# **Hierarchical Recursive Least Squares Space-Time Interference Cancellation for DS-WCDMA Systems**

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### Abstract

In this paper, a new training-based spatial-temporal receiver is proposed for an asynchronous wideband direct sequence code division multiple access (DS-CDMA) system. The presented receiver employs a hierarchical recursive least squares (HRLS) algorithm to reduce the computational complexity.

## I. Introduction

Space-time adaptive processing (see Fig.8) has been proposed for high data rate DS-WCDMA wireless communications to mitigate multiple access interference (MAI) and intersymbolic interference (ISI). The multipath signals arrive at the receiver with different angles-ofarrival (AOA) and with different delays, allowing an antenna array [1, 2, 3] to exploit this spatial and temporal signatures to combat deep fades that occur in the wireless channels. Space-time processing can improve significantly the signal-to-noise ratio (SNR).

In 3<sup>rd</sup> Generation cellular systems, pilot symbols (training) are available in the forward and reverse link and can be used for channel equalization, interference cancellation and antenna array adaptation. The training-based recursive least squares (RLS) adaptive filtering algorithm can be employed to perform this functions and reach the performance requirements of the new multimedia applications. In spite of presenting a fast convergence rate comparing to least mean squares adaptive (LMS) filtering algorithm, RLS algorithm presents a high computational complexity. In [4], a new RLS algorithm that employs a hierarchical structure was proposed. The author claims that the new method, called HRLS, can reduce convergence rate, improve performance and reduce complexity comparing with original RLS.

This paper addresses the problem of designing a spatialtemporal beamformer antenna array using HRLS algorithm to reduce the computational complexity. The objective is to find a weight vector that provides an appropriate beam pattern to the desired system user. The resulting algorithm allows combining coherently the desired signal multipath, canceling the interference signal, removing phase ambiguities present in blind algorithms and reduces the number of training symbols to obtain the filter coefficients. Comparison with training-based RLS [5] spatial-temporal beamformer is performed to confirm the improvements obtained.

The paper is organized as follows: system model is presented in section II; channel model is described in section III; RLS and HRLS beamspace-time algorithms are presented in sections IV and V, respectively; simulation results in section VI and conclusion in section VII

#### II. System Model

We consider the reverse link of an asynchronous direct sequence CDMA (DS-CDMA) system employing complex spreading [6] and OPSK data modulation to reduce peak-average ratio and achieve better bandwidth occupation. There are M users in the system and each user may transmit  $N_b$  data symbols packets over assumed stationary conditions. It is also assumed that the receiver employs an antenna array consisting of A identical elements equally-spaced by  $\lambda_{aut}/2$ .

Assuming that the inverse signal bandwidth is large compared to the travel time across the array, the complex envelopes of the signals received by different antenna elements from a given path are identical except for phase and amplitude differences that depend on the path angleof-arrival (AOA), array geometry and the element pattern [3]. The angle-of-arrival dependent phase and amplitude response of the *l*th multipath signal from the *m*th user is  $\theta_m^i$  and  $\mathbf{a}(\theta_m^i)$  is the array response vector to the multipath signal arriving from the direction  $\boldsymbol{\theta}^{l}$ with  $\mathbf{a}(\boldsymbol{\theta}_{m}^{T}) = [a_{1}(\boldsymbol{\theta}_{m}^{T}), \cdots, a_{A}(\boldsymbol{\theta}_{m}^{T})]^{T}.$ 

We can represent the reverse link baseband complex signal in the following vector form:

$$\mathbf{r}(t) = \sum_{m=1}^{M} \sum_{k=0}^{N_{s}-1} \sqrt{\gamma_{m}} \cdot b_{m}(k) \cdot \mathbf{h}_{m}(t-kT_{s}) + \mathbf{v}(t)$$
(1)

Where  $\mathbf{r}(t) = [r_1(t), \dots, r_A(t)]^T$ ;  $\gamma_m$  is the transmitted signal power of the *m*th user;  $T_s$  is the symbol duration;  $b_m(k) = (b_{m,k}^t + jb_{m,k}^o)/\sqrt{2}$  is the information symbol of the *m*th user at time *k* with  $b_{m,k}^t, b_{m,k}^o \in \{\pm 1, -1\}$ ;  $\mathbf{v}(t) = [\mathbf{v}^1(t), \dots, \mathbf{v}^A(t)]^T$  is a complex white Gaussian noise vector with variance  $\sigma^2$  and  $\mathbf{h}_m(t) = [h_m^1(t), \dots, h_m^A(t)]^T$  is the normalized complex signature waveform vector of the *m*th user, described by:

$$\mathbf{h}_{m}(t) = \sum_{n=0}^{G-1} c_{m}(n) \cdot \mathbf{p}_{m}(t - nT_{c}), \ 0 \le t \le T_{s}$$

Where  $T_c$  is the chip duration;  $G = T_s/T_c$  is the processing gain;  $c_m(n) = (v_{m,n}^t + jv_{m,n}^o)/\sqrt{2G}$  is the complex signature sequence (or spreading code) of the *m*th user at time *n* with  $v_{m,n}^t$ ,  $v_{m,n}^o \in \{+1, -1\}$  and  $\mathbf{p}_m(t) = [p_m^1(t), \dots, p_m^A(t)]^T$  is the chip waveform vector of the *m*th user that has been filtered at the transmitter and receiver and distorted by the multipath channel. We can model  $\mathbf{p}_m(t)$  as:

$$\mathbf{p}_{m}(t) = \sum_{l=0}^{L_{n}-1} \boldsymbol{\beta}_{m}^{l} \cdot \mathbf{a}(\boldsymbol{\theta}_{m}^{l}) \cdot \boldsymbol{\psi}(t-\boldsymbol{\tau}_{m}^{l})$$

Where  $L_m$  is the number of multipath components for the *m*th user;  $\beta_m^i$  and  $\tau_m^i$  are the complex gain and time delay of the *l*th path of the *m*th user; and  $\psi_{chip}(t)$  is the filtered chip waveform, which includes the effect of the transmitter and receiver filter. Finally, sampling the received signal at chip rate and assuming  $T_c = 1$ , we obtain the following discrete-time signal:

$$\mathbf{r}(n) = \sum_{m=1}^{M} \sqrt{\gamma_m} \sum_{k=0}^{N_n-1} b_m(k) \cdot \mathbf{h}_m(n-kG) + \mathbf{v}(n)$$
(2)

We consider that the receiver is in perfect synchronization with the strongest multipath component,  $l_m$ , of the desired user m ( $\tau_m^{l_m} = 0$ ) and that each of the A stacked impulse responses of  $\mathbf{p}_m(n)$  is FIR with order  $q_m$  such that  $\left[\tau_m^{\max}/T_c\right] \leq q_m \leq (L-1) \cdot G_p$ , where  $\tau_m^{\max}$  is the maximum delay spread experienced by the *m*th user and L is some integer. So, we can write the discrete-time received signal corresponding to the *k*th symbol as:

$$\mathbf{r}_{s}(k) = [\mathbf{H}(L-1), \cdots, \mathbf{H}(0)] \cdot [\mathbf{b}(k-L+1)^{T}, \cdots, \mathbf{b}(k)^{T}]^{T} + \mathbf{v}_{s}(k)$$
(3)

Where

$$\mathbf{r}_{s}(k) = \left[\mathbf{r}(k \cdot G)^{T}, \dots, \mathbf{r}((k+1) \cdot G - 1)^{T}\right]^{T}$$
$$\mathbf{H}(l) = \begin{bmatrix} \mathbf{h}_{1}(l \cdot G) & \cdots & \mathbf{h}_{M}(l \cdot G) \\ \vdots & \vdots \\ \mathbf{h}_{1}((l+1) \cdot G - 1) & \cdots & \mathbf{h}_{M}((l+1) \cdot G - 1) \end{bmatrix}$$

$$\mathbf{b}(k) = [b_1(k), \cdots, b_M(k)]^T$$
  
$$\mathbf{v}_s(k) = [\mathbf{v}(k \cdot G)^T, \cdots, \mathbf{v}((k+1) \cdot G - 1)^T]^T$$

In some cases, depending on the channel length, number of users (*M*), processing Gain (*G*) and number of antenna elements (*A*), it might be necessary to process more than one received vector at a time in order to estimate the *k*th symbol. Stacking  $\mu$  consecutive symbols ( $\mu \ge 2$ ), we can define the vector that will be processed,  $\mathbf{r}_u(k)$ , as:

$$\mathbf{r}_{\mu}(k) = \mathbf{H}_{\mu} \cdot \mathbf{b}_{\mu}(k) + \mathbf{v}_{\mu}(k); \qquad (4)$$

Where

$$\mathbf{r}_{\mu}(k) = \left[\mathbf{r}_{s}(k)^{T}, \dots, \mathbf{r}_{s}(k+\mu-1)^{T}\right]^{T}$$
$$\mathbf{b}_{\mu}(k) = \left[\mathbf{b}(k)^{T}, \dots, \mathbf{b}(k+\mu-1)^{T}\right]^{T}$$
$$\mathbf{v}_{\mu}(k) = \left[\mathbf{v}_{s}(k)^{T}, \dots, \mathbf{v}_{s}(k+\mu-1)^{T}\right]^{T}$$
$$\mathbf{H}_{\mu} = \begin{bmatrix}\mathbf{H}(L-1) & \cdots & \mathbf{H}(0) & \cdots & 0\\ \vdots & \ddots & \ddots & \vdots\\ 0 & \cdots & \mathbf{H}(L-1) & \cdots & \mathbf{H}(0)\end{bmatrix}$$

#### III. Channel Model

In this paper, we consider a microcellular environment. The microcellular multipath propagation channel for each user is represented by the geometrically based single bounced elliptical model (GBSBEM) [7]. In the GBSBEM, it is assumed that the scatterers between the base-station and each user are uniformly distributed within an ellipse. This model is suitable to microcell and picocell environments where antenna heights are low and multipath scattering can occur near the base station or near the mobile with same probability [7]. We can obtain  $\beta_m^i$ ,  $\tau_m^i$  and  $\theta_m^i$  of section II, using the procedures presented in [7]. The resulting joint probability density function of AOA and TOA (time of arrival) is given by:

$$f_{\tau,\theta}\left(\tau_{m}^{i},\theta_{m}^{i}\right) = \begin{cases} \frac{\left(d_{m}^{2}-\tau_{m}^{i}^{2}c_{v}^{2}\right)\cdot\left(d_{m}^{2}c_{v}-2\tau_{m}^{i}c_{v}^{2}d_{m}\cos(\theta_{m}^{i})+\tau_{m}^{i}^{2}c_{v}^{3}\right)}{4\pi a_{m}^{\max}b_{m}^{\max}\left(d_{m}\cos(\theta_{m}^{i})-\tau_{m}^{i}c_{v}\right)^{3}},\\ d_{m}^{2}/c_{v}<\tau_{m}^{i}\leq\tau_{m}^{\max}\\ 0, \qquad otherwise \end{cases}$$
(5)

Where  $c_v$  is the speed of light,  $d_m$  is the distance of the *m*th user to the base-station,  $a_m^{\max} = c_v \tau_m^{\max}/2$  and  $b_m = \sqrt{c_m^2 \tau_m^{\max^2} - d_m^2}$  are the major and minor axes of the ellipse containing the scatterers for the *m*th user.

In Fig.1, we present the joint AOA-TOA probability density function obtained by evaluating 100.000 scatterers for  $d_m = 500$  and  $\tau_m^{\text{max}} = 2 d_m / c_v$  [7]. The plot shows that

there is a high concentration of scatterers near the line of  $iv_{iv}$  sight with relatively small delays.



**Figure 1** Join AOA-TOA Probability Density Function ( $d_m = 500$ and  $\tau_m^{\text{max}} = 2d_m/c_v$ )

#### **IV. Recursive Least Squares Algorithm**

RLS algorithm can be used to obtain the spatial-temporal beamforming weight vector,  $\hat{\mathbf{w}}_m$ . In the following, we describe briefly the algorithm, for additional information see [5].

i. Initialize 
$$\mathbf{w}_m^0 = \mathbf{0}$$
 and  $\hat{\mathbf{R}}_{N_r}^{-1} = \delta^{-1} \cdot \mathbf{I}_{u \cdot A \cdot G_p}$  (6)

ii. Compute 
$$\mathbf{K}_{gain}(k) = \frac{\lambda^{-1} \cdot \hat{\mathbf{R}}_{N_{i}}^{-1} \cdot \mathbf{r}_{\mu}(k)^{*}}{1 + \lambda^{-1} \cdot r_{\mu}(k)^{T} \cdot \hat{\mathbf{R}}_{N_{i}}^{-1} \cdot \mathbf{r}_{\mu}(k)^{*}}$$
 (7)

iii. Determine 
$$\varepsilon(k) = b_m(k) - \mathbf{r}_\mu(k)^T \cdot \hat{\mathbf{w}}_m^{k-1}$$
 (8)

iv. Compute 
$$\mathbf{w}_{m}^{k} = \mathbf{w}_{m}^{k-1} + \mathbf{K}_{sain}(k) \cdot \varepsilon(k)$$
 (9)

v. Update 
$$\hat{\mathbf{R}}_{N_t}^{-1} = \lambda^{-1} \cdot \left\{ \hat{\mathbf{R}}_{N_t}^{-1} - \mathbf{K}_{gain}(k) \cdot \mathbf{r}_{\mu}(k)^T \cdot \hat{\mathbf{R}}_{N_t}^{-1} \right\}$$
 (10)

Where  $\hat{\mathbf{R}}$  is the autocorrelation matrix,  $\lambda$  is the forgetting factor and  $\delta$  is some small positive constant.

## V. Hierarchical Recursive Least Squares Algorithm

We can also employ HRLS algorithm [4] to obtain the spatial-temporal beamforming weight vector,  $\hat{\mathbf{w}}_m$ :

Do 
$$\eta = 1$$
 to  $N_{\eta}$  (Level)

Do 
$$\xi = 1$$
 to  $N_{\xi}^{\eta}$  (Group)

i. Initialize 
$$\mathbf{w}_{\xi}^{0} = \mathbf{0}$$
 and  $\hat{\mathbf{R}}_{\xi}^{\eta - 1} = \delta^{-1} \cdot \mathbf{I}_{u \cdot AG}$  (11)

ii. Compute 
$$\mathbf{K}_{\xi}^{\eta}(k) = \frac{\lambda^{-1} \cdot \mathbf{R}_{\xi}^{\eta-1} \cdot \mathbf{r}_{\mu}(k)^{*}}{1 + \lambda^{-1} \cdot r_{\mu}(k)^{T} \cdot \hat{\mathbf{R}}_{\xi}^{\eta-1} \cdot \mathbf{r}_{\mu}(k)^{*}}$$
 (12)

iii. Determine 
$$\varepsilon(k) = b_m(k) - \mathbf{r}_\mu(k)^T \cdot \hat{\mathbf{w}}_{\xi}^{\eta \ k-1}$$
 (13)

Compute 
$$\hat{\mathbf{w}}_{\varepsilon}^{\eta k} = \hat{\mathbf{w}}_{\varepsilon}^{\eta k-1} + \mathbf{K}_{\varepsilon}^{\eta}(k) \cdot \varepsilon(k)$$
 (14)

$$\nu. \quad \text{Update } \hat{\mathbf{R}}_{\xi}^{\eta} \stackrel{-1}{=} \lambda^{-1} \cdot \left\{ \hat{\mathbf{R}}_{\xi}^{\eta} \stackrel{-1}{=} -\mathbf{K}_{gain}(k) \cdot \mathbf{r}_{\mu}(k)^{T} \cdot \hat{\mathbf{R}}_{\xi}^{\eta} \stackrel{-1}{=} \right\} (15)$$

Where  $N_{\eta}$  is the number of hierarchical levels;  $N_{\xi}^{\eta}$  is the number of groups on the  $\eta$  level,  $\hat{\mathbf{R}}_{\xi}^{\eta}$  is the autocorrelation matrix of the group  $\xi$  on the  $\eta$  level.

#### **VI.** Simulation Results

In this section, we investigate the performance of the presented HRLS spatial-temporal beamformer in a microcell scenario using the GBSBEM model and we compare the results with the RLS spatial-temporal beamformer.

For the simulations, we consider an asynchronous DS-CDMA system with complex spreading operating at 2GHz. There are 5 QPSK modulated users (M=5) per cell, each one transmitting frames with 1000 symbols  $(N_b=1000)$ . The chip rate is 3.84 Mcps and the processing gain is 7 and 9 (G=7/9). The spreading sequences are randomly generated and are normalized to unit energy. It is considered that the frame duration is short compared with the coherence time of the channel to simplify the analysis. The cell ratio is 500 m, and the users are randomly positioned around the cell between 50 m and 500 m ( $50 \le d_m \le 500$ ) and with angles between  $-180^\circ$ and 180°. We assume for all the users that the propagation delay is 3.33  $\mu$ s (~13 chip) and  $L_m$ =4. The AOA of the strongest path of the desired signal is kept 150°  $(\theta_m^{1_m} = 150^\circ)$  for all the simulations. The base station employs a circular array antenna (see Fig.9) with equally spaced elements ( $\lambda_{ant}/2$ ) and the SNR at bit level for each antenna element is 7dB (Fig.2-5). The number of space and time elements is equal for all simulations. The results are obtained computing 1000 frames per evaluated parameter (SNR,  $N_t$  etc). For all simulations, we consider  $\mu$ =1, *L*=2,  $\lambda$ =0.99 and  $\delta$ =0.01.

In Fig.2 and Fig.4, a comparison of the bit error rate (BER) of the HRLS and RLS space-time beamformers varying the length of the training sequence for a 7 elements (A=7) and 9 elements (A=9) antenna array, respectively, is presented. In Fig.3 and Fig.5, we show the equivalent mean squares error (MSE). The results show that the proposed receiver presents a faster convergence rate although the RLS space-time beamformer presents, in the limit, better performance.

Comparison of the BER between RLS and HRLS spacetime receivers varying the SNR for a 7 ( $N_r$ =50) and 9 ( $N_r$ =100) elements antenna array (A=7/9) is presented in Fig.6 and Fig.7, respectively. For this particular condition, the performance of the proposed receiver outperforms the RLS space-time receiver.



**Figure 2** BER of space-time RLS and HRLS varying the number of training symbols for SNR=7dB (*M*=5 and *A*=7)



Figure 4 BER of space-time RLS and HRLS varying the number of training symbols for SNR=7dB (*M*=5 and *A*=9)



Figure 6 BER of space-time RLS and HRLS varying SNR (M=5, A=7 and  $N_{t}$ =50)



Figure 3 MSE of space-time RLS and HRLS varying the number of training symbols for SNR=7dB (M=5 and A=7)



**Figure 5** MSE of space-time RLS and HRLS varying the number of training symbols for SNR=7dB (*M*=5 and *A*=9)



Figure 7 BER of space-time RLS and HRLS varying SNR (M=5, A=9 and N=100)



Figure 8 Beamspace-Time Receiver



Figure 9 Circular Antenna Array

#### VII. Conclusion

In this paper, we have proposed a hierarchical recursive least squares (HRLS) spatial-temporal beamformer to reduce receiver complexity. Comparison against the recursive least squares (RLS) spatial-temporal beamformer in a microcellular environment with low mobility and high data rate users were obtained to investigate the performance of the proposed receiver.

We have performed simulations considering an asynchronous high data rate WCDMA system employing complex spreading and a circular antenna array. The results show that, although HRLS spatial-temporal beamformer do not perform join detection, the faster convergence rate compared with RLS spatial-temporal beamformer brings advantages for wireless applications. Also, the reduction in complexity of the HRLS spatial-temporal beamformer can be explored by real-time applications in next generation of wireless systems.

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