to attain the same BER (in the range from 10^{-5} to 10^{-3}) of the original PPM. This difference can be explained by the fact that the value of δ used was originally chosen to be equal to δ_{opt}, which minimizes $\int w_G(t)w_G(t-\delta)dt$. This is suitable for the first scheme but not for the second, whose best choice would be a value great enough to make $w_G(t-\delta)$ orthogonal to $w_G(t)$, but this would significantly modify the time slot required, as explained in section II.

The standard binary PSM scheme with two correlators (2a-BPSM2), had a performance near $1dB$ worse than scheme (1a-BPPM1) and thus approximately $4dB$ better than PPM with two correlators (1b-BPPM2). Differently from what happens with the two PPM schemes, the performances of the two PSM schemes (2a-BPSM2) and (2b-BPSM1) are very similar, the latter being only approximately $0.5dB$ better than the first. We highlight that the best binary scheme immune to the signal inversion problem is the standard binary PSM (2a-BPSM2) proposed in section III.

The standard quaternary orthogonal PSM scheme (3a-QOPSM4) with four correlators have a performance about $5dB$ better the non-orthogonal scheme (3c-QNPSM) and only $0.5dB$ worse the alternative 2-correlator scheme (3b-QOPSM2). The quaternary scheme (3a-QOPSM4) has BER equivalent to that of a binary scheme (2a-BPSM2) with an SNR $0.5dB$ lower or a binary PPM scheme (1aBPPM1) with a SNR approximately $2dB$ smaller. As a quaternary scheme provides twice the rate of a binary scheme, the above results mean that the *quaternary orthogonal PSM schemes* (3b-QOPSM2) and (3a-QOPSM4) are the *most efficient* among all simulated schemes, since using two binary channels with a given SNR would rise the total power in $3dB$.

VI. Conclusions

IR/UWB communications have proved to have great efficiency in the use of available power and bandwidth. The simulations presented showed that a careful choice of the modulation scheme can have great influence on the final system performance. The proposed new form of modulation based on Hermite pulses, PSM, presented a good performance, surpassing PPM, specially when the restrictions due to uncertainty about the received signal polarity are respected.

The introduction of orthogonality into IR/UWB systems by the use of orthogonal pulse shape boosted its natural qualities as it was evidenced by the superior performance achieved by the Quaternary Orthogonal PSM schemes, even when no information about the received signal polarity is used, when compared to any of the modulation schemes tested.

A method for obtaining sets of orthogonal IR/UWB pulses based on the Hermite functions was derived. This method was successfully applied to find sets of 3 and 4 pulses, and can be easily extended to greater sets. The resulting pulses have optimal characteristics for their utilization in IR/UWB systems as they have no DC component, similar frequency spectra and minimum duration-bandwidth products.

References

[1] Robert A. Scholtz, "Multiple access with time-hopping impulse modulation," in *MILCOM'93*, 1993.
[2] J. T. Conroy, L. LoCicero, and D. R. Ucci, "Communication techniques using monopulse waveforms," in *MILCOM'99*, 1999.
[3] John David Jackson, *Mathematics for Quantum Mechanics*, Lecture Notes and Supplements in Physics. W. A. Benjamin, Inc., 1962.
[4] Eugene Butkov, *Física Matemática*, Guanabara Koogan, 1968.
[5] Franz Hlawatsch, *Time-Frequency Analysis and Synthesis of Linear Signal Spaces*, The Kluwer International Series in Engineering and Computer Science. Kluwer Academic Publishers, 1998.
[6] L Cohen, *Time-Frequency Analysis*, New Jersey: Prentice-Hall PTR, 1995.