Adaptive LCMV Beamforming Avoiding DOA Estimation for Packet-like Wireless Systems

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Abstract — A new adaptive LCMV solution that avoids DOA estimation for uplink beamforming is proposed. The use of the uplink channel covariance matrix (UCCM) instead of DOA estimation is shown to be a suitable alternative for constraints selection. The resulting LCMV adaptive solutions are then based on the LMS and RLS versions with the UCCM estimation. The new method is then evaluated for packed-like wireless systems through computational simulations.

I. INTRODUCTION

Signal detection is a very interesting field in mobile communications due to the importance of correctly recovering the desired signal. This detection can be performed both in the uplink and downlink. However, due to some system features, the detection in uplink is preferable.

In the uplink context of a mobile communication system, where several users share the same resources, an important strategy to separate the signals of interest is the use of beamforming techniques. Some works have proposed the use of such a strategy with decoupled space-time processing [1]. The approach consists in providing interference cancelling by means of spatial processing, so that the recovered signal be feed into a temporal equalizer, since this signal is still subject to intersymbol interference (ISI).

In a previous work [2], two different optimization criteria for beamforming have been investigated: the so-called *Linearly Constrained Minimum Variance* (LCMV) and the *Summed Inverse Carrier to interference Ratio* (SICR). SICR demands an a priori knowledge about the uplink channel covariance matrix (UCCM), while LCMV needs a previous estimation of the angle or direction of arrival (DOA) of the desired users. One interesting result of [2] states that when one single eigenvectorial constraint (EV) is used for the LCMV solution, the performance of both criteria are practically the same, but LCMV provides adaptive solutions for performing beamforming.

The use of an adaptive approach is particularly interesting, since in a mobile system the users' characteristics can vary in according to the employed strategy. For instance, frequency hopping (FH) is a strategy for interference reduction largely applied in practical wireless systems. From one slot to another the spatial interference pattern of the desired user changes, since the mobile user that shares the same frequency of the desired user changes every slot. Slotted-packet networks causes the same change in the spatial interference pattern since users transmit over time slots sorted randomly (S-Aloha networks) [3].

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This work was supported by the Ericsson Research Brazilian Branch under the ERBB/UNI.33 technical cooperation contract. Both previously mentioned situations can be seen as *packet-like* wireless systems since the spatial interference pattern can suddenly change. In such systems, the tracking capability is a highly important issue in the design of good signal detection strategies.

In the present work, the application of the LCMV adaptive solutions (namely LMS and RLS) is investigated and compared with the LCMV batch solution. Also, an alternative strategy concerning the implementation of the adaptive LCMV algorithm with EV constraint is proposed. It consists in using the UCCM, instead of the DOA estimation, for constructing the matrix of constraints to be applied in the adaptation process. Simulations are performed for different scenarios of user speed in a packet-like environment, in order to evaluate the performance of the proposed method.

The rest of the paper is organized as follows. Section II is devoted to a recall on the batch solutions of the LCMV and SICR optimization criteria. Section III presents the novel LCMV method, which avoids DOA estimation by using UCCM to construct the matrix of constraints. Section IV shows the formulation of the corresponding adaptive versions of the proposed method. Section V presents simulations results to illustrate our analyze. Finally, conclusions are stated in Section VI.

II. RECALLS ON SICR X LCMV BATCH SOLUTIONS

Here we present some recalls about the most important characteristics of the constrained criteria shown in [2], in order to provide some background about them.

A. SICR Solution

The SICR was initially proposed by Zetterberg in [4] and discussed by Asté in [5] as a downlink criterion for the maximization of the carrier-to-interference ratio (CIR), for all co-channel mobile users. However, the resolution of the downlink problem also leads to the determination of the uplink beamforming weights [6].

In this work, we will use the SICR solution in uplink, where the beamforming weights can be independently performed as follows:

$$\mathbf{w}_{k} = \operatorname*{arg\,max}_{\mathbf{w}_{k}} \frac{\mathbf{w}_{k}^{H} \mathbf{R}_{k} \mathbf{w}_{k}}{\mathbf{w}_{k}^{H} \mathbf{R}_{k, \mathrm{int}} \mathbf{w}_{k}}$$
(1)

where \mathbf{R}_k is the uplink channel covariance matrix (UCCM) of the *k*th user and $\mathbf{R}_{k,\text{int}} = \mathbf{R} - \mathbf{R}_k = \sum_{i,i \neq k} \mathbf{R}_i + \sigma_n^2 \mathbf{I}$ is the *k*th's interferers plus noise uplink covariance matrix. **R** is the total (for all users) UCCM, \mathbf{w}_k is the uplink beamformer weight vector for the *k*th user and σ_n^2 is the thermal noise power per antenna element at the base station. This minimization procedure can be solved by using Lagrange multipliers. The solution is the unit norm generalized eigenvector of $[\mathbf{R}_k, \mathbf{R}_{k,\text{int}}]$ corresponding to the largest eigenvalue. Such criterion also corresponds to

$$\mathbf{w}_{k} = \operatorname*{arg\,min}_{\mathbf{w}_{k}} \left\{ \mathbf{w}_{k}^{H} \mathbf{R} \mathbf{w}_{k} \right\} \left| \mathbf{w}_{k}^{H} \mathbf{R}_{k} \mathbf{w}_{k} = c \qquad (2)$$

where c is an arbitrary constant.

B. LCMV Solution

The LCMV criterion minimizes the output power of the beamformer, by considering incident signals in all directions, under the constraint of fixed gains in some given values of DOA. So the array is set to receive from the desired user while minimizing interferers arriving from other directions. This criterion is expressed by:

$$\mathbf{w}_{k} = \operatorname*{arg\,min}_{\mathbf{w}_{k}} \left\{ \mathbf{w}_{k}^{H} \mathbf{R} \mathbf{w}_{k} \right\} \mid \mathbf{C}_{k}^{H} \mathbf{w}_{k} = \mathbf{f}_{k}$$
(3)

where C_k is the matrix of constraints for the *k*th user and f_k is the response vector for the *k*th user.

Such method just takes into account the corresponding DOAs for each user by introducing point constraints for each direction. The matrix C_k is then formed as follows:

$$\mathbf{C}_{k} = \begin{bmatrix} \mathbf{d}(\theta_{k,1}) & \mathbf{d}(\theta_{k,2}) & \cdots & \mathbf{d}(\theta_{k,L}) \end{bmatrix}$$
(4)

where $\mathbf{d}(\theta_{k,l})$ is the steering vector corresponding to the *l*th considered DOA of user *k* and *L* is the number of constraints. The response vector is given by:

$$\mathbf{f}_{k} = \begin{bmatrix} 1 & \sqrt{\gamma_{k,2}} & \cdots & \sqrt{\gamma_{k,L}} \end{bmatrix}^{T}$$
(5)

where $\gamma_{k,l}$ is the relative power of the *l*th path with respect to the first one, for user *k*.

Thus, this approach requires the knowledge of both DOA and power for each user multipath, in order to construct the constraints. Besides, in order to deal with non-null angular spread, two types of constraints may be posed:

• Point constraints: a number of point constraints is inserted in the angle bandwidth. This approach is limited due to the number of degrees of freedom, which depends on the number of antennas in the array [2].

• Eigenvectorial constraints (EV): cope with the number of degrees of freedom by using the most significative singular vectors (associated with the most significative singular values) of the matrix of constraints.

The resultant criterion for the LCMV with EV, denoted by $LCMV_{EV}$, is given by the singular value decomposition of the matrix of constraints used in (3).

Previous simulation results [2] have shown that the use of only one EV (called EV1) provides better results. Figure 1 shows the performance in terms of SNIR (Signal to Noise plus Interference Ratio). The CDF(SNIR) states for the cumulative function distribution of SNIR. It means the probability of the SNIR be lower than or equal to a given value (in the horizontal axis). So, better performance results are represented by curves located more to the right side.



Fig. 1. Comparison between the SICR and $\rm LCMV_{EV1}$ for $\rm AS=1^\circ,5^\circ$ and $10^\circ.$

III. LCMV AVOIDING DOA ESTIMATION

In order to compute the eigenvectorial constraint, the matrix of constraints is formed by the directional vectors corresponding to the estimated DOA of the desired signals, as stated in equation (4). Then, the singular value decomposition (SVD) may be directly applied to the matrix of constraints so that a more simple representation of \mathbf{C}_k^H be given by:

$$\mathbf{C}_{k}^{H} = \mathbf{U}_{k} \begin{bmatrix} \boldsymbol{\Sigma}_{k} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \mathbf{V}_{k}^{H}$$
(6)

with

$$\mathbf{U} = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \cdots & \mathbf{u}_L \end{bmatrix}$$
$$\mathbf{V} = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \cdots & \mathbf{v}_M \end{bmatrix}$$
(7)
$$\mathbf{\Sigma} = \operatorname{diag}\left(\sigma_1, \sigma_2, \dots, \sigma_P\right)$$

where U and V are unitary matrices, which contain respectively the left and right singular vectors; and Σ is a diagonal matrix composed by the singular values of \mathbf{C}_k^H sorted as $\sigma_1 \ge \sigma_2 \ge$ $\ldots \ge \sigma_P > 0$.

As mentioned before, the best performance is obtained with the use of only one eigenvectorial constraint [2]. That is, with the use of the singular vector associated with the maximum singular value. Thus, the EV constraint and the corresponding response element are:

$$\mathbf{C}_{k} = \mathbf{v}_{1}$$

$$\widetilde{\mathbf{f}}_{k} = \frac{1}{\sigma_{1}} \mathbf{u}_{1}^{H} \mathbf{f}_{k}$$
(8)

It is important to highlight that v_1 is the eigenvector associated with the maximum eigenvalue of $C_k C_k^H$. So, this matrix can be written as follows:

$$\mathbf{C}_{k}\mathbf{C}_{k}^{H} = \sum_{l=1}^{L} \mathbf{d}^{H}\left(\theta_{k,l}\right) \mathbf{d}\left(\theta_{k,l}\right)$$
(9)

TABLE I	
LCMV-LMS	

• Initialization

 $\mathbf{w}_k(0) = \mathbf{q}_k \tag{11}$

• Update the antenna weights

$$\mathbf{w}_k(n+1) = \mathbf{P}_k \left[\mathbf{w}_k(n) - \mu y(n) \mathbf{x}(n) \right] + \mathbf{q}_k \tag{12}$$

It is worth to notice the similarity between this matrix and the UCCM matrix of the k - th user:

$$\mathbf{R}_{k} = \sum_{l=1}^{L_{T}} g(\theta_{k,l}) \mathbf{d}^{H}(\theta_{k,l}) \mathbf{d}(\theta_{k,l})$$
(10)

where $g(\theta_{k,l})$ is the mean power of the received signal in direction $\theta_{k,l}$ and L_T is the total number of multipaths linking the mobile k and the base station.

Moreover, as far as the number of considered DOAs L in (4) and (9) becomes greater and close to L_T , the constraints will better express the spatial channel between the base station and the mobile. This leads to a better performance for the LCMV solution.

Therefore, we propose to take the UCCM matrix in (10) as the best representation of $\mathbf{C}_k \mathbf{C}_k^H$. So, the EV constraint $\widetilde{\mathbf{C}}_k$ in (8) becomes the eigenvector associated with the maximum eigenvalue of the matrix \mathbf{R}_k and the response element $\widetilde{\mathbf{f}}_k$ can be set to an arbitrary value.

Hence, the LCMV without DOA estimation solution, socalled LCMV_{CM} (LCMV Covariance Matrix), consists of the following steps for each user k:

1. Estimation of \mathbf{R}_k

2. Computation of C_k , the eigenvector associated with the maximum eigenvalue of R_k .

3. Computation of the beamforming weight:

$$\mathbf{w}_k = \mathbf{R}^{-1} \widetilde{\mathbf{C}}_k \left(\widetilde{\mathbf{C}}_k^H \mathbf{R}^{-1} \widetilde{\mathbf{C}}_k \right)^{-1}$$
, where $\widetilde{\mathbf{f}}_k$ was set to 1.

IV. ADAPTIVE LCMV

The results of our previous work [2] motivated the investigation of the performance of adaptive versions of the LCMV criterion, since they may be more suitable for tracking changes in the channel parameters. Furthermore, the LCMV_{CM} solution was used in order to derive the adaptive solutions.

A. LCMV-LMS

The LMS adaptive version of the LCMV criterion was firstly proposed by Frost in [7] and uses an instantaneous estimation of the total covariance matrix \mathbf{R} .

This algorithm is shown in table I for user k, where $\mathbf{x}(n)$ is the sampled signal at the antenna at instant n, y(n) is the array output defined by $y(n) = \mathbf{x}^{H}(n)\mathbf{w}_{k}(n)$, while the vector \mathbf{q}_{k} and the matrix \mathbf{P}_k are defined by:

$$\mathbf{q}_{k} \triangleq \widetilde{\mathbf{C}}_{k} \left(\widetilde{\mathbf{C}}_{k}^{H} \widetilde{\mathbf{C}}_{k} \right)^{-1} \widetilde{\mathbf{f}}_{k}$$
(13)

$$\mathbf{P}_{k} \triangleq \mathbf{I} - \widetilde{\mathbf{C}}_{k} \left(\widetilde{\mathbf{C}}_{k}^{H} \widetilde{\mathbf{C}}_{k} \right)^{-1} \widetilde{\mathbf{C}}_{k}^{H}$$
(14)

B. LCMV-RLS

The RLS version of the LCMV criterion was proposed by Resende in [8]. The algorithms is initialized by posing the covariance matrix as a diagonal one and then obtaining the initial values for the antenna weights, in according to the LCMV criterion. Afterwards, the weights are computed as indicated in table II for user k. More considerations on the role of each intermediary variable in such summarized table are found in [8].

TABLE II LCMV-RLS

• Initialization

$$\tilde{\mathbf{R}}(0) = \delta \mathbf{I}_M \tag{15}$$

$$\mathbf{Q}_{k}(0) = \widetilde{\mathbf{R}}^{-1}(0)\widetilde{\mathbf{C}}_{k} \left(\widetilde{\mathbf{C}}_{k}^{H}\widetilde{\mathbf{R}}^{-1}(0)\widetilde{\mathbf{C}}_{k}\right)^{-1}$$
(16)

$$\mathbf{w}_k(0) = \mathbf{Q}_k(0)\mathbf{f} \tag{17}$$

· Compute the adaptation gain

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$$\mathbf{g}(n+1) = \frac{\widetilde{\mathbf{R}}^{-1}(n)\mathbf{x}(n+1)}{\lambda + \mathbf{x}^{H}(n+1)\widetilde{\mathbf{R}}^{-1}(n)\mathbf{x}(n+1)}$$
(18)

$$\widetilde{\mathbf{R}}^{-1}(n+1) = \frac{1}{\lambda} \left[\widetilde{\mathbf{R}}^{-1}(n) - \mathbf{g}(n+1)\mathbf{x}^{H}(n+1)\widetilde{\mathbf{R}}^{-1}(n) \right]$$
(19)

• Update the matrix $\mathbf{Q}(n+1)$

$$\mathbf{u}_k(n+1) = \widetilde{\mathbf{C}}_k^H \mathbf{g}(n+1) \tag{20}$$

$$r_k^H(n+1) = \mathbf{x}^H(n+1)\mathbf{Q}_k(n)$$
(21)

$$\mathbf{Q'}_k(n+1) = \left[\mathbf{Q}_k(n) - \mathbf{g}(n+1)\mathbf{v}_k^H(n+1)\right] \cdot \left[\mathbf{Q}_k(n+1) - \mathbf{g}(n+1)\mathbf{v}_k^H(n+1)\right] \cdot \left[\mathbf{Q}_k(n+1) - \mathbf{g}(n+1)\mathbf{v}_k^H(n+1)\right] \cdot \left[\mathbf{Q}_k(n+1) - \mathbf{g}(n+1)\mathbf{v}_k^H(n+1)\right] \cdot \left[\mathbf{Q}_k(n) - \mathbf{Q}_k(n) - \mathbf{Q}_k(n)\right] \cdot \left[\mathbf{Q}_k(n) - \mathbf{Q}_k(n)\right] \cdot \left[\mathbf{Q}_k(n$$

$$\cdot \left[\mathbf{I}_{K} + \frac{\mathbf{u}_{k}(n+1)\mathbf{v}_{k}^{H}(n+1)}{1 - \mathbf{v}_{k}^{H}(n+1)\mathbf{u}_{k}(n+1)} \right]$$
(22)

$$\mathbf{Q}_{k}(n+1) = \mathbf{Q}_{k}'(n+1) + \widetilde{\mathbf{C}}_{k} \left[\widetilde{\mathbf{C}}_{k}^{H}\widetilde{\mathbf{C}}_{k}\right]^{-1} \cdot \left[\mathbf{I}_{K} - \widetilde{\mathbf{C}}_{k}^{H}\mathbf{Q}_{k}'(n+1)\right]$$
(23)

• Update the antenna weights

$$\mathbf{w}_k(n+1) = \mathbf{Q}_k(n+1)\mathbf{f} \tag{24}$$

V. SIMULATIONS

In order to access the performance of the LCMV-based algorithms, a scenario of two users sharing the same resources (frequency, time and code) in a mobile system was simulated. Such a model could represent a SCR (Same Cell Reuse) system where the users are separated by their spatial positions.

Moreover, the simulations are carried in a GSM context with an uplink carrier frequency $f_c=2$ GHz, a symbol rate $R_s = \frac{1}{T_s} = 270.833$ Mbauds and a rased cosine model, with rolloff 0.35, for both transmitter and receiver filters.

The base station has a 3 element antenna array, with $\frac{\lambda_c}{2}$ interelement distance, used to perform purely spatial processing as mentioned before. This leads to a higher SNIR in the temporal equalizer input.

The space-time channel between each user and the array is modelled as composed by 4 multipaths as described in table III, where the DOAs are given in degrees. The SNR (Signal to Noise Ratio) for each antenna element was set to 20 dB.

Firstly, we have tested the LCMV_{CM} to verify its performance when compared with the LCMV_{EV} solution. These simulations were carried out in a static environment (without fading) and 0° of angular spread. The total covariance matrix **R** was estimated over 7000 symbols and the users' UCCM was supposed to be known. As expected, the results have shown that the use of the UCCM matrix instead of the matrix of constraints leads to an equivalent performance. Figure 2 shows the reception radiation pattern for the previously mentioned channel.



Fig. 2. Reception radiation pattern for LCMV $_{\rm CM}$ method.

Two distinct cases were chosen to evaluate the adaptive versions of $LCMV_{\rm CM}$ in presence of fading or in a packet-like transmission.

A. Fading

The Jakes' model [9] is used in the non-static environment, so that fading is independently inserted in each multipath, for which the respective values of delay, DOA and mean power are assumed to be as stated in table III. The total covariance matrix \mathbf{R} was estimated over 15000 symbols for the batch solution and

TABLE III Space-Time Channel Parameters.

Parameter	User 1				User 2			
Delay $(\times T_s)$	0	0.5	1	2	0	0.5	1	2
DOA	0	10	20	30	-40	-30	-20	-10
Mean power	0.45	0.4	0.1	0.05	0.45	0.4	0.1	0.05



Fig. 3. Comparison between LCMV $_{\rm CM}$ Batch Solution (solid lines), LCMV $_{\rm CM}$ -LMS (dotted lines) and LMCV $_{\rm CM}$ -RLS (dashed lines) for 15 km/h.

the users' UCCM was supposed to be known for all solutions.

Figure 3 shows the SNIR evolution for the two users, each one with a speed of 15 km/h and 3° of angular spread. The LCMV_{CM}-LMS convergence factor was $\mu = 5 \cdot 10^{-4}$. The LCMV_{CM}-RLS convergence parameters were δ =10 and λ =0.999999. As it can be seen, the RLS version slightly outperforms the LMS one. Both tracks the channel variations due to fading, but the LMS version has a poorer tracking capability and a higher misadjustment.

Figure 4 shows the SNIR evolution for both users, now with a speed of 100 km/h. The other parameters were the same as in the previous case. Once again, the RLS version sightly outperforms the LMS one, with a better tracking capability. Due to the instantaneous estimation of the adaptive algorithms, they have a better performance than the batch solution in some intervals of time.

It is worth to mention that the convergence parameters were chosen in order to provide good performance in both situations, i.e. 15 km/h and 100 km/h. Although, these parameters could be adjusted in each case to improve the performance.

B. Packet-like simulations

In this case, a sudden change in the interferer characteristics is forced in a given time instant. This change concerns the interferer's DOAs. As previously mentioned in Section I we call in this paper *packet-like system* both FH or slotted-packet systems.

Simulations were carried out in a static environment but with 3° of angular spread. At 7000 samples the interferer's multipath DOAs changed from [-40, -30, -20, -10] to [-10, -3, 40, 60], while all other channel parameters remains the same.

Figure 5 shows the SNIR evolution for the desired user for the LCMV_{CM} batch solution, LCMV_{CM}-LMS and LCMV_{CM}-RLS adaptive solutions. The estimation of the total covariance matrix \mathbf{R} for the batch solution was done each 15000 samples. The desired user's UCCM was again assumed to be known. Parame-



Fig. 4. Comparison between $LCMV_{\rm CM}$ Batch Solution (solid lines), $LCMV_{\rm CM}\text{-}LMS$ (dotted lines) and $LMCV_{\rm CM}\text{-}RLS$ (dashed lines) for 100 km/h.

ters μ =10⁻³ for the LMS version and δ =10 and λ =0.999 for the RLS one were used.

The batch solution has two sudden changes in the SNIR level, the first one (at 7000) is due to the interferer channel change and the second (at 15000) corresponds to the second window used for estimating total covariance matrix.

As it can be seen in figure 5, the adaptive solutions are able to track variations in the interference pattern, becoming more suitable than the batch solution.



Fig. 5. Comparison between LCMV $_{\rm CM}$ Batch Solution, LCMV $_{\rm CM}$ -LMS and LMCV $_{\rm CM}$ -RLS in a packet-like context.

VI. CONCLUSIONS

In this paper a novel approach based on the LCMV criterion that avoids DOA estimation was proposed. Eigenvectorial constraints have allowed the use of an alternative matrix of constraints formed by the maximum eigenvector of the estimated uplink channel covariance matrix. The proposed approach provides a better criterion since the use of the uplink covariance matrix brings to the method more information about the spatial channel than DOA estimation. Besides, high computational complexity algorithms for DOA estimation are replaced by low complexity covariance matrix estimation.

Moreover, both LMS and RLS adaptive versions of this novel approach were studied and investigation about the performance evaluation of the proposition was done by means of a packet-like wireless system. Simulations have shown that adaptive versions could outperform batch solutions in the situations where the interferers' characteristics change suddenly, as in FH systems or slotted-packet systems.

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