Adaptive Approaches for Blind Equalization Based on Multichannel Linear Prediction

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Abstract – **In this work, the problem of blind multichannel equalization is considered. A strategy for saving computation in zero-forcing equalizers design is proposed and its performance is evaluated under an adaptive implementation of an algorithm based on multichannel forward linear prediction. Moreover, a cascade structure based on forward and backward linear prediction is regarded: an adaptive implementation of such a structure is also proposed and its performance is verified through computer simulations.**

I. INTRODUCTION

INTERSYMBOL interference (ISI) is a major impairment in digital communications. Equalization is often considered as a suitable countermeasure for ISI. Usually, equalizer coefficients are adapted with a training sequence, which is required to be periodically sent. However, trained equalizers present important drawbacks such as wasted bandwidth and the possibility of fading occurrence during the training period. Blind equalization is then an interesting alternative, so that a training sequence is no longer needed.

Blind algorithms that make use of higher-order statistics (HOS) are divided into explicit HOS-based algorithms and implicit HOS-based algorithms, which include the so-called *Bussgang Algorithms*. Both the implicit and the explicit HOS-based algorithms suffer from a slow convergence rate. Blind algorithms based on second-order statistics (SOS) are believed to overcome such a limitation. SOS-based algorithms exploit the *cyclostationarity* of the received signal. Such a property is preserved when the incoming signal is sampled at a rate higher than the symbol rate. It can be shown that such oversampling leads to a *multichannel model*.

According to the Gardner's pioneer work [4], identification of both magnitude and phase of communication channels with SOS is possible due to the cyclostationary properties of modulated signals. Tong et al. proposed the use of cyclostationary SOS for blind channel identification and equalization [5]. Most of recently SOS-based blind identification and equalization algorithms deal with a multichannel model, so that cyclostationarity is exploited indeed in an implicit way.

SOS-based blind techniques can be broadly divided into two main approaches, namely the *subspace* methods and the *linear prediction* methods. The linear prediction approach was first proposed by Slock [1]. A zero-forcing (ZF) solution based on multichannel forward linear prediction is proposed in [1] and further elaborated in [2] and [3].

A cascade structure of a multichannel forward prediction filter and a multichannel backward prediction filter [2, 3] also provides a ZF solution. While the ZF equalization based only on forward prediction leads to an equalization delay equal to zero, the use of a cascade structure makes possible the tuning of equalization delay, which may lead to a lower steady-state estimation error.

The present paper deals with the linear prediction method for SOS-based blind equalization. Two original contributions are proposed: First, a strategy to calculate a ZF solution based on multichannel linear prediction is derived. Such strategy reduces the computation involved in the ZF equalizer algorithm based on multichannel forward prediction [1-3], since the estimate of a forward prediction error variance matrix is completely avoided. An important issue is that an adaptive version of the algorithm can straightly derived.

The second proposition consists in an adaptive implementation of a multichannel forward/backward cascade structure. As pointed out, such algorithm allows dealing with arbitrary equalization delays. The performance of the proposed technique is evaluated by means of computer simulations.

The remainder of the paper is organized as follows. The multichannel model and used notation are presented in Section II. The original approach for ZF equalizers design based on linear prediction [1]-[3] is briefly presented in Section III. The proposed strategy for ZF equalizers design is presented in Section IV. A description of the adaptive implementation procedure proposed for the forward/backward prediction cascade structure is presented in Section V. Finally, Section VI is devoted to our conclusions.

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II. BACKGROUND ON MULTICHANNEL EQUALIZATION

The considered baseband model for the received signal uniformly sampled at symbol rate *1/T* is given by

$$u(kT) = \sum_i h(iT)x(kT - iT) + v(kT), \qquad (1)$$

where $\{x(\cdot)\}$ is the transmitted symbol sequence, $\{h(\cdot)\}$ stands for the total channel impulse response comprised of the transmission and reception filters and transmission channel, and $\{v(\cdot)\}$ represents the additive white zero-mean gaussian noise. The channel is modeled as a finite impulse response (FIR) filter.

The received signal uniformly sampled at a rate P times higher than the symbol rate – *oversampling* – is given by:

$$u\left(k\frac{T}{P}\right) = \sum_i h\left(i\frac{T}{P}\right)x\left(kT - i\frac{T}{P}\right) + v\left(k\frac{T}{P}\right) \qquad (2)$$

Oversampling is also known as *fractional sampling*. The resulting oversampled or *fractionally-spaced* sequence $\{u(kT/P)\}$ may be divided into P symbol-rate sequences $\{u_p(kT)\}$, $p=0, \ldots, P-1$. From now on, the temporal indices kT and iT will be respectively represented by k and i, for the sake of simplicity. The k-*th* sample of the $(p+1)$-*th* sequence is written as

$$u_p(k) = \sum_{i=0}^{N-1} h_p(i)x(k-i) + v_p(k) \qquad (3)$$

The fractionally spaced channel impulse response is assumed to have length NP. It is worth noting that the sequence $\{h_p(0) \ \ldots \ h_p(N-1)\}$ represents the $(p+1)$-*th subchannel*, so that the oversampled channel is comprised of P symbol-rate subchannels also modeled as FIR filters. In vector form, (3) can be written as:

$$\mathbf{u}(k) = \sum_{i=o}^{N-1} \mathbf{h}(i)x(k-i) + \mathbf{v}(k), \qquad (4)$$

where $\mathbf{u}(k) = [u_o(k) \ \cdots \ u_{P-1}(k)]^T$ is the vector of P received samples at time k, $\mathbf{v}(k) = [v_0(k) \ \cdots \ v_{P-1}(k)]^T$ is the corresponding vector of P noise samples and $\mathbf{h}(i) = [h_0(i) \ \cdots \ h_{P-1}(i)]^T$, $i = 0, \ldots, N-1$, is a vector with the $(i+1)$-*th* samples of each subchannel. Equation (4) describes a *single-input multiple-output* (SIMO) system.

A vector of L successive samples of $\mathbf{u}(k)$ is given by

$$\mathbf{U}_L(k) = \mathbf{H}\,X_{L+N-1}(k) + \mathbf{V}_L(k) \qquad (5)$$

where $\mathbf{U}_L(k) = [\mathbf{u}^T(k) \ \cdots \ \mathbf{u}^T(k-L+1)]^T$ is the received signal, $X_{L+N-1}(k) = [x(k) \ \cdots \ x(k-L-N+2)]^T$ is the transmitted symbol vector, and $\mathbf{V}_L(k) = [\mathbf{v}^T(k) \ \cdots \ \mathbf{v}^T(k-L+1)]^T$ is the associated noise vector. $\mathbf{H}$ is the *channel convolution matrix*, which is a $LP \ X \ L+N-1$ block-Toeplitz matrix given by:

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}(0) & \cdots & \mathbf{h}(N-1) & & \mathbf{0} \\ & \ddots & & \ddots & \\ \mathbf{0} & & \mathbf{h}(0) & \cdots & \mathbf{h}(N-1) \end{bmatrix} \qquad (6)$$

An estimate of the transmitted sample is then obtained by filtering the received sample vector by a *fractionally spaced equalizer*:

$$\hat{x}(k-d) = \mathbf{F}_L^H(k)\mathbf{U}_L(k), \qquad (7)$$

where d is the *equalization delay* and $\mathbf{F}_L(k)$ is the $LP \ X \ 1$ vector with all the equalizer taps at instant k. The operator $(\cdot)^H$ denotes *Hermitian transposition*.

Regarding the multichannel model, each subchannel is associated with a *subequalizer* comprised of L coefficients. The equalizer vector is given by:

$$\mathbf{F}_L = [\mathbf{f}^H(0) \ \cdots \ \mathbf{f}^H(L-1)]^H, \qquad (8)$$

where the $P \ X \ 1$ vector $\mathbf{f}(l)$, $l = 0, \ldots, L-1$, defined as $\mathbf{f}(l) = [f_0(l) \ \cdots \ f_{P-1}(l)]^T$, contains the $(l+1)$-*th* coefficients of each subequalizer. Fig. 1 illustrates the multichannel model and also the multichannel equalizer.

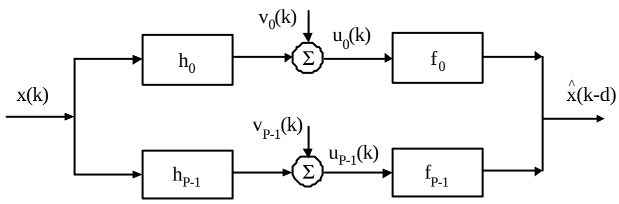

Fig. 1: Multichannel model

In the absence of additive noise ($\mathbf{v}(k)=\mathbf{0}$), perfect equalization is attainable according to the *Bezout Identity* [3], provided that the P subchannels have no common zeros. By generalizing this result, it is possible to obtain a ZF equalizer that leads to a combined channel-equalizer response equal to a delayed Dirac funcion. This is equivalent to pose:

$$\mathbf{F}_L^H\mathbf{H} = [\mathbf{0}_{1xd} \quad 1 \quad \mathbf{0}_{1xL+N-d-2}] \qquad (9)$$

Indeed, (9) is a linear system of $L+N-1$ equations and LP unknowns [3]. For a solution to exist, the condition $LP > L+N-1$ holds, which imposes a condition to the length L of each subchannel.

III. ZF EQUALIZATION BASED ON MULTICHANNEL LINEAR PREDICTION

The guidelines for ZF equalizers design based on multichannel linear prediction [1]-[3] are now briefly summarized. First, the deduction of a ZF equalizer with equalization delay equal to zero (*zero-delay* equalizer) based on multichannel forward linear prediction will be shown. All of the deduction was based on the assumption of absence of additive noise.

A multichannel one-step forward prediction error over the received signal is defined as

$$\mathbf{e}_f(k) = \mathbf{u}(k) - \mathbf{A}_{L-1}^H \mathbf{U}_{L-1}(k-1) = \\ = \left[\mathbf{I}_P \quad -\mathbf{A}_{L-1}^H\right]\mathbf{U}_L(k) \tag{10}$$

where $\mathbf{A}_{L-1}$ is the $(L\text{-}1)P \ X \ P$ matrix with the optimal multichannel forward prediction error coefficients and $\mathbf{I}_P$ is the $P \ X \ P$ identity matrix. The $P \ X \ 1$ forward prediction-error variance matrix is shown [3] to be given by:

$$E\left[\mathbf{e}_f(k)\mathbf{e}_f^H(f)\right] = s_x^2\mathbf{h}(0)\mathbf{h}^H(0), \tag{11}$$

where s_x^2 is the variance of the transmitted symbol sequence.

Both $\mathbf{A}_{L-1}$ and $E\left[\mathbf{e}_f(k)\mathbf{e}_f^H(f)\right]$ can be extracted from the autocorrelation matrix of the received signal $\mathbf{U}_L(k)$. It is shown in [3] that the following relation holds:

$$\left[\mathbf{I}_P \quad -\mathbf{A}_{L-1}^H\right]H = \mathbf{h}(0)\left[1 \quad 0 \quad \cdots \quad 0\right] \tag{12}$$

Therefore, one may notice that

$$\mathbf{h}^{\#}(0)\left[\mathbf{I}_P \quad -\mathbf{A}_{L-1}^H\right]H = \left[1 \quad 0 \quad \cdots \quad 0\right] \tag{13}$$

where $\mathbf{h}^{\#}(0) = \mathbf{h}^H(0) / \left\|\mathbf{h}(0)\right\|^2$. An estimate of $\mathbf{h}(0)$ may be obtained from (11). The rightmost term of (13) is then indeed the ideal combined channel-equalizer response corresponding to $d = 0$. Hence, one can withdraw the ideal ZF equalizer from (13):

$$\mathbf{F}_{ZF,0}^H = \mathbf{h}^{\#}(0)\left[\mathbf{I}_P \quad -\mathbf{A}_{L-1}^H\right] \tag{14}$$

An equalization delay equal to zero may lead to a poor steady-state estimation-error performance for some channels. A ZF equalizer obtained from a cascade of a forward predictor and a backward predictor [2],[3] provides an adjustable equalization delay (*d-delay* equalizer). Once again, absence of additive noise is assumed for deduction of the *d-delay* ZF equalizer.

A (d_f +1)-step forward prediction error over the received signal is written as

$$\mathbf{e}_f(k) = \mathbf{u}(k) - \mathbf{A}_{Lf}^H\mathbf{U}_{Lf}\left(k-1-d_f\right), \tag{15}$$

where $\mathbf{A}_{Lf}$ is a $PL_f \ X \ P$ matrix with the optimal multichannel forward prediction coefficients. A one-step backward prediction over the *forward prediction error signal* is defined as

$$\mathbf{e}_b(k) = \mathbf{e}_f(k - M_b) - \mathbf{B}_{Mb}^H \begin{bmatrix} \mathbf{e}_f(k) \\ \mathbf{e}_f(k-1) \\ \vdots \\ \mathbf{e}_f(k - M_b + 1) \end{bmatrix} \tag{16}$$

where $\mathbf{B}_{Mb}$ is a $PM_b \ X \ P$ matrix with the optimal multichannel backward prediction coefficients. The $P \ X \ 1$ backward prediction-error variance matrix is shown [3] to equal:

$$E\left[\mathbf{e}_b(k)\mathbf{e}_b^H(k)\right] = s_x^2\mathbf{h}(d_f)\mathbf{h}^H(d_f) \tag{17}$$

The ZF equalizer is then obtained from the convolution of forward and backward prediction-error filters and is expressed by

$$\mathbf{F}_{ZF,d}^H = \\ \mathbf{h}^{\#}(d_f)\left[-B_{Mb}^H \quad \mathbf{I}_P\right] \begin{bmatrix} \mathbf{I}_P & \mathbf{0}_{P\,x\,P\cdot df} & -\mathbf{A}_{Lf}^H & & & \mathbf{0} \\ & \ddots & & & \ddots & \\ \mathbf{0} & & & \mathbf{I}_P & \mathbf{0}_{P\,x\,P\cdot df} & -\mathbf{A}_{Lf}^H \end{bmatrix} \tag{18}$$

where $\mathbf{h}^{\#}(d_f) = \mathbf{h}^H(d_f) / \left\|\mathbf{h}(d_f)\right\|^2$. An estimate of $\mathbf{h}(d_f)$ may be obtained from (17). The estimate of the transmitted symbol at instant k is given by

$$\hat{x}(k - d_f - M_b) = \mathbf{F}_{ZF,d}^H(k)\mathbf{U}_L(k) \tag{19}$$

The equalization delay is $d = d_f + M_b$. The length of each subequalizer is $L = L_f + M_b + d_f + 1$. Equalization delay depends on an appropriate selection of the orders of the forward and backward predictors.

IV. Proposed Strategy for Obtaining a ZF Solution Based on Linear Prediction

The aforementioned ZF equalizer based on multichannel forward linear prediction (*zero-delay* equalizer) depends both on the optimal forward prediction coefficients and on the forward prediction error variance matrix. The proposed strategy also leads to a ZF solution based on forward linear prediction and makes possible to avoid estimation of the forward prediction error variance matrix.

A. Proposed Solution

The proposed strategy consists of the following steps:

- the forward-prediction-coefficients matrix is obtained so as to satisfy (10);

- the first coefficient of each subequalizer is set to unity: $\mathbf{f}(0) = \left[1 \quad \cdots \quad 1\right]^T$ *cf.* (8);

- finally, the remainder of the equalizer vector is given by the sum of the columns of the forward-prediction-coefficients matrix: if one defines the optimal forward prediction coefficients matrix as $\mathbf{A}_{Lf} = \left[\mathbf{a}_0 \quad \cdots \quad \mathbf{a}_{P-1}\right]$, therefore

$$\begin{bmatrix} \mathbf{f}(1) \\ \vdots \\ \mathbf{f}(L-1) \end{bmatrix} = -\sum_{p=0}^{P-1} \mathbf{a}_p \qquad (20)$$

One may notice that the proposed ZF equalizer is given by summing up the P columns of $\begin{bmatrix} I_P \\ -A_{Lf} \end{bmatrix}$. Regarding (12), the above sum-of-columns operation corresponds to summing up the lines of $\begin{bmatrix} \mathbf{I}_P & -A_{Lf}^H \end{bmatrix}$. Therefore, (12) becomes

$$\mathbf{F}_{ZF,0}^H H = \left[\left(\sum_{i=0}^{P-1} h_p(0) \right) \; 0 \; \cdots \; 0 \right] \qquad (21)$$

$\mathbf{F}_{ZF,0}$ is then a ZF solution up to a gain factor. By obtaining a ZF equalizer with such a procedure, it is no longer necessary to obtain an estimate of a forward prediction error variance matrix (11).

B. Adaptive Implementation

A number of adaptive algorithms based on multichannel linear prediction have been recently proposed. For example, the adaptive algorithm described in [6] involves both a forward and a backward prediction over the received signal in order to provide an estimate of a ZF solution. Another adaptive algorithm [7] estimates a ZF solution from two forward prediction operations over the incoming signal in a similar fashion.

Now, based on the proposed solution described above, an adaptive version of the ZF equalizer algorithm design is implemented and its performance is tested through simulations. The forward prediction error vector is estimated at each iteration and the forward prediction coefficients matrix may be adapted with either the recursive least squares (RLS) or the least mean squares (LMS) algorithm, as in [6] and [7]. For an adaptive implementation of ZF equalizer algorithm, a recursive estimation of the forward prediction-error variance matrix (11) can be carried out, every iteration, by:

$$E_f(k) = \lambda E_f(k-1) + (1-\lambda)\mathbf{e}_f(k)\mathbf{e}_f^H(k) \qquad (22)$$

where $0 \ll \lambda < 1$ acts as a *forgetting factor.*

An estimate of $\mathbf{h}(0)$ is obtained from (22) by taking the column of (22) with largest norm [6].

The adaptive procedure can be then summarized as follows:

- at each iteration k, the forward prediction error vector is obtained and the forward prediction matrix (10) is adapted with either RLS or LMS algorithm;

- an estimate of $\mathbf{h}(0)$ is calculated from (22) and the ZF equalizer is obtained through (14) or, equivalently;

- an estimate of ZF equalizer is obtained with the proposed strategy (21), therefore avoiding the intermediate step (22) and saving some computation.

C. Simulation Settings

The adaptive version of zero-delay ZF equalizer algorithm is tested through a simulation example of channel equalization. The transmitted symbol sequence is drawn from a unit-variance, uniformly distributed 16-QAM constellation. The multichannel coefficients are shown in Table I. This example consists of two subchannels ($P=2$) each of them with $N=4$. It is worth noting that the channel coefficients were normalized [8] so that the received signal variance is set to unity.

TABLE I
MULTICHANNEL COEFFICIENTS

coefficient index i	$h_0(i)$	$h_1(i)$
0	0.4219	0.3375
1	-0.2953 + 0.3375i	-0.1688 - 0.6329i
2	0.0127 + 0.2700i	-0.8649 + 0.0211i
3	-0.2418 - 0.1114i	0.2801 + 0.3173i

The criterion described in [8] to help evaluate the performance of a blind algorithm is adopted. When a *symbol error rate* (SER) of about 0.04 is achieved, it is possible to transfer to a decision-directed operation mode. For a unit-variance 16-QAM signal, the above SER level corresponds to a *mean-squared error* (MSE) about 0.08 [9]. So, the blind algorithm is considered successful if it is able to achieve such a MSE *transfer level*.

The length of each subequalizer is $L = 8$. Additive white Gaussian-distributed complex noise is applied in all simulations, at transmitted signal-to-noise (SNR) ratios from 10 to 40 dB. Both adaptation strategies are considered – either with or without error variance matrix estimation (proposed strategy). Moreover, phase correction and automatic gain control are applied after equalization [10]. For each adaptation strategy and at each SNR point, a Monte Carlo simulation of 50 trials is performed and the final steady-state MSE at 30000 samples is shown in the MSE X SNR curves.

D. Simulation Results

Fig. 2 shows the MSE X SNR curves for both adaptations strategies with RLS algorithm. Both strategies have similar steady-state MSE performance and reach the transfer level only for SNR values greater than about 20 dB.

Fig. 3 shows the MSE curves for SNR = 25 dB. Both strategies have similar convergence speed performance and reach the transfer level with less than 1000 samples. Although the proposed strategy reaches the transfer level with fewer samples, it leads to a slightly higher steady-state MSE.

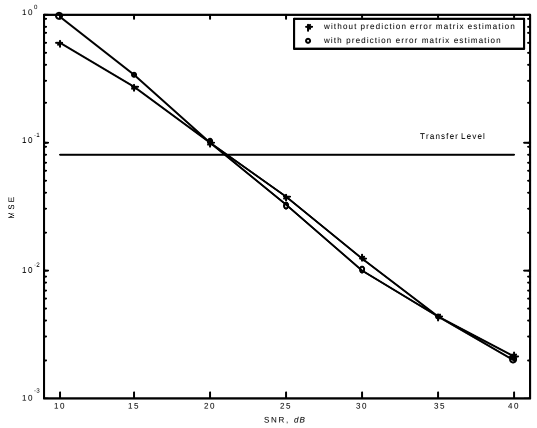

Fig. 2: zero-delay equalizer, adaptation with RLS

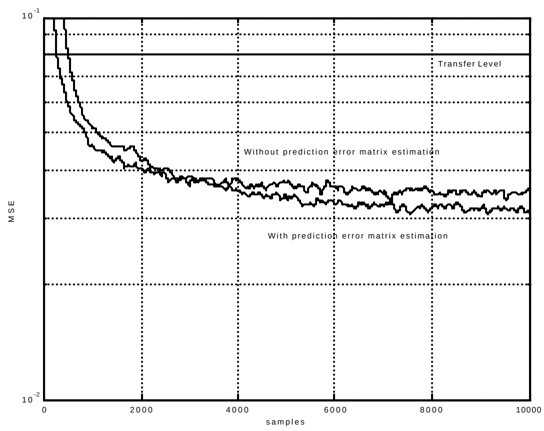

Fig. 3: zero-delay equalizer, adaptation with RLS, SNR = 25 dB

V. PROPOSED ADAPTIVE IMPLEMENTATION OF A ZF SOLUTION

The proposed adaptive implementation of a forward/backward predictor cascade structure follows the same procedure used in the previous section for derivation of an adaptive version of a zero-delay ZF equalizer. The main steps of the procedure are summarized below:

- at each iteration, $\mathbf{A}_{Lf}$ is adapted with either RLS or LMS – forward *(d_f+1)*-step prediction;

- one-step backward prediction is performed over forward prediction error signal with adaptation of B_{Mb} ;

- an estimate of backward prediction-error variance matrix is calculated as in (21)

$$E_b(k) = l\,E_b(k-1) + (1-l)\mathbf{e}_b(k)\mathbf{e}_b^H(k) \qquad (23)$$

- from (23), an estimate of **h**(df) is obtained by taking the column of (23) with highest norm;

- an estimate of *d-delay* ZF equalizer may be obtained with (18).

A. Simulation Settings

All the simulation framework of the previous section is again considered. Once again, the subequalizer length is made $L = 8$. The orders of multichannel forward and backard predictors are $Lf = 3$ and $Mb = 2$, respectively. The forward predictor step parameter is $d_f = 2$, hence the equalization delay is $d = 4$. Either both forward and backward prediction matrices are adapted with RLS or are adapted with LMS. For the sake of comparison, the zero-delay ZF equalizer is employed with estimation of forward prediction-error variance matrix (22).

B. Simulation Results

Fig. 4 shows the MSE X SNR curves for both zero-delay and *d*-delay (cascade structure) with RLS algorithm. The *d-delay* equalizer leads to a lower steady-state MSE, and reaches transfer level within a lower SNR condition.

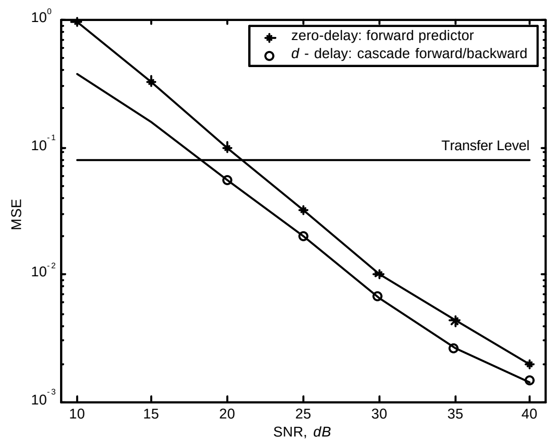

Fig. 4: adaptation with RLS, zero-delay and d-delay ZF equalizers

Fig. 5 shows the convergence curves for both ZF equalizers with RLS at SNR = 25 dB. The *d*-delay equalizer presents a lower steady-state MSE and a faster convergence speed. Also, it reaches the transfer level with fewer samples, compared to the zero-delay ZF equalizer.

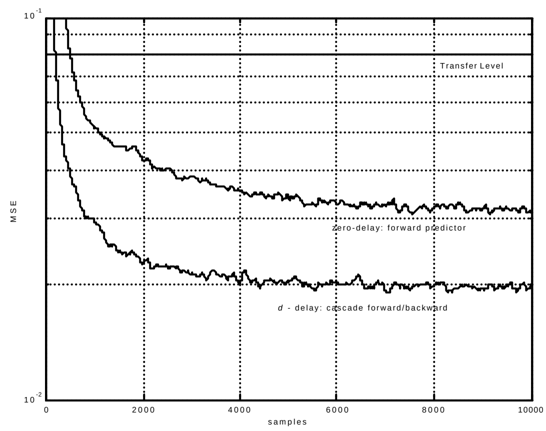

Fig. 5: adaptation with RLS, SNR = 25 dB, zero-delay and d-delay ZF equalizers

Fig. 6 presents the MSE x SNR curves for the cascade structure with adaptation driven by RLS and LMS. Fig. 7 compares the convergence curves for the cascade structure with adaptation of forward and backward predictors, driven by RLS and LMS, both with SNR = 25 dB

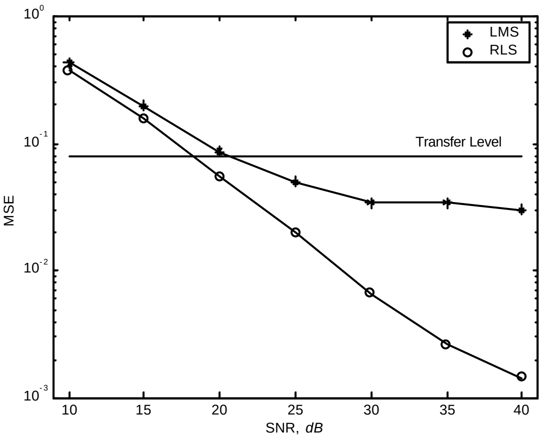

Fig. 6: d-delay equalizer, adaptation with RLS and LMS

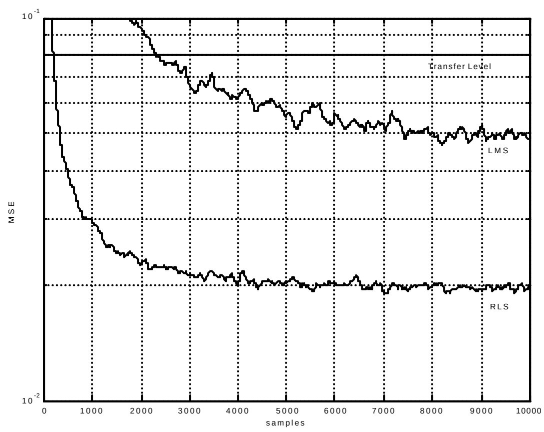

Fig. 7: d-delay equalizer, adaptation with RLS and LMS, SNR = 25 dB

When the predictors are adapted with LMS, both convergence speed and steady-state MSE performances are severely affected. MSE for adaptation with LMS could be somewhat improved at the expense of an even worse convergence speed performance.

VI. CONCLUSIONS

In this paper, a strategy for obtaining a blind ZF equalizer based on multichannel forward linear prediction was proposed and its performance was evaluated under an adaptive implementation. The proposed strategy was shown to save some computation, since it avoids the estimation of a prediction error variance matrix, and leads to an effective adaptive version.

The cascade structure comprised of a forward and a backward multichannel prediction error filters is known to result in a blind ZF equalizer. An adaptive implementation of such a structure was also proposed and so was its performance evaluated through computer simulations.

The deduction of blind ZF equalizers based on multichannel linear prediction relies on the assumption of absence of additive noise. Simulations suggest that the performance of the above algorithms is to be harmfully affected in low SNR environments. Intense research effort is currently performed in order to improve the performance of such algorithms regarding to robustness to additive noise and convergence speed.

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