An Integrated Design for Topologies of Optical Networks

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Abstract - **Traditional approaches to wavelength routing network design divide it into two separate problems: virtual topology design (VTD), in which best connections among nodes are derived from traffic demand; and routing-and-wavelength assignment (RWA), in which physical paths are accommodated in the physical topology to support the requested connections. We propose an iterative linear programming approach to solve both problems jointly under multiple objectives such as congestion avoidance, fiber load and wavelength pool minimization. The solution of the VTD problem generates a request for a set of paths to be supplied by the physical topology. Physical paths are then allocated in order to minimize some objective function that is akin to a linear programming formulation.**

I. INTRODUCTION

Wavelength Division Multiplexing (WDM) shares the large bandwidth available in optical fibers into multiple channels, each one operating at different wavelengths and at specific data rates (up to 40Gbps). Due to current advances in WDM and high-speed eletronic routing/switching, it is likely to be the case that next-generation broadband networks will employ a hybrid, layered architecture, using both optical WDM and electronic switching technologies. In such networks, a significant gap exists between the huge transmission capacity of WDM fibers and the eletronic switching capacity. This is commonly called the *eletronic bottleneck* [1,7].

Although the combination of the emerging ATM technology with the new SDH (Synchronous Digital Hierarchy) transport network constitutes a widening of this bottleneck, it is not enough to eliminate it. At this moment, it seems that the viable technology to bridge this speed gap is WDM. In a networking environment, enabled by optical crossconects and a whole new family of emerging photonic devices, the wavelengths may then be all-optically routed to different destinations in the network.

Modulated on different wavelengths, different packets can share the fiber without any risk of collision, but requiring a coordination of the wavelength tunings of transmission at the source node and of reception at the destination node, so that connectivity is provided between them. For this purpose, wavelength conversion may also have to be provided and dynamically assigned at intermediate nodes.

In order to engineer such wavelength coordination, many studies are being accomplished for design of optical WANs, especially in the problem of designing the virtual topology (VTD) to be overlayed on optical networks. The architectural framework assumes transparent clear channels called *lightpaths*, so named because they traverse several physical links without ever leaving the optical domain from end to end [4,5].

In general, VTD and RWA problems can be formulated as optimization problems aimed at maximizing network throughput or other performances measures of interest. Typically, the exact solution can be easily shown to be NP-hard, and heuristic approaches are needed to find realistic good solutions. For this purpose, the problem can be decomposed into two subproblems. The first is to decide what virtual topology to embed on a given physical topology, that is, what are the lightpaths to be implemented. The second is the routing–and-wavelength assignment (RWA) of these lightpaths. The routing of packet traffic on the lighpaths is also usually seen to be a part of the VTD problem.

In the following we present the solution approach. Section 2 explains the virtual and physical topology. Sections 3 and 4 show a precise formulation for the VTD and RWA. Section 5 exemplifies the application on a network example, and finally section 6 draws some conclusions.

II. PROBLEM STATEMENT

Although lightpaths underlay SDH networks in a natural way, packet- and cell- switching client networks, like ATM and IP, would be better served by more packet-oriented WDM layer mechanisms and protocols. However, current optical packet-switching technologies do not yet deliver the same performance that is possible in electronic networks. By looking for the best possible circuit configuration for the traffic demand at any given time, optimizing the virtual topology with constraints of physical topology mitigates the impairments caused by the inability to switch packets on individual basis.

A physical topology is a graph representing the physical interconnection of the wavelength routing nodes by means of fiber-optic cables. In Fig. 1 is show a physical topology of a six-node- wide-area network. The wavelength routing nodes are numbered from 0 to 5. We consider an edge in the physical topology to represent a pair of fibers, one in each direction.

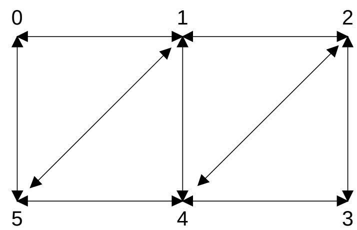

Fig. 1. Illustrative example of a six-node network, physical topology.

The set of all unidirectional lightpaths (calleds b_{ij}*'s*) set up among the access nodes is the virtual topology. For example, Fig. 2 shows a possible virtual interconnection. There is an edge in the virtual topology between node 2 and node 0 when the data or packets from node 2 to node 0 traverse the optical network in the optical domain only, i.e., undergo no electronic conversion in the intermediate wavelength routing nodes. Edges in a virtual topology are called virtual links.

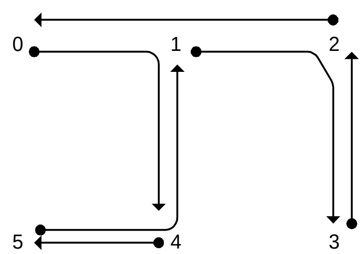

Fig.2. Virtual topology

For example, in Fig. 2 data from node 2 to node 0 are sent on lightpath b_{20} through the wavelength routing node at 1. Simultaneously, we can send a packet from node 1 to node 3 through the wavelength routing node at 2. We see that even though in the physical topology there is a fiber connection between node 2 and node 4, to send a packet from node 2 to node 4 we would have to use two virtual links b_{20} and b_{04} . We say that *hop length* of virtual link b_{20} is two as it traverses two physical edges, (2,1) and (1,0). See [3,9] for a detailed explanation of this type of routing.

Ideally in a network with N nodes, we would like to setup lightpaths between all the $N(N-1)$ pairs. However this is usually not possible because of two reasons. First, the number of available wavelengths imposes a limit on how many lightpaths can be set up. (this is also a function of the traffic distribution). Secondly, each node can be the source and sink of a limited number of lightpaths [6].

III. Formulation of the Problem

We formulate the joint VTD and PTD (same as RWA) problems as an optimization problem. The problem of embedding a desired virtual topology on a given physical topology (fiber network) was formally stated as an **exact linear programming** formulation.

A) Notation:

- i and j denote *originating* and *terminating* nodes, respectively, in a lightpath.
- m and n denote endpoints of a physical link that might occur in a lightpath.

B) Given:

- Number of nodes in the network: N.
- Number of transmitters at node i: T_i ($T_i \geq 1$). Number of receiver at node i: R_i ($R_i \geq 1$).
- Traffic matrix Λ_{sd}: Which denotes the average rate of traffic flow from s to node d.
- Capacity of each channel: C (normally expressed in bits/second).
- Maximum loading per channel: β, $0<\beta<1$. β restricts the queuing delay on a lightpath from getting unbounded by avoiding excessive link congestion.
- Number of wavelengths available: $\varsigma = 1, 2.... W$
- Physical Topology (P_{mn}): Denotes the number of fibers interconnecting node m and n. $P_{mn} = 0$ for nodes which are note physically adjacent to each other. $P_{mn} = P_{nm}$ indicates that there are equal number of fibers joining two nodes in different directions. Note that there may be more than one fiber link connecting adjacent nodes in the network. $\sum_{mn} P_{mn} = M$ denotes the total number of fiber links in the network.

C) Variables:

- Load: L is the maximum load needed in any fiber.
- Lightpath: The variable: $b_{ij} = 1$ if there exists a lightpath from node i to node j in the virtual topology; $b_{ij} = 0$ otherwise. Note that this formulation is general since lightpaths are not necessarily assumed to be bidirectional, i.e., $b_{ij} = 1 \nRightarrow b_{ji} = 1$. Moreover, there may be multiple lightpaths between the same source-destination pair, i.e., $b_{ij} > 1$, for the case when traffic between nodes i and j is greater than a single lightpaths's capacity.
- Traffic routing: The variable λ_{ij}^{sd} denotes the amount of traffic flowing from node s to node d and employing b_{ij} as an intermediate virtual link.

- Physical topology route: The variable p_{mn}^{ij} denotes the number of lightpaths between nodes *i* and *j* being routed though fiber link *m-n*.
- $b_{ij\varsigma}$ = Number of lightpaths between node *i* and node *j* uses wavelength ς; for $\varsigma = 1,2,3,...,W$.
- Wavelength assignment variables: $p_{mn\varsigma}^{ij}$ = 1, if the lightpath between node *i* and *j* uses wavelength ς; and is routed through physical link *m-n*.

D) Virtual Topology Design (VTD)

- Objective:

Minimize: $$\frac{1}{\sum_{sd} \Lambda_{sd}} \sum_{ij} \sum_{sd} \lambda_{ij}^{sd} \qquad (3.1)$$

- On virtual topology connection matrix:

$$\sum_j b_{ij} \leq T_i ,....\forall_i \qquad (3.2)$$

$$\sum_i b_{ij} \leq R_j ,....\forall_j \qquad (3.3)$$

- On virtual topology traffic variables:

$$\sum_j \lambda_{sj}^{sd} = \Lambda_{sd} \qquad (3.4)$$

$$\sum_i \lambda_{id}^{sd} = \Lambda_{sd} \qquad (3.5)$$

$$\sum_i \lambda_{ik}^{sd} = \sum_j \lambda_{kj}^{sd} ....se....k \neq s,d \qquad (3.6)$$

$$0 \leq \lambda_{ij}^{sd} \leq \Lambda_{sd}.\{1+\text{sgn}(b_{ij} - 0{,}5)\}/2 \qquad (3.7)$$

$$\sum_{sd} \lambda_{ij}^{sd} \leq \beta.C.b_{ij} \qquad (3.8)$$

Int b_{ij}

E) Physical Topology Design (PTD)

- E.1) Routing on physical topology p_{mn}^{ij} :

$$\sum_m p_{mk}^{ij} = \sum_n p_{kn}^{ij} ,....if....k \neq i,j \qquad (3.9)$$

$$\sum_n p_{in}^{ij} = b_{ij} \qquad \forall i,j \qquad (3.10)$$

$$\sum_m p_{mj}^{ij} = b_{ij} \qquad \forall i,j \qquad (3.11)$$

$$\sum_{ij} p_{mn}^{ij} \leq L.P_{mn} \qquad \forall m,n \qquad (3.12)$$

Int p_{mn}^{ij}

- E.2) On coloring lightpaths:

$$\sum_n p_{\ln\varsigma}^{ij} = \sum_m p_{ml\varsigma}^{ij}, \qquad \forall i,j,\varsigma \ if \ l \neq i,j \qquad (3.13)$$

$$\sum_n p_{in\varsigma}^{ij} = b_{ij\varsigma} \qquad \forall i,j,\varsigma \qquad (3.14)$$

$$\sum_m p_{mj\varsigma}^{ij} = b_{ij\varsigma} \qquad \forall i,j,\varsigma \qquad (3.15)$$

$$\sum_{ij} p_{mn\varsigma}^{ij} \leq P_{mn} \qquad \forall m,n,\varsigma \qquad (3.16)$$

$$\sum_{\varsigma} b_{ij\varsigma} = b_{ij} \qquad \forall i,j \qquad (3.17)$$

$$\sum_{\varsigma} p_{mn\varsigma}^{ij} = p_{mn}^{ij} \qquad \forall i,j,m,n. \qquad (3.18)$$

Int $p_{mn\varsigma}^{ij}$, $b_{ij\varsigma}$

F) Explanation

In VTD the objective function minimizes the average packet hop distance in the network. The (3.1) is a linear objective function. Eqs. (3.2) and (3.3) ensure that the number of lightpaths emerging from a node is constrained by the number of transmitters at that node, while the number of lightpaths terminating at a node are constrained by the number of receivers at that node. Eqs. (3.4)-(3.6) are multicommodity-flow equations governing the flow of traffic through the virtual topology: (3.7) ensures that traffic can only flow through an existing lightpath, while (3.8) specifies the capacity constraint in the formulation.

In PTD, section E.1, (3.9)-(3.11) are multicommodity-flow equations governing the routing of lightpaths from source to destination. Eq. (3.12) ensures that the number of lightpaths flowing through a fiber link does not exceed *L*.

In PTD, section E.2, (3.13)-(3.15) ensure that a wavelength ς is conserved at every node for a lightpath. In (3.16) we are assured that there is no wavelength clash at physical link, i.e., no two virtual links traversing through the physical link will be assigned the same wavelength. Eq. (3.17) requires that a lightpath be one color only and (3.18) the number of wavelengths present in each physical link is equal to the number of lightpaths traversing it.

IV. Approach for Integrated Design

The full problem was decomposed into two subproblems, VTD and RWA. The fully integrated problem is solved by the algorithm below:

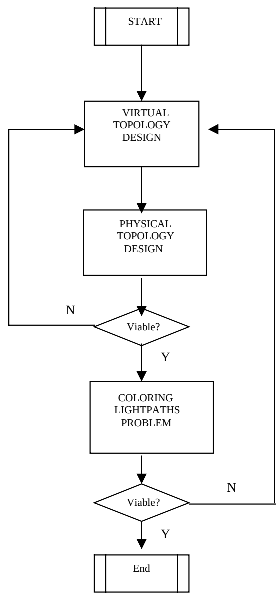

Fig. 3. Heuristic MinW

Notice that solving VTD is the same as finding the values of the b_{ij}*'s* variables. Solving for the physical topology is the same as finding the values of p^{ij}_{mn} variables, and to decide the coloring of the lightpaths means to find of $p^{ij}_{mn\varsigma}$ variables. If each one, separately, results in viable solutions, the joint problem is solved. Otherwise, if one of subproblems is not feasible, a new virtual topology must be designed and the algorithm is executed again.

It is interesting to observe that each one of the problems above must be formulated to optimize a different objective function. The VTD, as expressed in (3.1), will be formulated to minimize the average number of virtual hops in the network, which is the same as the maximization of efficiency of resources in the network, as well as the minimization of electronic processing (switching speed) at the nodes. The design of the physical topology has the objective of minimizing L, and, if there is *wavelength-continuity constraint*, the lightpath coloring problem aims at minimizing W [2].

The viability of the solutions is characterized by the minimum values of L and W. Respectively, to the maximum load of fibers and to the pool of available wavelengths.

Each time that will be evidenced the unfeasibility of the solution of the physical topology or of the coloring, the rollback to the problem of the virtual topology asks for the next best solution, after the ones that resulted in unfeasible solutions in lower layers. Therefore, the algorithm used for the solution of the VTD must be capable to list solutions for b_{ij}*'s* in the order of their closeness to the optimal value of the objective function.

V. Numerical Results

For simulation, using the described strategy in the previous section of the following form:

1.) <u>VTD</u>: With the HVDT (Heuristic for the Virtual Topology Design), to find the b_{ij}*'s* for one given matrix of traffic Λ_{sd}. The heuristic attempts to establish lightpaths between source-destination pairs with the highest Λ_{sd} values, subject to constraints of the virtual degree. We find more interesting to decide the VTD as the HVDT, abandoning the equations (3.1)-(3.8), because the HVDT is a classic heuristic [6]. However, the traffic can easily be routing through b_{ij}*'s* obtained in this step through multicommodity [8].

2.) <u>PTD</u>: Solve (3.9)-(3.12) with a objective function (3.20): Min L. *Our objective is to minimize the maximum load needed in any fiber in the network in order to establish a certain set of lightpaths for a given physical topology*.

3.) <u>Coloring</u>: Given L, re-optimize (3.9)-(3.18) with a objective function (3.20): Min $\sum p^{ij}_{mn}$. It is need because maximum fiber load may be oblivious to the persistence of cycles in paths, which may even be dismembered from the source-to-destination link sequence. These anomalies may be eliminated by re-optimizing the solution using the total number of hops as a new objective function. Moreover the coloring is carried through. Therefore we add the equations (3.13)-(3.18). We assume a certain availability of wavelengths and get the minimum number of wavelengths necessary to color the lightpaths.

Then:

Consider the traffic matrix Λ_{sd} of Table 1, obtained through the generation of random numbers between 0 and 1 with a gaussian distribution of $\mu = 0.5$ e $\sigma = 0.1$.

Λ_{sd}	*0*	*1*	*2*	*3*	*4*	*5*
0	-	0.90	0.62	0.51	0.28	0.52
1	0,53	-	0,39	0,92	0,26	0,15
2	0,47	0,31	-	0,34	0,21	0,14
3	0,29	0,48	0,34	-	0,99	0,36
4	0,15	0,44	0,14	0,84	-	0,99
5	0,48	0,19	0,99	0,75	-	0,18

Table.1. Traffic matrix

We then get the following virtual links matrices of b_{ij}*'s*, for the virtual degree 1 and 2:

b_{ij}	*0*	*1*	*2*	*3*	*4*	*5*
0	-	1	-	-	-	-
1	-	-	-	1	-	-
2	1	-	-	-	-	-
3	-	-	-	-	1	-
4	-	-	-	-	-	1
5	-	-	1	-	-	-

Table. 2. Degree 1

b_{ij}	*0*	*1*	*2*	*3*	*4*	*5*
0	-	2	-	-	-	-
1	-	-	-	2	-	-
2	2	-	-	-	-	-
3	-	-	-	-	2	-
4	-	-	-	-	-	2
5	-	-	2	-	-	-

Table. 3. Degree 2

Fig. 4 shows the cycles formed for virtual degree “1”. This justifies the re-optimization previously proposal in steep 3. Moreover the shortest path always is the chosen one, for example b_{34} only needs a physical hop after of the re-optimization. These two factors guarantee a minimum number of *hop length* in the network. Therefore, the success of the re-optimization is guaranteed.

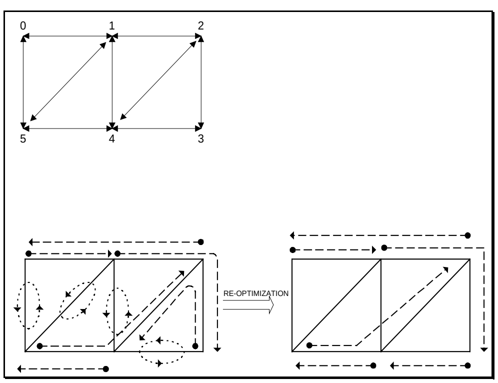

Fig. 4. Re-optimization, for degree virtual “1” . Only one wavelength is necessary.

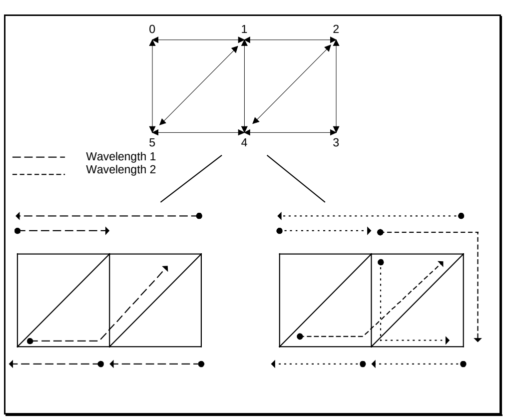

Fig. 5. Degree virtual “2”.

In Fig. 5 the wavelengths had been separate in subnetworks for better visualization. For example, there are two parallels lightpaths from node 0 to node 1, implying that b_{011}=1 and b_{012}=1.

In Fig. 6 we observe the minimum number of wavelengths as a function of the virtual degree. If for one degree we have availability of less wavelengths of that the one shown, there is no feasible solution.

Fig. 6 also shows a physical topology degree bound. This bound is derived from the simple consideration that each node in the virtual topology must source a number of lightpaths equal to the virtual degree of the virtual topology Δ_l. Considering the node with the minimum physical degree δ_p in the physical topology, there must be sufficient wavelengths to allow Δ_l lightpaths to be realized over δ_p physical links, that is, the number of wavelengths required is bounded from below as [6,9]:

$$W \geq \lceil \Delta_l / \delta_p \rceil \qquad (3.21)$$

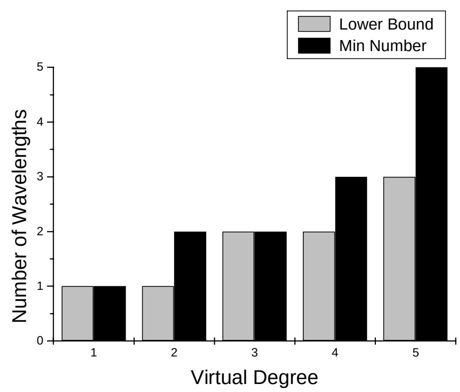

Fig. 6. Number of wavelengths x virtual degree

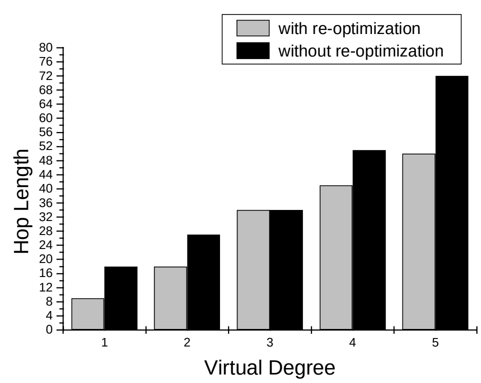

Fig. 7. Number of hop length x virtual degree

In Fig. 7 we observe the efficiency of the re-optimization to eliminate the cycles of the network and to find shorter paths for lightpaths, decreasing sufficiently the number of *hop lenght*. Notice that for degree 3, the number of *hop length* remained constant. In this case the re-optimization would not be necessary. All results, including maximum fiber load, are summarized in the following table:

Δ_l	*L*	W_{min}	*HL*	*HL/O*
1	1	1	9	18
2	2	2	18	27
3	2	2	34	34
4	3	3	41	51
5	4	5	50	72

Table. 4

Legend:

- Δ_l **:** Virtual degree.
- ***L*****:** Maximum fiber load.
- W_{min} **:** Minimum number of wavelengths necessary.
- ***HL*****:** Number of *hop length* with the re-optimization.
- ***HL/O*****:** Number of *hop length* without the re-optimization.

VI. CONCLUSIONS

In this paper we propose an iterative linear programming approach to solve virtual and physical topology design.

The solution of the VTD problem generates a request for a set of paths to be supplied by the physical topology. Physical paths are then allocated in order to minimize the maximum fiber load. This may be oblivious to the persistence of cycles in paths, which may even be dismembered from the source-to-destination link sequence.

These anomalies may be eliminated and the mean *hop length* reduced by re-optimizing the solution using the total number of hops as a new objective function, subject to the minimal value of maximum fiber load that was determined in the previous optimization step. The final design phase is the assignment of wavelengths to paths.

In spite of the complexity of the design described above, it only serves to get the best static solution. For the case of dynamic traffic, one needs to set up a connection for each request as it arrives, and the lightpath is released after some finite, random amount of time. The routing and wavelength assignments must then preserve enough open capacity to avoid blocking of future requests. Traffic statistics must then be taken into account, raising the need for good traffic models that incorporate emerging self-similarity features generated by the Internet.

Our formulation was with no wavelength changers. But a formulation with resources of wavelength conversion, will be provided in a next paper.

ACKNOWLEDGMENTS

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