Q-GA: Performance Analysis in Low-pass Filter Equalization Design

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Abstract - **The Q-GA (modified genetic algorithm) has been proposed to design phase equalizers by using the symmetry error criterion of the impulse response. In this approach, the search space is partitioned into subspaces in which a small population evolves, regarding a reduced number of generations. This approach presents a considerable computational complexity gain, as compared with the use of a conventional GA. In this paper, we assess such an algorithm for performance in equalizer design of low-pass filters.**

I. Introduction

The symmetry error of the impulse response, introduced in [1], has been proposed successfully for phase equalization. This approach has presented itself as a very good alternative to the ones that use group and phase delay functions [2]. Moreover, for cases of baseband signal (e.g. PAM, PCM) processing [3], the impulse response symmetry error criterion has shown to be more efficient than other criteria presented in the open literature. In addition, by using such a criterion, one obtains a lower order for the equalizer, as compared with other procedures.

On the other hand, independently of the approach used, generally the equalization algorithms make use of optimization techniques for determining the equalizer coefficients. Due to simplicity and robustness, the DownHill Simplex Method (DHSM) [1-2,4-6] has been largely used for solving optimization questions in phase equalizer designs. In spite of its robustness, its performance is much dependent on the starting estimates, leading in some cases to local minima. This has been verified in several cases, when used for equalizing other different types of approximation functions [1-2]. For the impulse response symmetry error criterion, the performance surface presents a multimodal behavior, being a convex function just for a region around the optimal point, as depicted in Fig. 1. In this figure the (x, y) values represent the 2nd–order equalizer coefficients. Thus, an efficient and robust algorithm is necessary to overcome such difficulties. Based on the principles of the conventional genetic algorithms (GA), a modified genetic algorithm (Q-GA) to design phase equalizers, using the impulse response symmetry error criterion, has been proposed in [7]. Genetic algorithms have become a powerful and robust tool for solving complex optimization problems [8]. On the other hand, genetic algorithms present a large computational effort, as carried out in their conventional formulation. This is mainly due to the initial diversity inherent to these algorithms.

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Thus, to overcome this drawback, the Q-GA approach becomes an alternative strategy for improving the search process. This is accomplished by systematic division of the search space into subspaces, in which a small population evolves, regarding a reduced number of generations. Through a mechanism of competition between the subspaces, one obtains (with a reduced number of evaluations of the objective function) the convergence region. Through the examples of phase equalizer designs, for low-pass filters, the Q-GA approach is assessed and compared with the one based on the conventional GA for performance.

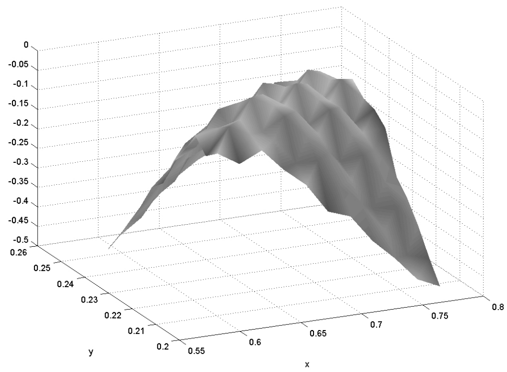

Fig. 1. Detail of the performance surface of the impulse response symmetry error criterion for a low-pass filter equalization.

II. Modified Genetic Algorithm

The determination of the convergence region or feasible region (in which the optimal solution is contained) is the main obstacle for the search algorithms, when one does not have the knowledge of the performance surface. Another difficulty found is the possibility of occurring local minima (usual for objective functions in phase equalizer design), which give rise to stationary points, making it difficult to use algorithms based on derivatives. Thus, an efficient procedure within a given search space must provide the convergence region, overcome the local minima and lead to a better possible solution. GA represent a good alternative to carry out this task. Although, they are an efficient optimization and search tool, their performance is strongly dependent on the initial diversity. Thus, a great number of individuals should evolve in a given number of generations. Such a characteristic leads to a large computational effort, rendering the algorithm less competitive as compared with other search procedures. Through the Q-GA approach, we have an algorithm more adequate to solve the optimization problems in phase equalizer design. This approach is unlike the one proposed in [8], in which competition is stimulated among several populations within the same region.

A. Segmentation Process and Competition

In the Q-GA approach, the segmentation process is based on the quadtree decomposition [9] along with a search procedure, which is performed within each subspace. First, the search space limits are determined. This determination is somewhat heuristic and it is based on the designer's expertise. Figure 2 shows a limited search space, which is divided into quadrants (subspaces), named (I), (II), (III) and (IV). In this same figure, the successive partitions of the subspaces are also illustrated, which are denoted by (1), (2), (3) and (4). Such a systematic division denotes the evolution of the quadtree structure [8], whose main characteristic is the search of the homogeneity of a target parameter under inspection. In this case, the optimal point is located within the quadrant (I) of the first partition (1). Thus, the search algorithm must be capable of selecting the quadrant (I) as the winner space for the next step. This process is repeated until a stopping criterion is reached.

B. Computational Complexity Analysis

To obtain an adequate solution within the whole search space via a conventional GA a great deal of individuals (Ni) is needed, evolving for a large number of generations (Ng). By considering the GA a stochastic process, $\Re$ realizations are needed for assessing its performance. Then, the complexity of this algorithm is $\Re \times (Ni \times Ng)$ evaluations of the objective function. In the Q-GA approach, we have used a small number of individuals by quadrant (ni), evolving along a reduced number of generations (ng). To find the convergence region, ne successive partitions are made on the search subspaces. Thus, we obtain $ne \times (ni \times 4 \times ng)$ evaluations of the objective function, which is less than the one required by a conventional GA.

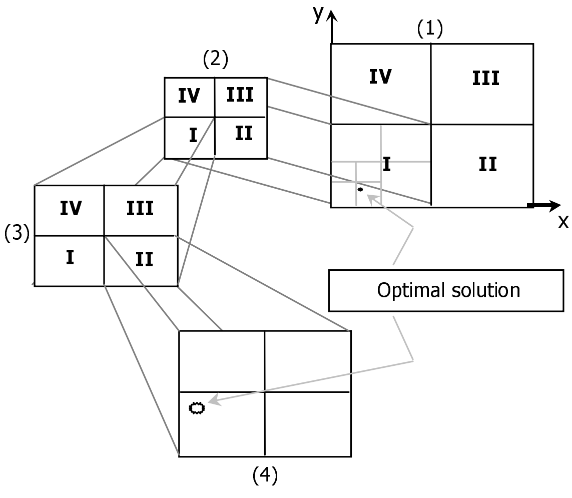

Fig. 2. Q-GA search process.

C. Criterion for Determining the Winner Subspace

In the quadtree decomposition, the search by homogeneity is the principal aim of the process. It can be represented by the variance of the parameter under investigation. In the Q-GA approach, we have used the homogeneity criterion to explore the performance surface, which is obtained from the impulse response symmetry error function [1].

In the GA, through the fitness function, one can assess the performance of the optimization procedure such that we can distinguish the regions that present a high variance (non-homogeneous regions) of the ones that exhibit a low variance (homogeneous regions).

In the studied cases, since the convergence region is found, we have verified that the variance of the parameter under investigation tends to a minimum. On the other hand, the mean of this parameter tends to a maximum. Thus, through the evolution of the better individual, which is based on the maximum, mean and variance of the fitness function, for a given partition level (φ) in each quadrant $(Q = 1, \ldots, 4)$, one can extract the following parameter sets:

- Maximum fitness:

$$\gamma(\varphi, Q) = \left\{ \max \left[\eta_i(\varphi, Q) \right] \right\}, \quad i = 1, 2, \ldots, g \tag{1}$$

- Mean fitness:

$$\mu(\varphi, Q) = \left\{ \frac{1}{g} \sum_{i=1}^{g} \eta_i(\varphi, Q) \right\} \tag{2}$$

- Normalized fitness:

$$\beta_i(\varphi, Q) = \left\{ \frac{\eta_i(\varphi, Q) - \mu(\varphi, Q)}{\mu(\varphi, Q)} \right\}, \quad i = 1, 2, \ldots, g \tag{3}$$

- Variance of the normalized fitness:

$$\sigma_\beta^2(\varphi, Q) = \left\{ \frac{1}{g-1} \sum_{i=1}^{g} \beta_i^2(\varphi, Q) \right\} \tag{4}$$

where $\eta_i(\cdot)$, for $i = 1, 2, \ldots, g$, is the fitness evolution of the best individual and g is the generation number. The winner quadrant, for a given partition level (φ) in each quadrant $(Q = 1, \ldots, 4)$, is determined by a set of selection criteria $(\chi(\cdot))$, defined as follows:

$$\chi(1, \varphi, Q) = \left\{ \gamma(\varphi, Q) / \max \left[\gamma(\varphi) \right] \right\} \tag{5a}$$

$$\chi(2, \varphi, Q) = \left\{ \mu(\varphi, Q) / \max \left[\mu(\varphi) \right] \right\} \tag{5b}$$

$$\chi(3, \varphi, Q) = \left\{ \frac{\sigma_\beta^2(\varphi, Q) \cdot \sqrt{1/\varphi}}{\max[\sigma_\beta^2(\varphi)]} \right\} \tag{5c}$$

$$\chi(4, \varphi, Q) = \left\{ \frac{\gamma(\varphi, Q) - \frac{1}{4} \sum_{j=1}^{4} \gamma(\varphi, j)}{\frac{1}{4} \sum_{j=1}^{4} \gamma(\varphi, j)} \right\} \tag{5d}$$

In the selection criterion (5c), the factor $\sqrt{1/\varphi}$ was introduced for weighting the variance of the normalized fitness. The aim of this factor is to reduce the variance weight in the quadrant choice while the process progresses to the convergence region.

The selection criterion, $\|\chi(\varphi, Q)\|$, for a given partition level (φ) in each quadrant $(Q = 1, \ldots, 4)$, is then defined as the normalized sum of the sets given by (5), as follows:

$$\|\chi(\varphi, Q)\| = \left\{ \frac{\sum_{j=1}^{4} \chi(j, \varphi, Q)}{\max \left[\sum_{j=1}^{4} \chi(j, \varphi) \right]} \right\}. \tag{6}$$

III. Impulse Response Symmetry Error

In [1], the phase equalization approach based on the impulse response symmetry error is discussed. The error function is defined as

$$\varepsilon_h(k) = \sum_{\ell=1}^{L} \varepsilon_\ell^2(k), \quad (7)$$

where $\varepsilon_\ell(k) = h[(T_0 + \ell\,\Delta t), k] - h[(T_0 - \ell\,\Delta t), k],\ \ell = 1, \ldots, L$; $h(t)$ is the impulse response; L is the length of the sampled impulse response symmetrically distributed around a reference value T_0; T_0 is the time instant, characterizing the maximal point of $h(t)$; Δt is the sampling period defined by T_0 / L; and k denotes the k^{th}-iteration.

The equalization procedure considers that the whole system is composed of the original filter and equalizer connected in cascade. In this approach, only the equalizer order is modified. The cost function of the optimization process in the k^{th}-iteration is the variance of the error function defined as

$$\sigma_{\varepsilon h}^2(k) = \frac{1}{L-1} \sum_{\ell=1}^{L} [(\varepsilon_\ell(k) - \bar{\varepsilon}(k)]^2, \quad (8)$$

where $\bar{\varepsilon}(k)$ denotes the mean value of the symmetry error.

IV. Results

The Q-GA approach was applied to the following phase equalizer designs: Chebyshev and elliptic low-pass filters, both of 8^{th}-order. The features of these filters are: passband distortion $A_{max} = 0.5\,\text{dB}$, and selectivity factor $\omega_s / \omega_p = 2.0$, for the elliptic filter. Figure 3 illustrates the normalized impulse responses for the Chebyshev and elliptic filters without equalization. The variances of the symmetry error $(\sigma_{\varepsilon h}^2)$ are 10.94×10^{-4} and 13.45×10^{-4}, respectively.

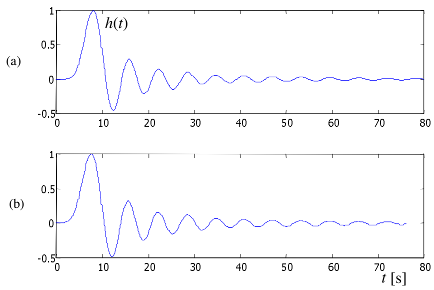

Fig. 3. Normalized impulse responses. (a) Chebyshev; (b) elliptic.

A. Exhaustive Search Results

In order to assess the performance of Q-GA and GA, we have obtained the optimal solution (for a predefined accuracy), by using a two-dimensional exhaustive search (EXS) process over the domain $\{x, y \in [0.01; 5.01]\}$ with 256 points in each direction (x, y). Table I shows the results of this simulation, which are the optimal equalizer coefficients and variances of the symmetry error.

TABLE I
EQUALIZATION RESULTS FOR EXHAUSTIVE SEARCH PROCESS

Filters	Equalizer coefficients $[x, y]$	$\sigma_{\varepsilon h}^2 \times 10^{-4}$
Chebyshev	[0.657, 0.226]	1.1648
Elliptic	[0.63, 0.20]	1.5695

B. Parameters of the Modified GA (Q-GA)

The Q-GA parameters used for the phase equalizer design are:

- population size: 20 individuals/quadrant/decomposition;
- number of generations: 10;
- chromosome or bit string: 12 bits per variable;
- single point crossover probability (p_c): 0.8;
- mutation probability (p_m): 1/(population size);
- selection method: stochastic remainder without replacement [7];
- fitness function defined as

$$\eta(x_\ell, y_\ell) = \frac{1}{\sigma_{\varepsilon h}^2(x_\ell, y_\ell) + \alpha}, \qquad \ell = 1, 2, \ldots, \frac{N}{2}, \quad (9)$$

where N is the equalizer order and $\alpha = 1 \times 10^{-10}$ is a constant used for avoiding that the fitness function tends to infinity when the variance tends to zero.

The use of 12 bits per variable for the chromosome coding is adequate for the quantization levels of the coefficients. The used parameters lead to 200 evaluations of the objective function per quadrant. To assess the process consistency, we have run it 100 times ($\Re$=100).

B.1 Evaluation Methods

The evaluation of the obtained solutions, from $\Re$ realizations, considering the winner quadrant (Q_v) in each decomposition (φ), has been performed by using the following measurements:

- Mean value of the impulse response symmetry error:

$$\bar{\sigma}_{\varepsilon h}^2(\varphi, Q_v) = \frac{1}{\Re} \sum_{i=1}^{\Re} \frac{1}{\gamma_i(\varphi, Q_v)} \quad (10)$$

- Variance of the impulse response symmetry error:

$$\Omega_{\sigma_{\varepsilon h}^2}(\varphi, Q_v) = \frac{1}{\Re - 1} \sum_{i=1}^{\Re} \left(\frac{1}{\gamma_i(\varphi, Q_v)} - \bar{\sigma}_{\varepsilon h}^2(\varphi, Q_v) \right)^2 \quad (11)$$

These measures are normalized with respect to the impulse response symmetry error obtained from the filter without equalization ($\sigma_{\varepsilon h}^2(\varphi(0))$).

B.2 Q-GA Equalization Results

Considering the exhaustive search results (EXS) shown in Table I, the Q-GA decompositions should find the following sequence of winner quadrants: [I, I, II, I, IV], from the five decomposition levels (Fig. 4). Such decompositions are named **Solution 1**.

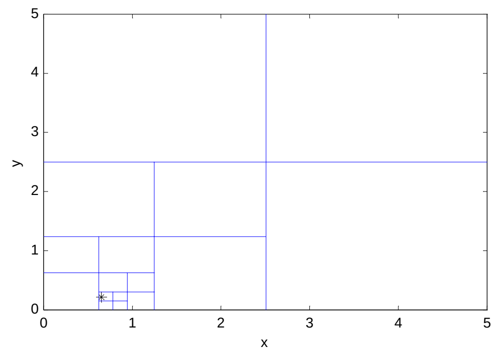

Fig. 4 Decomposition sequence considering the exhaustive search results (EXS); sequence also obtained by the Q-GA algorithm.

By analyzing the performance surface characteristics (Fig. 1) along with the GA operation theory, there exists a nonzero probability that the process finds the following alternative sequence of winner quadrants: [I, I, I, II, III] (Fig. 5). These decompositions are here called **Solution 2.** Such a solution also leads to the region where the optimum point is located.

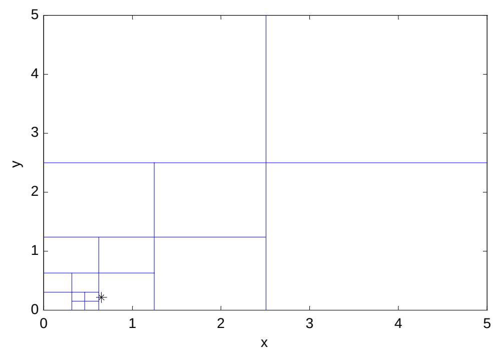

Fig. 5. Alternative decomposition sequence.

For all the realizations, in the case of the Chebyshev filter equalizer design, the Q-GA process found 48% as Solution 1 and 52% as Solution 2. In the elliptic filter case, the algorithm found 40% as Solution 1 and 60% as Solution 2.

Figure 6 and 7 show the mean value (Eq.10) of each solution for both the equalizer designs.

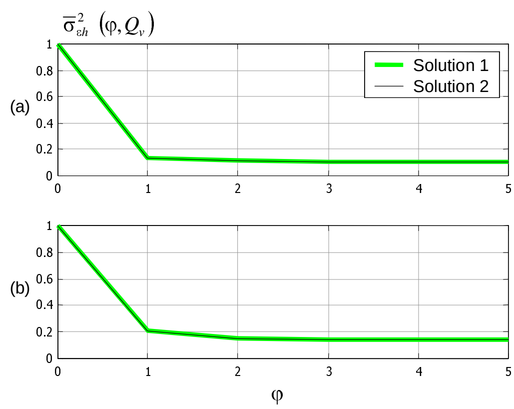

Fig. 6. Normalized mean value of the impulse response symmetry error. (a) Chebyshev; (b) elliptic.

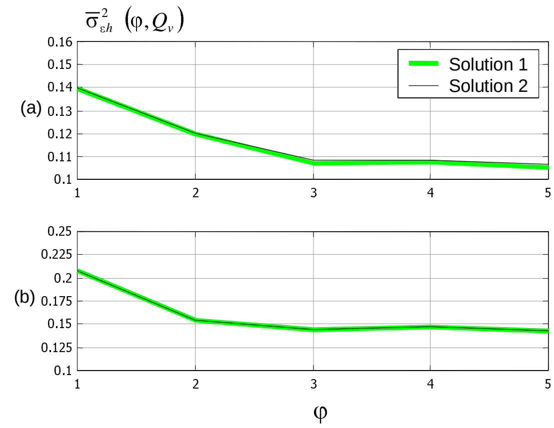

Fig. 7. Normalized mean value (details) of the impulse response symmetry error. (a) Chebyshev; (b) elliptic.

Figures 8 and 9 show the variance of the impulse response symmetry error from the $\Re$ realizations. The obtained values confirm that the used approach presents a satisfactory performance for both solutions in both equalizer designs.

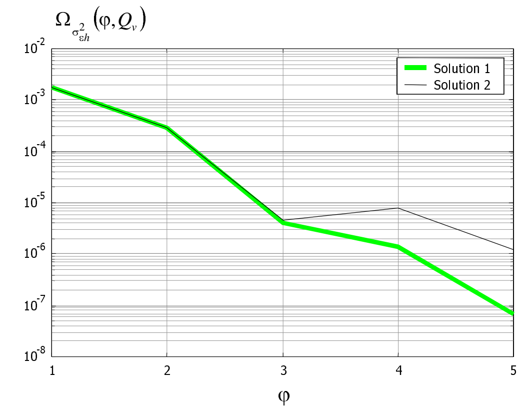

Fig. 8. Impulse response symmetry error variance for the Chebyshev filter equalizer design.

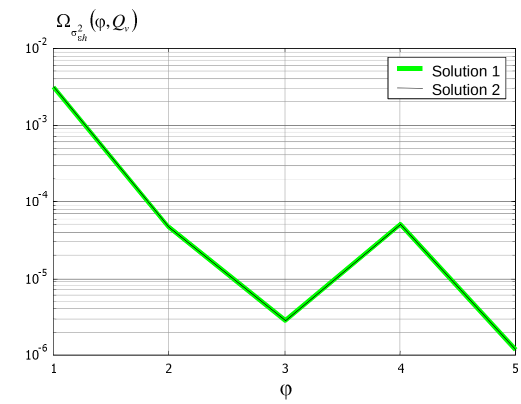

Fig. 9. Impulse response symmetry error variance for the elliptic filter equalizer design.

By analyzing the mean value of the impulse response symmetry error depicted in Fig. 6, one can verify a significant decrease in this value for the first decomposition. In the next decompositions, this value is slightly decreasing, as shown in Fig. 7. Such a characteristic plays an important role in the computational complexity analysis. For the Q-GA approach, the computational effort is strongly related to the decomposition sequence. As previously presented (Section 2.2), the algorithm complexity is $ne \times (ni \times 4 \times ng)$. Considering a constant number of individuals and generations, then, the number of objective function evaluations increases proportionally to the established number of decompositions. Thus, we can measure an attenuation coefficient, between the impulse response symmetry error and the defined number of decompositions, given by

$$\bar{\sigma}^2_{\varepsilon h}(\varphi, Q_v)\Big|_{dB} = 10 \times \log_{10}\left(\frac{\bar{\sigma}^2_{\varepsilon h}(\varphi, Q_v)}{\sigma^2_{\varepsilon h}(\varphi(0))}\right) \quad (12)$$

where $\sigma^2_{\varepsilon h}(\varphi(0))$ denotes the impulse response symmetry error obtained from the original filter to be equalized.

Figure 10 shows the attenuation coefficient values, considering 5 decompositions, for both solutions in both designed equalizers.

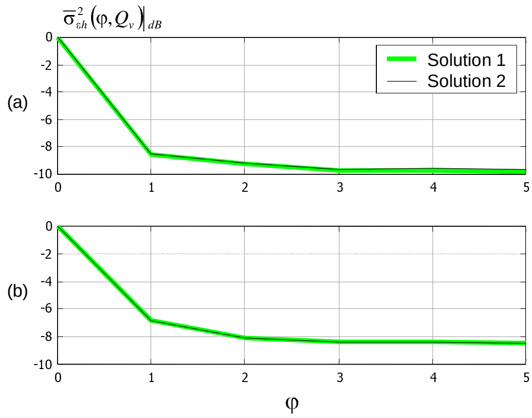

Fig. 10. Attenuation coefficient. (a) Chebyshev; (b) elliptic.

Based on the variance values of the impulse response symmetry error (Figs. 8 and 9), and attenuation coefficient values (Fig. 10), we conclude that a reduced number of decompositions can be used. This choice decreases the computational load without affecting the algorithm performance in the solution search.

For the studied cases, we randomly select 6 possible solutions in $\mathfrak{R}$ realizations, considering at most 3 decompositions. Thus, after 2400 evaluations (800 per decomposition) of the objective function, the equalizer coefficients and the impulse response symmetry error values have been obtained and are shown in Tables II, III and IV. Figures 11, 12 and 13 depict the impulse responses of the equalized filters superimposed on the ones obtained by EXS (Table I). For all cases, the convergence region that contains the optimal solution has always been found.

TABLE II
EQUALIZATION RESULTS BY Q-GA (1ST DECOMPOSITION)

Filters	Equalizer coefficients $[x, y]$	$\sigma^2_{\varepsilon h} \times 10^{-4}$
Chebyshev	[0.794, 0.253]	1.31
Elliptic	[0.468, 0.184]	2.22

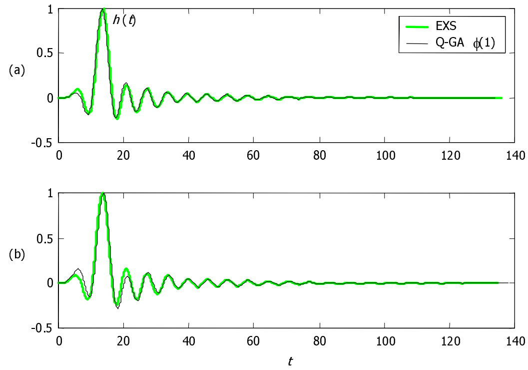

Fig. 11. Normalized impulse response after the equalization via EXS and Q-GA (1st decomposition). (a) Chebyshev; (b) elliptic.

TABLE III
EQUALIZATION RESULTS BY Q-GA (2nd DECOMPOSITION)

Filters	Equalizer coefficients $[x, y]$	$\sigma^2_{\varepsilon h} \times 10^{-4}$
Chebyshev	[0.7112, 0.234]	1.19
Elliptic	[0.5784, 0.1956]	1.59

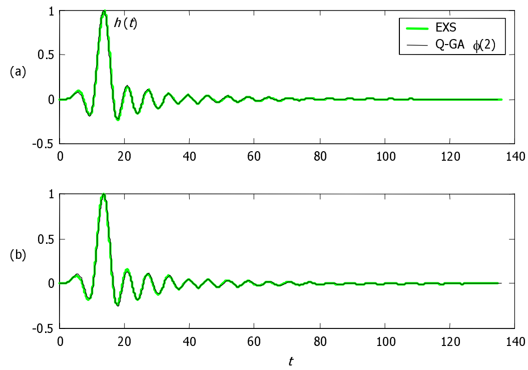

Fig. 12. Normalized impulse response after the equalization via EXS and Q-GA (2nd decomposition). (a) Chebyshev; (b) elliptic

TABLE IV
EQUALIZATION RESULTS BY Q-GA (3rd DECOMPOSITION)

Filters	Equalizer coefficients $[x, y]$	$\sigma^2_{\varepsilon h} \times 10^{-4}$
Chebyshev	[0.607, 0.21]	1.17
Elliptic	[0.633, 0.20]	1.58

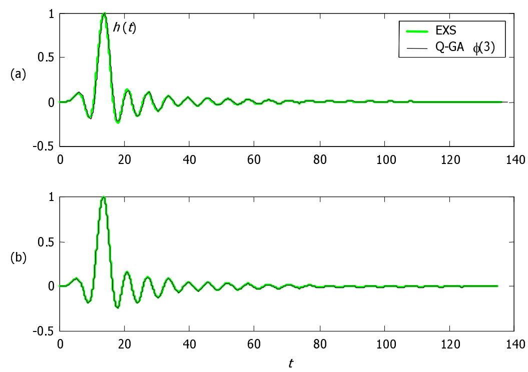

Fig. 13. Normalized impulse response after the equalization via EXS and Q-GA (3rd decomposition). (a) Chebyshev; (b) elliptic.

By considering the obtained results of the 2nd and 3rd decompositions, we can verify that the impulse response symmetry errors do not present a significant difference. The impulse responses of the equalized systems (filter+equalizer), considering the obtained results from the 2nd decomposition (Fig. 12), are very close to the ones obtained by EXS procedure. Therefore, only two decompositions are needed to obtain a satisfactory impulse response. In this way, 1600 evaluations of the objective function are carried out.

B.3 Conventional GA results

To ratify the excellent performance of the Q-GA approach, we have compared it with the conventional GA [8], for the equalizer design of the previously specified filters. The conventional GA parameters used for simulation are:

- population size: 50 individuals;
- number of generations: 50;
- number of realizations: 50;
- chromosome or bit string: 12 bits per variable;
- single point crossover probability (p_c) : 0.8;
- mutation probability (p_m) : 1/(population size);
- selection method: stochastic remainder without replacement [8];

For each realization, 2500 evaluations of the objective function have been carried out. The mean and variance of the fitness function, for the best individual, normalized with respect to expected fitness (inverse of the impulse response symmetry error found via EXS), are obtained for 50 realizations. These results are depicted in Figs. 14 and 15. Analyzing such results, we can verify the poor performance of the conventional GA, as compared with Q-GA approach.

V. Conclusions

This paper has accomplished a performance analysis for the Q-GA approach – a modified genetic algorithm for designing phase equalizers. For such, we have used different assessment criteria, which showed the effectiveness of the Q-GA approach for equalizer designs. Thus, within a given search space, it provides the convergence region; overcomes the local minima, which are widespread in the objective function; and leads to a useful set of possible solutions.

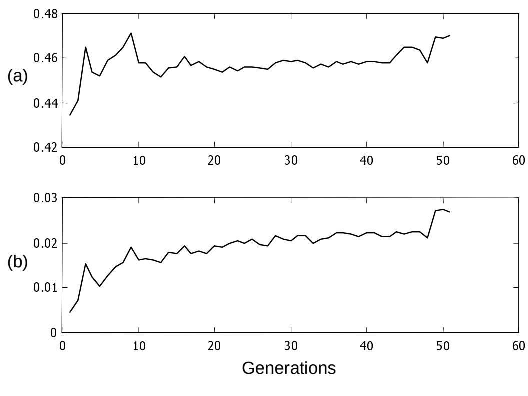

Fig. 14. (a) Mean and (b) variance of the fitness function for the conventional GA applied to Chebyshev filter equalization.

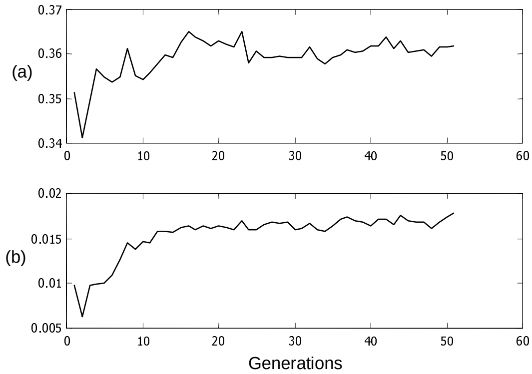

Fig. 15. (a) Mean and (b) variance of the fitness function for the conventional GA applied to elliptic filter equalization.

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