Multirate Performance in Multiuser MMSE and Decorrelating Detectors using Random Spreading Sequences

Gustavo Fraidenraich, Renato Baldini F. and Celso de Almeida
Departamento de Comunicações - DECOM
Faculdade de Engenharia Elétrica e de Computação in the Universidade Estadual de Campinas - Unicamp
P. O Box 6.101 13081-970 Campinas S.P. Brazil (e-mail: gf@decom.fee.unicamp.br)

Abstract— **This paper presents simplified expressions for the mean bit error probability using random spreading sequences on AWGN and multipath Rayleigh fading channels using the multiuser MMSE and decorrelating detectors. It is assumed the multi processing gain schemes of multirate.**

Keywords—**Multiuser, CDMA, multirate, MMSE, decorrelating.**

I. Introduction

THE new third generation mobile system will allow multirate schemes. There will be different modulation schemes supporting multiple data rates. This paper focus on the multi processing gain scheme for multiuser MMSE and decorrelating detectors.

II. Performance on AWGN channels

Is it possible to devise a good approximation for the exact performance expression of the BPSK single rate CDMA synchronous system with K users, processing gain G_p and perfect power control. This approximation can be made by assuming that the multiple access interference is Gaussian with zero mean and variance proportional to the number of users. Thus the bit error probability can be written as:

$$P_b = \mathrm{Q}\left(\sqrt{\frac{1}{\frac{1}{2\gamma_b}+\frac{K-1}{G_p}}}\right), \tag{1}$$

where Q(.) is the complementary Gaussian error function and γ_b is the signal-to-noise ratio.

III. Multi Processing Gain Systems

Distinct users using different data rates can be allocated in the same system bandwidth B, if the processing gain is variable. Thus, high data rate implies low processing gain for a given user.

It will be assumed a multi processing gain system with K users and rates $R_1 > R_2 > ... > R_n$, with no loss of generality. All users have the same energy per bit to noise ratio γ_b and the processing gain is defined as $G_{p_i} = B/R_i$, where R_i is the bit rate for the *i*-th group with K_i users. The system is defined in a such way that $\sum_{i=1}^{n} K_i = K$. It is also assumed that random spreading sequences are utilized for all users. The performance of user *k* with rate R_k in a synchronous CDMA system using matched filter detector can be expressed as [1]*:*

$$P_{b_{ij}} = \mathrm{Q}\left\{\left[\frac{1}{2\gamma_b}+\frac{1}{G_{p_j}}\left(\sum_{i=1}^{n}\frac{R_i}{R_j}K_i-1\right)\right]^{-1/2}\right\}. \tag{2}$$

IV. Bit Error Probability of decorrelating and MMSE detectors in a single rate system on AWGN channel

In this work the result given in (2) is extended to the multiuser MMSE and decorrelating detectors. As showed in [2, 3, 4], equivalent expressions to (1) were derived as a function of the processing gain and the number of users.

The mean bit error probability for the decorrelating detector with perfect power control in a synchronous system on AWGN channel can be expressed as [3]:

$$P_b = \mathrm{Q}\left(\sqrt{2\gamma_b\left(1-\frac{(K-1)}{G_p}\right)}\right), \tag{3}$$

and for the MMSE detector P_b is given by [4]

$$P_b = \mathrm{Q}\left(\sqrt{2\gamma_b-\frac{1}{4}F\left(2\gamma_b,\frac{(K-1)}{G_p}\right)}\right), \tag{4}$$

with

$$F(x,y) = \left(\sqrt{x\left(1+\sqrt{y}\right)^2+1}-\sqrt{x\left(1-\sqrt{y}\right)^2+1}\right)^2. \tag{5}$$

V. Multiuser performance in a multirate system on AWGN channel

The mean bit error probability for the decorrelating detector in a multirate synchronous CDMA system using random spreading sequences with multi processing gain on AWGN channel is given by:

$$P_b = \mathrm{Q}\left(\sqrt{2\gamma_b\left(1-\frac{\left(\sum_{i=1}^{n}\frac{R_i}{R_j}K_i-1\right)}{G_p}\right)}\right), \tag{6}$$

and for the MMSE detector P_b can be expressed as:

$$P_b = \mathrm{Q}\left(\sqrt{2\gamma_b-\frac{1}{4}F\left(2\gamma_b,\frac{\left(\sum_{i=1}^{n}\frac{R_i}{R_j}K_i-1\right)}{G_p}\right)}\right). \tag{7}$$

VI. Numerical Results & Comparisons

Simulations were carried out in order to validate the approximations. The results of the comparisons are shown in Figures 1-2. The graphics show comparisons between the analytic expressions and simulations results. All the simulations were made

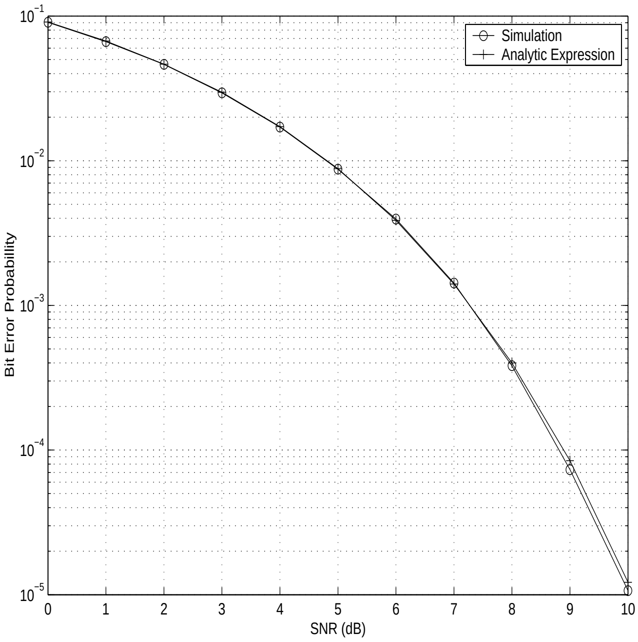

Fig. 1. Decorrelating on AWGN channel with 5 users with Gp=64 and 5 users with Gp=128.

using $K = 10$ users. The system was subdivided into two subgroups, the first with five users and processing gain $G_p = 64$ and the other with five users and $G_p = 128$. The results show that this system is equivalent to a system with 15 users and a processing $G_p = 128$ or with a fictitious 7.5 users and processing gain $G_p = 64$.

Figure 1 shows the comparison between the simulation and the analytic expression (6) for the decorrelating detector. Figure 2 shows the comparison between the simulation and the analytic expression (7) for the MMSE detector. Note that it is possible to define the parameter *equivalent number of users*, which allows to represent a multirate system as an equivalent single rate system. For instance, for the parameters used in this paper the equivalent number of users is 15.

VII. CONCLUSIONS

Approximated expressions for the mean bit error probability with random spreading sequences on AWGN channel using the multiuser MMSE and decorrelating detectors were presented. Those analytic expressions have shown to be a very good approximation when compared to the simulations.

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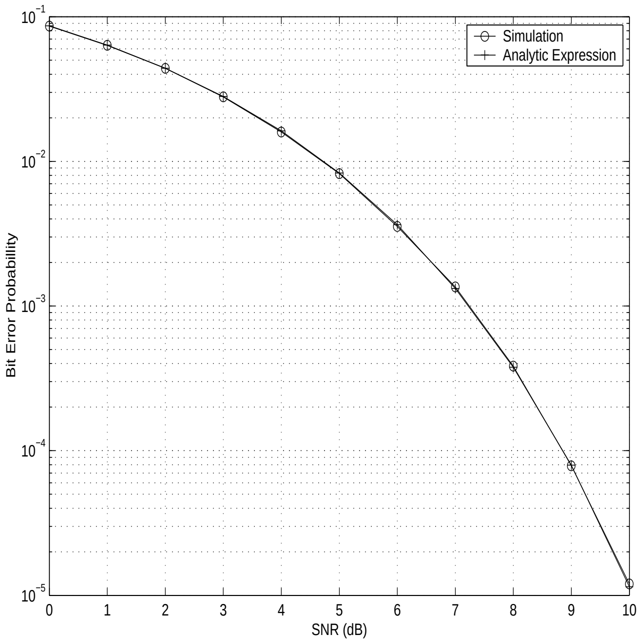

Fig. 2. MMSE on AWGN channel with 5 users with Gp=64 and 5 users with Gp=128.