SUBSPACE-BASED ESTIMATION METHODS SUITABLE FOR DOWNLINK DS-CDMA BLIND DETECTORS

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Abstract— **In subspace-based blind linear multiuser detection we may use a subspace tracking algorithm and a blind method for estimating the composite code vector, which is the convolution of the user code and the multipath channel.**

In this paper we propose two subspace tracking algorithms based on the Power method, one with $O(N^2L)$ and the other with $O(NL)$ operations per update (where N is the dimension of sampled received signal and L is the rank of the estimated subspace). We also propose a blind method for estimating the composite code vector, which uses the estimated signal subspace bases. This method requires $O(N^2L)$ operations per update (considering that L is larger than the number of multipath gains).

The proposed algorithms are suitable for subspace-based blind linear multiuser detectors for DS-CDMA system downlink. Computer simulations show superior performance of the methods proposed in comparison with other methods presented in the literature.

I. Introduction

It has been demonstrated that multiuser detection provides substantial performance gain over conventional detection techniques used in multiple-access channels. The multiuser detectors with optimal-performance was first presented in [1]. As its implementation presents high computational complexity and it is necessary to know the channels and the codes of all users, several sub-optimal solutions have been proposed [2], [3].

Subspace-based blind multiuser linear MMSE detectors (S-MMSE) [3] are suitable for downlink communication in direct-sequence code-division multiple-access (DS-CDMA) systems. It can demodulate the desired-user signal with only prior knowledge of his own code and synchronization at chip level.

The S-MMSE detectors use the bases of the space spanned by the covariance matrix of the received signal to implement a Wiener filter that estimates the desired-user symbol from the received signal.

Our implementation of the S-MMSE detector uses a subspace tracking algorithm to identify the signal subspace components and a blind adaptive method for estimating the composite code, which is the convolution of the channel and the user code. This implementation of the S-MMSE employs only one antenna (single-channel detector) as opposed to multichannel detectors which employ multiple antennas. Single-channel detectors are more interesting for the donwlink, because they are, usually, less complex, cheaper and smaller.

Due to the capacity and flexibility requirements for future communications systems, the S-MMSE detectors are among those technologies considered to be used in the future.

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We propose novel low-complexity S-MMSE detectors for DS-CDMA systems. We develop new subspace tracking adaptive algorithms based on the Power method [4], one with $O(N^2L)$ and another with $O(NL)$ operations per update, where N is the dimension of sampled received signal and L is the rank of the estimated subspace. We also propose a new method to estimate the composite code with complexity $O(N^2L)$, considering that L is larger than the number of multipath gains.

In spite of the application chosen herein (DS-CDMA multiuser detection), there are several other classical problems where the proposed algorithms could be used, such as data compression and filtering, system identification and pattern recognition.

This paper is organized as follows. In section II the signal model is introduced. In section III reduced-rank blind linear MMSE multiuser detectors are reviewed. In section IV we introduce novel subspace tracking algorithms. In section V we propose a new approach for estimating the composite code. In section VI we describe our proposed subspace-based blind detectors that combine the subspace tracking algorithms and the method for estimating the composite code introduced in sections V and VI. In section VII the performance of these proposed methods are evaluated in a subspace-based blind linear MMSE detection scheme and compared to other methods presented in the literature. Section VIII contains the conclusions.

II. Signal Model

In a synchronous DS-CDMA system scenario with K users, the received signal in a downlink channel, after the chip rate sampling at $1/T_c$ forms an N-vector for every symbol interval NT_c, given by

$$\begin{aligned}\mathbf{r} &= \textstyle\sum_{k=1}^{K} A_k b_k \sum_{j=1}^{J} h_j \mathbf{g}_{k,j} + \mathbf{n} \\ &= \textstyle\sum_{k=1}^{K} A_k b_k \mathbf{s}_k + \mathbf{n},\end{aligned} \tag{1}$$

where A_k is the power of user k, $b_k \in \{-1, +1\}$ is the binary independent and equiprobable data of user k, $\{h_j\}_{j=1}^{J}$ are multipath gains and J is the number of paths. Vector $\mathbf{n}$ is a complex Gaussian noise vector with mean $\mathbf{0}$ and covariance matrix $\sigma^2\mathbf{I}_N$[1], $\mathbf{g}_{k,j}$ is the user code delayed by jT_c, such that

$$\mathbf{g}_{k,1} = (1/\sqrt{N})\left[\beta_0^k \quad \beta_1^k \quad ..., \beta_{N-1}^k\right]^T \tag{2}$$

is the code vector of user k , and $\mathbf{s}_k$ is the received composite code vector also for user k given by

$$\mathbf{s}_k = \textstyle\sum_{j=1}^{J} h_j \mathbf{g}_{k,j} = \mathbf{Q}_k \mathbf{h}. \tag{3}$$

[1] $\mathbf{I}_N$ denotes the $N \times N$ identity matrix

In equation (3) $\mathbf{Q}_k$ is the transmitted code matrix and $\mathbf{h}$ is the channel vector, given, respectively, by

$$\mathbf{Q}_k = [\mathbf{g}_{k,1}\ \ \mathbf{g}_{k,2} \dots \mathbf{g}_{k,J}]_{N \times J}$$
$$= \begin{bmatrix} \beta_0^k & 0 & \dots & 0 \\ \beta_1^k & \beta_0^k & \dots & 0 \\ \beta_2^k & \beta_1^k & \ddots & 0 \\ \vdots & \vdots & & \\ \beta_{N-1}^k & \beta_{N-2}^k & \dots & \beta_{N-(J-1)}^k \end{bmatrix}$$

and

$$\mathbf{h} = [h_1\ \ h_2 \dots h_J]^T_{J\times 1}.$$

In this model the composite code vector has the same dimension of the code vector. It neglects intersymbol interference (ISI) between data symbols to avoid changing the dimension of the vectors for different number of multipath gains.

An alternative model is used in [2], where the composite code vector has a larger dimension due to the convolution of the code vector with the channel. Computational complexity requirements with this augmented model are higher than that for our model and, in addition, simulations have shown that the relative performance of the algorithms is the same regardless of the model used.

Extension of the results presented herein for asynchronous DS-CDMA system models should also be possible. In [5] it was shown that continuous asynchronous DS-CDMA system models can be reduced to equivalent digital synchronous system models. Therefore the results of this paper should apply to this context as well.

The received signal is assumed ciclostationary, so its statistical properties do not change on time intervals of duration T and the sequence $\{\mathbf{r}\}$ is stationary.

Throughout this work $[\cdot]^H$, $||[\cdot]||$ and $E\{[\cdot]\}$ denote transpose complex conjugate, norm and expected value of $[\cdot]$, respectively. Furthermore, the functions $sgn([\cdot])$ and $Re\{[\cdot]\}$ return the sign and the real part of $[\cdot]$, respectively.

A. Subspace Concept

The correlation matrix of the received signal is defined as $\mathbf{C} = E\{\mathbf{r}\mathbf{r}^H\}$ and its eigendecomposition (ED) is given by

$$\mathbf{C} \;=\; \mathbf{U}\boldsymbol{\Lambda}\mathbf{U}^H = \begin{bmatrix} \mathbf{U}_s & \mathbf{U}_n \end{bmatrix} \begin{bmatrix} \boldsymbol{\Lambda}_s & \\ & \boldsymbol{\Lambda}_n \end{bmatrix} \begin{bmatrix} \mathbf{U}_s^H \\ \mathbf{U}_n^H \end{bmatrix}$$

Matrix $\boldsymbol{\Lambda}_s$ is formed by the P largest eigenvalues of $\mathbf{C}$, $\lambda_1 \geq \lambda_2 \ldots \geq \lambda_P$, corresponding to the orthonormal eigenvectors of $\mathbf{U}_s = [\mathbf{u}_1\ \mathbf{u}_2 ... \ \mathbf{u}_P]$, and matrix $\boldsymbol{\Lambda}_n$ is formed by $N-P$ real eigenvalues equal to σ^2 corresponding to the orthonormal eigenvectors of $\mathbf{U}_n = [\mathbf{u}_{P+1} ... \ \mathbf{u}_N]$.

Matrix $\mathbf{C}$ can be rewritten as

$$\mathbf{C} = \lambda_1 \mathbf{u}_1 \mathbf{u}_1^H + \hat{\mathbf{C}}, \tag{4}$$

where

$$\begin{aligned} \hat{\mathbf{C}} &= \sum_{i=2}^{P} \lambda_i \mathbf{u}_i \mathbf{u}_i^H + \sum_{i=P+1}^{N} \sigma^2 \mathbf{u}_i \mathbf{u}_i^H \\ &= \mathbf{C} - \lambda_1 \mathbf{u}_1 \mathbf{u}_1^H \\ &= \mathbf{C} - \mathbf{u}_1 (\mathbf{u}_1^H \mathbf{C} \mathbf{u}_1) \mathbf{u}_1^H, \end{aligned} \tag{5}$$

is a reduced-rank approximation of $\mathbf{C}$.

Matrix $\hat{\mathbf{C}}$ can also be obtained using the deflation technique [6]. This technique consists in removing the projection of the vector $\mathbf{r}$ on $\mathbf{u}_1$, i.e.,

$$\mathbf{r}_2 = \mathbf{r} - \mathbf{u}_1 \mathbf{u}_1^H \mathbf{r}, \tag{6}$$

since the eigenvectors are orthonormal. It can be verified that $\hat{\mathbf{C}} = E\{\mathbf{r}_2 \mathbf{r}_2^H\}$.

III. Subspace-based Blind Linear Multiuser MMSE Detectors

For simplicity, denote x_k=$\{A_k b_k\}_{k=1}^K$. The problem at hand is to estimate x_k from the received signal $\mathbf{r}$ assuming prior knowledge of only the composite code $\mathbf{s}_k$.

Let $\hat{x}_k$ be the estimate of x_k, and the symbol transmitted by user k be calculated as

$$\hat{b}_k = sgn(Re\{\hat{x}_k\}). \tag{7}$$

The constrained optimal linear estimator of x_k is a vector $\mathbf{m}$ which minimizes the mean-squared-error (MSE) function given by

$$\Phi(\mathbf{m}) = E\{(x_k - \mathbf{m}^H \mathbf{r})^2\}, \tag{8}$$

subjected to $\mathbf{m}^H \mathbf{s}_k = 1$. The solution of this constrained minimization problem is $\mathbf{m}_k$, given by [3]

$$\mathbf{m}_k \;=\; \frac{1}{\mathbf{s}_k^H \mathbf{U}_s \boldsymbol{\Lambda}_s^{-1} \mathbf{U}_s^H \mathbf{s}_k} \mathbf{U}_s \boldsymbol{\Lambda}_s^{-1} \mathbf{U}_s^H \mathbf{s}_k. \tag{9}$$

Since $\mathbf{s}_k$ is orthogonal to the noise subspace, $\mathbf{U}_n^H \mathbf{s}_k = 0$, we could express $\mathbf{m}_k$ using only the signal subspace parameters $\{\mathbf{U}_s, \boldsymbol{\Lambda}_s\}$.

Minimizing the MSE is equivalent to maximizing the signal-to-interference-plus-noise-ratio (SINR), because the transmitted symbols are assumed independent. In spite of the bit error rate (BER) being the performance measurement of interest, the SINR is more easily calculated and is more convenient to compare linear detectors. Moreover, in certain conditions the SINR is a good predictor of the BER [7].

The principal characteristic of the S-MMSE detectors is that the algorithm estimates the subspace parameters in order to construct the filter $\mathbf{m}_k$ in its closed form.

In order to implement reduced-rank S-MMSE detectors it is necessary:

- to estimate the signal subspace parameters $\{\mathbf{U}_s, \boldsymbol{\Lambda}_s\}$. These parameters can be estimated using traditional methods such as: ED and SVD decompositions [4], or with subspace tracking adaptive algorithms [4], [6], [8], [9], that have lower computational complexity;
- to estimate the signal subspace rank [3]. This avoids the estimation of noise subspace bases, that are unnecessary;
- to estimate the composite code $\mathbf{s}_k$. This parameter can be estimated using only the code of the user of interest and the signal subspace bases [2], [3].

IV. Subspace-Tracking Algorithms

We propose novel algorithms to estimate the bases of the subspace spanned by the autocorrelation matrix of a vector sequence to be used in S-MMSE detectors. In the context of signal detection, we will use a sequence of vectors $\mathbf{r}$ from our model of DS-CDMA systems.

The class of algorithms proposed here is based on the Power method, but differently from other similar algorithms also based on the Power method, like the Natural Power method [10], the proposed algorithms estimate only one principal eigenvector of a reduced-rank estimate of the correlation matrix.

In the next sections we will derive and present the two algorithms suitable for subspace tracking, namely, Sequential Power Algorithm versions one and two.

A. Sequential Power Algorithm: Version 1 *(SP1)*

The estimated correlation matrix of $\mathbf{r}$ is obtained recursively with

$$\mathbf{C}(n) = \tau\mathbf{C}(n-1) + \mathbf{r}(n)\mathbf{r}(n)^H, \tag{10}$$

where τ is a constant $\in (0,1]$ called forgetting factor and $\mathbf{r}(n)$ is the nth received vector. This is a rank-1 updating version of an exponentially weighted sample correlation matrix suitable for nonstationary scenarios.

Define matrices $\hat{\mathbf{C}}_1(n) = \mathbf{C}(n)$ and $\hat{\mathbf{C}}_{i+1}(n)$ as a reduced-rank approximation of $\hat{\mathbf{C}}_i(n)$ given by (see equation (5))

$$\begin{aligned}\hat{\mathbf{C}}_{i+1}(n) = & \ \hat{\mathbf{C}}_i(n) - \left(\mathbf{w}_i^H(n)\hat{\mathbf{C}}_i(n)\mathbf{w}_i(n)\right)\mathbf{w}_i(n)\mathbf{w}_i^H(n) \\ = & \ \hat{\mathbf{C}}_i(n) - \hat{\lambda}_i(n)\mathbf{w}_i(n)\mathbf{w}_i^H(n),\end{aligned}$$

where $\mathbf{w}_i(n)$ and $\hat{\lambda}_i(n)$ are, respectively, estimates of the principal eigenvector and the principal eigenvalue of $\hat{\mathbf{C}}_i(n)$. Define also the matrix

$$\mathbf{W}(n) = [\mathbf{w}_1(n) \ \ \mathbf{w}_2(n)\ldots\mathbf{w}_L(n)]_{N\times L},$$

If $L = P$ this matrix is an estimate of $\mathbf{U}_s$.

The Power method is used to estimate the principal eigenvector of $\hat{\mathbf{C}}_i(n)$, so

$$\begin{aligned}\mathbf{w}'_i(n) &= \hat{\mathbf{C}}_i(n)\mathbf{w}_i(n-1) \\ \mathbf{w}_i(n) &= \mathbf{w}'_i(n)/\|\mathbf{w}'_i(n)\|.\end{aligned} \tag{11}$$

Considering that the vectors $\{\mathbf{w}'_i(n)\}_{i=1}^L$ are orthogonal and vary slowly with n, the estimate of the eigenvalue can be obtained as

$$\|\mathbf{w}'_i(n)\|^2 = \mathbf{w}'_i(n)^H\mathbf{w}'_i(n) \tag{12}$$

because

$$\begin{aligned}\mathbf{w}'_i(n)^H\mathbf{w}'_i(n) &= \mathbf{w}'_i(n-1)^H\hat{\mathbf{C}}_i^2(n)\mathbf{w}'_i(n-1) \\ &\approx \mathbf{w}'_i(n)^H\hat{\mathbf{C}}_i^2(n)\mathbf{w}'_i(n) = \hat{\lambda}_i^2(n).\end{aligned}$$

The algorithm SP1 is summarized below:

SP1

Choose $\mathbf{C}(0)$ and $\{\mathbf{w}_i(0)\}_{i=1}^L$ suitably
FOR $n = 1, 2, \cdots$ **DO**

$$\begin{aligned}\mathbf{C}(n) &= \tau_1\mathbf{C}(n-1) + \mathbf{r}(n)\mathbf{r}(n)^H \\ \hat{\mathbf{C}}_1(n) &= \mathbf{C}(n)\end{aligned}$$

FOR $i = 1, 2\cdots, L$ **DO**

$$\begin{aligned}\mathbf{w}'_i(n) &= \hat{\mathbf{C}}_i(n)\mathbf{w}_i(n-1) \\ \hat{\lambda}_i(n) &= \|\mathbf{w}'_i(n)\| \\ \mathbf{w}_i(n) &= \mathbf{w}'_i(n)/\hat{\lambda}_i(n) \\ \hat{\mathbf{C}}_{i+1}(n) &= \hat{\mathbf{C}}_i(n) - \hat{\lambda}_i(n)\mathbf{w}_i(n)\mathbf{w}_i(n)^H\end{aligned}$$

The algorithm SP1 requires $(4N^2 + 2N)L + 3N^2$ real aritmetic operations per update of $\mathbf{W}(n)$.

As the algorithm SP1 uses the Power method to obtain $\hat{\lambda}_i(n)$, the convergence rate of each $\mathbf{w}_i(n)$ is governed by the ratio of the two largest eigenvalues of $\hat{\mathbf{C}}_i(n)$ [4].

The Natural Power algorithms proposed in [10], that are also based on the Power method, are different implementations of the following equation:

$$\mathbf{W}(n) = \frac{\mathbf{C}(n)\mathbf{W}(n-1)}{\sqrt{\mathbf{W}(n-1)^H\mathbf{C}^2(n)\mathbf{W}(n-1)}}. \tag{13}$$

These algorithms are natural extensions of the Power method to estimate matrices of eigenvectors.

B. Sequential Power Algorithm: Version 2 (SP2)

A second, and less computationally demanding version of the sequential Power algorithm can be derived as follows:

Notice that

$$\begin{aligned}\mathbf{w}'_i(n) &= \hat{\mathbf{C}}_i(n)\mathbf{w}'_i(n-1) \\ &= \tau\hat{\mathbf{C}}_i(n-1)\mathbf{w}'_i(n-1) + \\ &\quad\ \mathbf{r}_i(n)\mathbf{r}_i(n)^H\mathbf{w}'_i(n-1).\end{aligned}$$

Thus, using the approximation

$$\begin{gathered}\hat{\mathbf{C}}_i(n-1)\mathbf{w}'_i(n-1) \approx \\ \hat{\mathbf{C}}_i(n-1)\mathbf{w}'_i(n-2) = \mathbf{w}'_i(n-1)\end{gathered} \tag{14}$$

the vector $\mathbf{w}'_i(n)$ can be obtained as:

$$\mathbf{w}'_i(n) = \tau\mathbf{w}'_i(n-1) + \mathbf{r}_i(n)\mathbf{r}_i(n)^H\mathbf{w}'_i(n-1). \tag{15}$$

Similarly, notice that

$$\begin{aligned}\hat{\lambda}_i(n) &= \mathbf{w}_i(n)^H\hat{\mathbf{C}}_i(n)\mathbf{w}_i(n) \\ &= \tau\mathbf{w}_i(n)^H\hat{\mathbf{C}}_i(n-1)\mathbf{w}_i(n) + \\ &\quad\ \mathbf{w}_i(n)^H\mathbf{r}_i(n)\mathbf{r}_i(n)^H\mathbf{w}_i(n).\end{aligned} \tag{16}$$

Thus, using the approximation

$$\begin{gathered}\mathbf{w}_i(n)^H\hat{\mathbf{C}}_i(n-1)\mathbf{w}_i(n) \approx \\ \mathbf{w}_i(n-1)^H\hat{\mathbf{C}}_i(n-1)\mathbf{w}_i(n-1) = \hat{\lambda}_i(n-1),\end{gathered} \tag{17}$$

$\hat{\lambda}_i(n)$ can be obtained as:

$$\hat{\lambda}_i(n) = \tau\hat{\lambda}_i(n-1) + \mathbf{w}_i(n)^H\mathbf{r}_i(n)\mathbf{r}_i(n)^H\mathbf{w}_i(n). \tag{18}$$

In order to estimate $\mathbf{w}_{i+1}(n)$ we must remove the projection of $\mathbf{r}_i(n)$ along the direction of $\mathbf{w}_i(n)$ forming $\mathbf{r}_{i+1}(n)$, where $\mathbf{r}_1(n) = \mathbf{r}(n)$. This process is repeated until L eigenvectors are estimated.

The algorithm SP2 is summarized bellow:

SP2

Choose $\{\mathbf{w}_i(0)\}_{i=1}^L$ and $\{\hat{\lambda}_i(0)\}_{i=1}^L$ suitably
FOR $n = 1, 2, \cdots$ **DO**

$$\mathbf{r}_1(n) = \mathbf{r}(n)$$

FOR $i = 1, 2\cdots, L$ **DO**

$$\begin{aligned}\gamma(n) &= \mathbf{r}_i(n)^H\mathbf{w}_i(n-1) \\ \mathbf{w}'_i(n) &= \tau\mathbf{w}'_i(n-1) + \gamma(n)\mathbf{r}_i(n) \\ \hat{\lambda}_i(n) &= \tau\hat{\lambda}_i(n-1) + \gamma^2(n) \\ \mathbf{w}_i(n) &= \mathbf{w}'_i(n)/\hat{\lambda}_i(n) \\ \mathbf{r}_{i+1}(n) &= \mathbf{r}_i(n) - \mathbf{w}_i(n)\mathbf{w}_i(n)^H\mathbf{r}_i(n)\end{aligned}$$

This algorithm requires $(10N + 2)L = O(NL)$ real aritmetic operations per update of $\mathbf{W}(n)$.

V. Channel and Composite code estimation

In this section we propose a subspace-based blind method to estimate the multipath gains and we use this to estimate the composite code, which is needed in S-MMSE detector.

This method works as follows: the multipath gains are estimated minimizing a projection of a vector in the noise subspace. Then these gains are used to construct the estimated composite code.

Let $\hat{\mathbf{s}}_k$ and $\hat{\mathbf{h}}$ be estimates of $\mathbf{s}_k$ and $\mathbf{h}$, respectively. These estimates are related according to

$$\hat{\mathbf{s}}_k = \mathbf{Q}_k \hat{\mathbf{h}}. \tag{19}$$

The proposed method consists in adapting vector $\hat{\mathbf{h}}$ to minimize the projection of $\hat{\mathbf{s}}_k$ in the noise subspace. In other words we wish to minimize

$$\begin{aligned} \Theta(\hat{\mathbf{s}}_k) &= \|\hat{\mathbf{s}}_k - \mathbf{U}_s\mathbf{U}_s^H\hat{\mathbf{s}}_k\|^2 \\ &= tr(\hat{\mathbf{s}}_k\hat{\mathbf{s}}_k^H - \hat{\mathbf{s}}_k^H\mathbf{U}_s\mathbf{U}_s^H\hat{\mathbf{s}}_k). \end{aligned} \tag{20}$$

Substituting $\hat{\mathbf{s}}_k$ by equation (19), we obtain

$$\Theta(\hat{\mathbf{h}}) = tr(\hat{\mathbf{h}}^H\mathbf{Q}_k^H\mathbf{Q}_k\hat{\mathbf{h}} - \hat{\mathbf{h}}^H\mathbf{Q}_k^H\mathbf{U}_s\mathbf{U}_s^H\mathbf{Q}_k\hat{\mathbf{h}}). \tag{21}$$

In this work we wish to minimize $\Theta(\hat{\mathbf{h}})$ in (21) subjected to the constraint that $\|\hat{\mathbf{h}}\| = 1$. Note that the channel estimate has an arbitrary phase ambiguity, as any other subspace-based blind method also has, which can be easily solved by differentially encoding and decoding the data symbols [3].

Equation (21) is similar to that used in methods derived from the algorithm MUSIC [11] when we substitute $\mathbf{U}_n\mathbf{U}_n^H$ by $\mathbf{I}_N - \mathbf{U}_s\mathbf{U}_s^H$.

The gradient of the function $\Theta_1(\hat{\mathbf{h}})$ with respect to $\hat{\mathbf{h}}$ is given by

$$\begin{aligned} \nabla\Theta_1(\hat{\mathbf{h}}) &= 2(\mathbf{Q}_k^H\mathbf{Q}_k - \mathbf{Q}_k^H\mathbf{U}_s\mathbf{U}_s^H\mathbf{Q}_k)\hat{\mathbf{h}} \\ &= 2\mathbf{Q}_k^H(\mathbf{I}_N - \mathbf{U}_s\mathbf{U}_s^H)\mathbf{Q}_k\hat{\mathbf{h}}. \end{aligned} \tag{22}$$

Our proposed method for composite code estimation, called herein Code Projection (CP) method, is described below. It is based on the steepest descent method to obtain iteratively an estimate $\hat{\mathbf{h}}$ of the multipath gains of the channel. The method uses the signal subspace bases that have been obtained previously with algorithms SP1 or SP2 presented in the previous section.

CP

Choose $\hat{\mathbf{h}}(0)$ suitably
FOR $n = 1, 2, \cdots$ **DO**

$$\begin{aligned} \mathbf{W}(n) &= \text{Estimate of } \mathbf{U}_s \text{ at iteration } n \\ \mathbf{W}_n(n) &= \mathbf{I}_N - \mathbf{W}(n)\mathbf{W}^H(n) \\ \hat{\mathbf{h}}'(n) &= \hat{\mathbf{h}}(n-1) - 2\mu\mathbf{Q}_k^H\mathbf{W}_n(n)\hat{\mathbf{s}}_k'(n-1) \\ \hat{\mathbf{h}}(n) &= \hat{\mathbf{h}}'(n)/\|\hat{\mathbf{h}}'(n)\| \\ \hat{\mathbf{s}}_k'(n) &= \mathbf{Q}_k\hat{\mathbf{h}}(n) \\ \hat{\mathbf{s}}_k(n) &= \hat{\mathbf{s}}_k'(n)/\|\hat{\mathbf{s}}_k'(n)\| \end{aligned}$$

The constant μ is the convergence factor, and the vectors $\hat{\mathbf{h}}(n)$ and $\hat{\mathbf{s}}_k(n)$ are the estimates of $\mathbf{h}(n)$ and $\mathbf{s}_k(n)$ at the iteration n, respectively.

The number of real aritmetic operations required per update by the method CP is $(2NL + 2N + 4J + 1)N + 4J$.

VI. Subspace-based Blind Linear Multiuser MMSE Detectors

Subspace-based blind linear multiuser MMSE detectors can be proposed by combining the method CP and any of two algorithms presented in section IV to estimate the bases of the signal subspace. The choice between the two alternatives depends on the acceptable computational complexity and on the performance requirements. Henceforth the detector that uses algorithm SP1 will be called S-MMSE-SP1, and the detector that uses algorithm SP2 will be called S-MMSE-SP2. The rank of the signal subspace can be estimated in an adaptive way based on estimated eigenvalues [3], but here we assume that the number of active users is known. In the next section we compare the performance of these detectors with other methods proposed in the literature.

VII. Simulations

A. Performance Comparison of Subspace Tracking Algorithms

This simulation compares the algorithms SP1 and SP2 with the RO-FST algorithm [9] used in the S-MMSE detector proposed in [2], and with the algorithm PASTd used in the S-MMSE detector proposed in [8]. Algorithms RO-FST and PASTd require $O(NL)$ operations per update.

Example 1: We assumed a synchronous DS-CDMA system that uses Hadamard sequences with $N = 16$ and the channel modeled as a tapped delay line with $J = 3$ and impulse response given by

$$h_1(t) = -0.2067\delta(t) + 0.6133\delta(t-1) + 0.2667\delta(t-2).$$

There were 6 independent users ($K = 6$) with the same signal to noise ratio $(SNR) = 30dB$, i.e., $\{A_k^2/\sigma^2 = 1000\}_{k=1}^K$.

The performance measurement utilized was the distance [4] between the subspace spanned by $\mathbf{W}(n)$ obtained with the algorithms SP1, SP2, RO-FST and PASTd and the subspace spanned by $\mathbf{U}_s$, estimated offline from the SVD of

$$\mathbf{C}(n) = \frac{1}{n}\sum_{i=1}^{n}\mathbf{r}(n)\mathbf{r}^H(n) \tag{23}$$

after $n = 12000$ received vectors.

The forgetting factors used in algorithms SP1 and SP2 were $\tau_1 = 0.9997$ and $\tau_2 = 0.997$, respectively. For the algorithm RO-FST we used forgetting factors equal to $\tau_3 = 0.9997$ and $\tau_3 = 0.997$, and for the algorithm PASTd we used forgetting factor $\tau_4 = 0.997$.

The algorithms were initialized with $\hat{\mathbf{C}}(0) = \mathbf{I}_N$, $L = 6$ and $\mathbf{W}(0)$ formed by the first L columns of $\mathbf{I}_N$.

Figure 1 presents an average over 200 simulations of the measured distances. Observe that the subspace estimate provided by the algorithm SP1 was closer to the true subspace and converged faster. The algorithms SP2 and PASTd presented almost the same performance and the algorithm RO-FST with $\tau_3 = 0.9997$ presented slightly better performance than both, but still inferior to that of the algorithm SP1.

The columns of the matrix $\mathbf{W}(n)$ obtained with the PASTd algorithm are not normalized, what can be a disadvantage in some applications. Which can be disadvantage of the FST algorithm is that it does not estimate the eigenvalues.

The convergence rate of the algorithm RO-FST with $\tau_3 = 0.9997$ was slow. When the forgetting factor was reduced to

$\tau_3 = 0.997$, the convergence rate improved, but at the expense of much larger distance from the optimum.

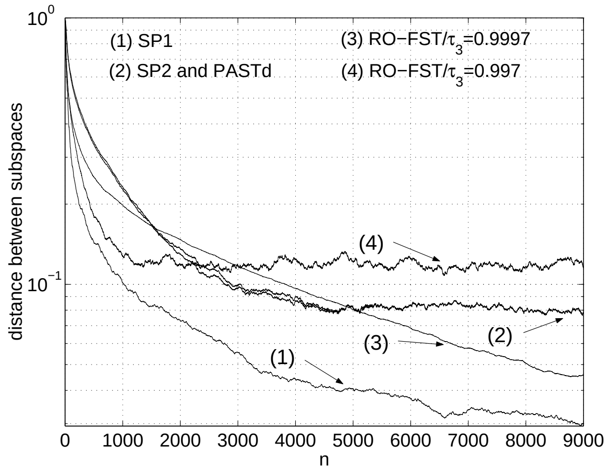

Fig. 1. Distance between the subspace spanned by $\mathbf{U}_s$ and the subspace spanned by $\mathbf{W}(n)$ obtained using the algorithms SP1, SP2, RO-FST and PASTd.

Example 2: In this example we compare the algorithms when the channel changes abruptly. The channel was

$$h_2(t) = 0.73\delta(t) + 0.21\delta(t-1) - 0,1\delta(t-2)$$

for $n \leq 5000$ and was switched to the same channel $h_1(t)$ used in example 1 above for $n > 5000$.

The initial conditions were the same as those used in example 1. The forgetting factors $\tau_1 = 0.9993$, $\tau_2 = \tau_4 = 0.997$ and $\tau_3 = 0.9985$ were set to make the distance obtained with all algorithms very close when $n = 9000$.

Figure 2 presents an average over 200 simulations of the distance measurements. We can observe again the superior performance of the algorithm SP1. The distance achieved was smaller than those achieved by the other algorithms.

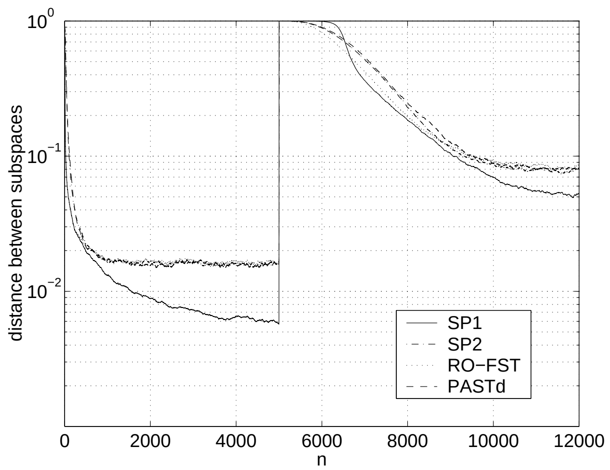

Fig. 2. Distance between the subspace spanned by $\mathbf{U}_s$ and the subspace spanned by $\mathbf{W}(n)$ obtained using the SP1, SP2, RO-FST and PASTd algorithms; The channel is $h_2(t)$ for $n \leq 5000$ and is $h_1(t)$ for $n > 5000$.

B. *Performance Comparison of Composite Code Estimators*

We compared the method CP with the subspace-based blind methods for estimating the composite code used in [3], called Eq-Wang, and the method used in [2], called Eq-Song. The method Eq-Song was derived using the the alternative model described in section I. Methods Eq-Song and Eq-Wang require $O(N^2L)$ operations per update.

For the same DS-CDMA system of simulation 1 and channel $h_1(t)$, we wanted to estimate the composite code of user 1 using the methods CP, Eq-Wang and Eq-Song. The subspace bases were estimated with the algorithm SP1 and the performance measurement was the MSE between the true composite code and the nth estimated composite code, given by

$$\Xi(n) = \|\mathbf{s}_1 - \hat{\mathbf{s}}_1(n)\|^2. \tag{24}$$

The constants $L = 6$ and $\tau = 0.9997$ and the initial conditions used in the algorithm SP1 were maintained from the previous simulation. For the method CP we used $\mu = 0.02$, for the Eq-Wang method we used $\mu = 0.2$ and $c = 1$. It is not necessary to use any convergence factor in the Eq-Song method, for the Power method is used to find the maximal eigenvector of a matrix.

All the elements of the initial condition vectors used in methods CP and Eq-Wang were equal to 0.001 and we used the code of the user of interest for the initial conditions on method Eq-Song.

Figure 3 presents an average over 200 simulations of the $\Xi(n)$ for these 3 methods. We can observe that the Eq-Song method converges faster than the others. Is is because the choice of the initial conditions and the use of the Power method for optimization, but the estimation was not as good as the others after convergence. The CP and Eq-Wang methods presented almost the same steady state performance, but the CP method converged faster.

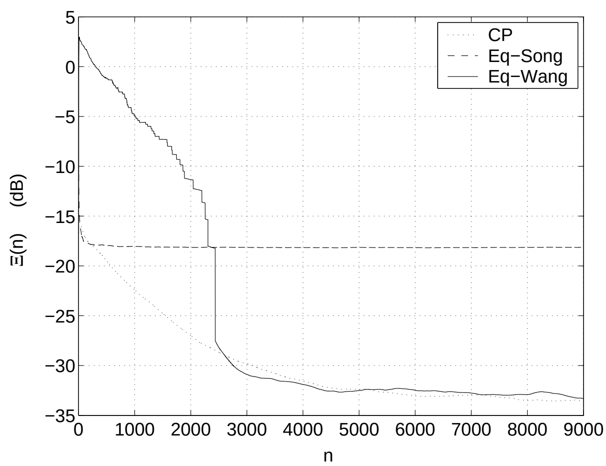

Fig. 3. MSE between the composite code vector and the estimated code vector obtained with methods CP, Eq-Wang and Eq-Song. The channel is $h_1(t)$ and the bases are estimated with the algorithm SP1.

The method CP has shown a good compromise between convergence speed, accuracy of the estimate and computational complexity. Although all three methods require $O(N^2L)$ computational complexity per update (considering $L \geq J$), the number of adapted parameters in the method CP does not change with the size of the code, and this is an important advantage when the user codes are increased.

C. Performance Comparison of S-MMSE detectors

We compared the detectors S-MMSE-SP1 and S-MMSE-SP2 with the S-MMSE detector proposed in [3], called S-MMSE-Wang and with an approximation of the S-MMSE detector proposed in [2], called S-MMSE-Song.

The detector S-MMSE-Wang uses the PASTd algorithm for subspace tracking and the Eq-Wang method for composite code estimation, and the detector S-MMSE-Song uses the RO-FST algorithm for subspace tracking and the Eq-Song method for composite code estimation.

We wanted to recover $M = 9000$ transmitted data symbols transmitted by the user 1 in the same DS-CDMA system of simulation 1 and channel $h_1(t)$.

All initial conditions, constants and convergence factors were the same as in the previous simulations, except the convergence factor used in the method Eq-Wang that we had to reduce to $\mu = 0.08$, because the PASTd algorithm used in the S-MMSE-Wang detector was slower than the algorithm SP1. In simulation 2 we used the method Eq-Wang with the estimated bases given by the algorithm SP1.

The performance measurements is the approximated SINR, given by

$$SINR(n) = \frac{\bar{\mu}(n)^2}{\frac{1}{N_r-1}(\sum_{i=1}^{N_r} |\mathbf{m}_{1,i}^H(n)\mathbf{r}_i(n) - \bar{\mu}(n)|)^2},$$

where

$$\bar{\mu}(n) = \frac{1}{N_r}\sum_{i=1}^{N_r} |\mathbf{m}_{1,i}^H(n)\mathbf{r}_i(n)|,$$

$\mathbf{m}_{1,i}(n)$ is the filter $\mathbf{m}_1$ of ith repetition in iteration n and N_r is the number of repetitions done for each experiment.

Figure 4 presents the $SINR(n)$ with $Nr = 300$. Notice the superior performance of the S-MMSE-SP1 as compared to the other detectors tested. The detector S-MMSE-SP2 provided similar performance as compared to the detector S-MMSE-Wang.

Although the computational complexity of the algorithm SP1 is $O(N^2L)$ and the algorithm SP2, PASTd and RO-FST require only $O(NL)$, all detectors require $O(N^2L)$ operations per update. Notice that the computational requirements for the subspace tracking algorithm and the composite code estimator are used in tandem to implement the filter $\mathbf{m}_k$.

VIII. Conclusion

We proposed two low complexity algorithms to estimate the bases of the subspace spanned by the autocorrelation matrix of a vector sequence and a blind subspace-based method to estimate the composite code to be used in subspace-based blind multiuser linear detection. In all simulations the proposed algorithm SP1 for subspace tracking and method CP for composite code estimation outperformed the others and showed a good trade off between performance and complexity.

To evaluate all methods proposed, we used them in S-MMSE detectors and compared with others described in the literature. The performance gainned by using a better subspace tracking algorithm, like algorithm SP1, in the detection was substantial.

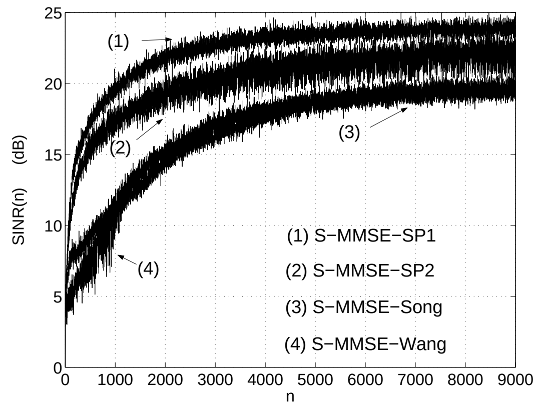

Fig. 4. $SINR(n)$ obtained using the detectors S-MMSE-SP1, S-MMSE-SP2, S-MMSE-Wang and S-MMSE-Song; channel $h_1(t)$ and $SNR = 30dB$.

The proposed detector has a computational complexity comparable to that of other detectors known in the literature, converges faster and to a higter SINR.

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