Adaptive Minimum BER Channel Equalisation in DMT Systems

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Abstract— **In this paper we investigate the use of adaptive minimum bit error rate (MBER) algorithms for channel equalisation in Discrete Multitone (DMT) systems. These algorithms approximate the bit error rate (BER) from training data using linear equaliser structures operating in the frequency domain. A comparative analysis of DMT systems with linear equalisers, employing minimum mean squared error (MMSE) and MBER algorithms is carried out. Computer simulation experiments show that the MBER algorithms outperform the LMS approach and can save transmitting power for the same BER performance.**

I. Introduction

Recent advances in multimedia applications and the development of the Internet have increased the demand for high-speed digital communications, which require broadband channels. The spectral shaping of these broadband communication channels imposes several limitations to data transmission such as intersymbol interference (ISI) and fading [1], [2]. Amongst the previously reported approaches to mitigate ISI we have full channel equalisation and multicarrier modulation (MCM). Full channel equalisation combats the spectral shaping effect of a channel using a filter which is called an equaliser. Although linear equalisers are easy to implement, their computational complexity under high sampling rates is too much complex for commercial applications. Multicarrier modulation is one possible solution for high-speed digital communications. In contrast to single carrier modulation, multicarrier modulation avoids full equalisation of a channel, uses available bandwidth efficiently by controlling the power and number of bits in each subchannel, is robust against impulsive noise and fast fading due to its long symbol duration, and avoids narrowband distortion by simply disabling one or more subchannels [1],[2].

In multicarrier modulation the original hostile communication channel, that usually exhibit deep spectral nulls, is partitioned into several small bandwidth subchannels. If a subchannel is narrow enough so that the channel gain in the subchannel is approximately a constant, then no ISI would occur in this subchannel. Thus, information can be transmitted over these narrowband subchannels without ISI, and the total number of bits transmitted is the sum of the number of bits transmitted in each subchannel. If the available power were distributed over the subchannels using the SNR of each subchannel, then high spectral efficiency could be achieved. One of the most efficient ways to partition a channel into a large number of narrowband channels is the fast Fourier transform (FFT). Multicarrier modulation implemented via a FFT is called Discrete Multitone (DMT) modulation or Orthogonal Frequency Division Multiplexing (OFDM). In transmission, the key difference between the two methods is in the assignment of bits to each subchannel. Indeed, DMT and OFDM differ in the loading algorithm, since OFDM puts an equal number of bits on all subchannels, rather than optimising the number of bits and the energy in each subchannel, as in DMT. Moreover, DMT is deployed in wireline applications such as telephone lines, whereas OFDM is used for wireless applications [1], [2].

Channel equalisers in DMT systems employing the minimum mean squared error (MMSE) [3]-[4] criterion have become rather successful, since they usually show good performance and have simple adaptive implementation [3]-[4]. However, it is well known that the MSE cost function is not optimal in digital communications applications, and the most appropriate cost function is the bit error rate (BER) [5],[6]. The approximate minimum bit error rate (AMBER) [5] and the least bit error rate (LBER) [6] are two of the most successful and suitable algorithms for adaptive implementation. However, these minimum bit error rate (MBER) algorithms usually require long training sequences to converge to lower bit error rates than those achieved by the techniques that employ the MSE cost function. Since DMT systems require an initialisation period to adjust their parameters and perform power allocation on the subchannels, these systems can cope with long training sequences and are suitable to MBER algorithms.

In this work, we investigate the use of adaptive MBER algorithms for channel equalisation in DMT systems using QAM signal constellations. Firstly, the channel is estimated using a binary training sequence to adjust its parameters via stochastic gradient algorithms. Then, we perform power allocation using Chow´s algorithm [1] on the subchannels and determine the modulation scheme (no transmission, 4-QAM or 16-QAM) to be used on each subchannel. We conduct a comparative analysis of linear equalisers, employing the LMS [7], the AMBER [5] and the LBER [6]. Computer simulation experiments show that the MBER approaches outperform the traditional MMSE solution via the LMS algorithm, whilst requiring no additional computational complexity.

This paper is organised as follows. Section II briefly describes the DMT communication system model. The channel equalisation problem and stochastic gradient algorithms are detailed in Sections III and IV. Section V presents and discusses the simulation results and Section VI gives the concluding remarks of this work.

II. DMT System Model

We assume a DMT communication system based on digital filter banks with M channels created by the Fast Fourier Transform (FFT) that transmits modulated symbols through a telephone-type communication channel and followed by an adaptive equaliser, as shown in Fig. 1. To describe the symbol mapping in a DMT system based on the FFT, we assume that the symbol sequence $x_n(k)$, where $n = 0,\ 1,\ 2, \ldots, M-1$. In the case of real time-domain signals, there is a restriction that implies that there are $M/2$ complex dimensions when M is even [1],[2]. To represent QAM symbols, an in-phase component $x_{c,n}(k)$ and a quadrature component $c_{s,n}(k)$ are employed to form a complex sequence $x_n(k)$ with M points, as expressed by:

$$x_n(k) = \begin{cases} 0 & n = 0 \\ x_{c,n}(k) \ + \ jx_{s,n}(k) & 1 \leq n \leq \frac{M}{2} - 1 \\ 0 & n = \frac{M}{2} \\ x_{c,n}(k) \ - \ jx_{s,n}(k) & \frac{M}{2} + 1 \leq n \leq M - 1 \end{cases} \tag{1}$$

The inverse FFT (IFFT) is then applied to the complex sequence given in (1) and tranformed into another set given by:

$$u_n(k) = \frac{1}{M} \sum_{i=0}^{M-1} x_i(k) e^{-jin2\pi/M}, \quad i = 0, 1, \ldots, M-1 \tag{2}$$

Note that this approach ensures that the IFFT yields a real sequence. The M signals are then interpolated and interleaved in time, resulting in a composite signal $u(k)$ which operates at a rate M times higher.

Consider a discrete time communication channel H, where $u(k)$ is the composite transmitted signal in the frequency domain, $H_n(k)$ is the channel frequency response in subchannel n for the DMT tone k and $e_n(k)$ is additive white gaussian noise (AWGN) with power spectrum density σ^2. The output signal $v_n(k)$ of the channel n is expressed by:

$$r_n(k) = H_n(k) u_n(k) + e_n(k) = s_n(k) + e_n(k) \tag{3}$$

where $s_n(k)$ is the channel output without noise for subchannel n.

At the receiver the discrete-time signal is equalised in the frequency domain, forming the set of signals $v_n(k)$, deinterleaved and decimated. Applying the FFT to these M signals we obtain the estimated signals $\hat{x}_n(k)$ as expressed by:

$$\hat{x}_n(k) = \sum_{i=0}^{M-1} v_i(k) e^{-jin2\pi/M}, \quad i = 0, 1, \ldots, M-1 \tag{4}$$

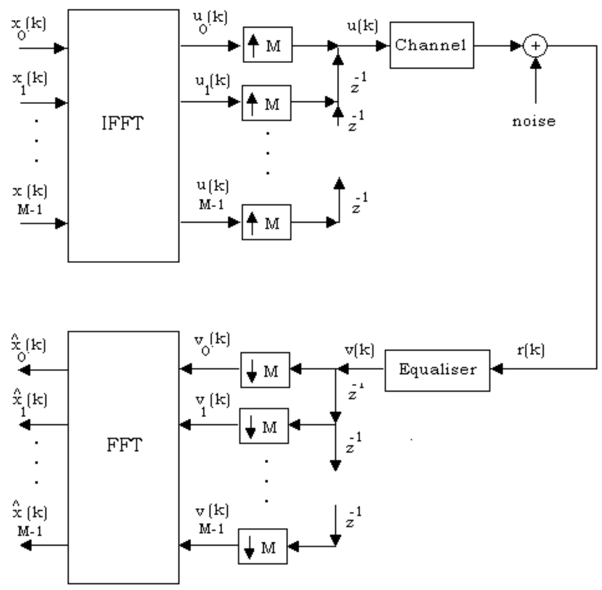

Fig. 1. Block diagram of the DMT system.

where M is the number of subchannels. Note that after the complex symbols $\hat{x}_n(k)$ are estimated, the decoder performs the demapping and detection of the QAM symbols.

III. Channel equalisation

The adaptive channel equalisation problem involves the application of a receiving filter, that adjusts its coefficients in order to minimise a given objective function [7]. The equaliser must be adaptive in order to track the signal variations imposed by the channel, however, it requires a desired signal taken from a training sequence to adjust its parameters, as shown in Fig. 2.

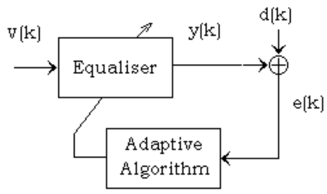

Fig. 2. Block diagram of the adaptive equalisation problem.

The linear transversal equaliser consists of a linear filter with $N+1$ taps described by the vector $\mathbf{w} = \left[w_0 \ldots w_N\right]^T$. The linear equaliser output is given by:

$$v_n(k) = \mathbf{w}_n^T \mathbf{r}_n(k) \tag{5}$$

where $\mathbf{r}_n(k) = \left[r_n(k) \ \ldots \ r_n(k-N)\right]$ is the observed output signal vector of the channel of subchannel n. The decision on the transmitted symbol $x(k)$ is determined by the equaliser output signal.

IV. Stochastic gradient algorithms

In this section, we describe stochastic gradient algorithms that adjust the parameters of the receivers based on the minimisation of the mean square error (MSE) and the bit error rate (BER) cost functions. Note that during the training period, the DMT system employs binary signalling and after the training period, the system uses QAM modulation, since it is simpler to use MBER algorithms in this situation.

A. The LMS algorithm

The adaptive equalisation solution for the linear equaliser via the LMS algorithm [7] is based on the MMSE error criterion formed by the error signal $e_n(k) = d_n(k) - y_n(k)$, and is described by:

$$\mathbf{w}_n(k+1) = \mathbf{w}_n(k) + \mu e_n(k)\mathbf{r}_n(k) \quad (6)$$

where d_n is the desired signal taken from the training sequence, $\mathbf{r}_n(k)$ is the observation vector for the linear equaliser, $y_n(k)$ is the estimated symbol after demapping and μ is the algorithm step size.

B. The AMBER algorithm

Given a transmitted binary training sequence $\mathbf{d}$, the bit error probability $P(\epsilon|\mathbf{d})$ is expressed by:

$$P\big(\epsilon|\mathbf{d}\big) = P\big(d_n(k)sgn(y(k)) = -1\big)$$

$$P\big(\epsilon|\mathbf{d}\big) = P\big(sgn(d_n(k)y_n(k)) = -1\big) = P\big(y_n(k) < 0\big) \quad (7)$$

where $y_n(k)$ is the decoded symbol or bit for channel n and $d_n(k)$ is the desired symbol taken from the training sequence.

The equaliser solution that minimises the BER criterion via the AMBER algorithm [5] for a single channel system employs the vector function $g(\mathbf{w}(k))$ [5] to approximate an expression for a coefficient vector $\mathbf{w}(k)$ that achieves a MBER performance with linear receiver structures, as described by:

$$g(\mathbf{w}(k)) = E\left[Q\left(\frac{d(k)\mathbf{w}^T(k)\mathbf{s}(k)}{\parallel \mathbf{w}(k) \parallel \sigma}\right)d(k)\mathbf{s}(k)\right] \quad (8)$$

where $d(k)$ is the desired transmitted symbol taken from the training sequence and $Q(.)$ is the Gaussian error function. A simple stochastic solution for $\mathbf{w}_n(k)$ can be derived by using $g(\mathbf{w}(k))$ and adjusting the receiver weights by:

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu g(\mathbf{w}(k)) \quad (9)$$

Note that for linear receiver structures the quantity $Q\left(\frac{d(k)\mathbf{w}^T(k)\mathbf{s}(k)}{\|\mathbf{w}(k)\|\sigma}\right)$ inside the expected value operator in (8) corresponds to the conditional bit error probability given the product $d(k)\mathbf{s}(k)$. This quantity can be replaced in (8) by an error indicator function $i_d(k)$ given by:

$$i_d(k) = \frac{1}{2}(1 - sgn(d_n(k)y(k))) \quad (10)$$

where $y(k)$ is the decoded symbol and $d(k)$ is the desired signal provided by the training sequence.

Following this approach, the AMBER algorithm, as devised for linear equalisers [5], is described by the following equalities:

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu E\left[Q\left(\frac{d(k)\mathbf{w}^T(k)\mathbf{s}(k)}{\parallel \mathbf{w}(k) \parallel \sigma}\right)d(k)\mathbf{s}(k)\right]$$

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu E\Big[E\big[i_d(k) \mid d(k)\mathbf{s}(k)\big]d(k)\mathbf{s}(k)\Big]$$

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu E\big[i_d(k)d(k)\mathbf{s}(k)\big]$$

Since $\mathbf{s}(k) = \mathbf{r}(k) - \mathbf{n}(k)$, and $i_d(k)$ and $d(k)$ are statistically independent, we have $E[i_d(k)d(k)\mathbf{n}(k)] = E[d(k)]E[i_d(k)\mathbf{n}(k)] = 0$ and thus:

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu E\Big[i_d(k)d(k)\mathbf{r}(k)\Big] \quad (11)$$

The AMBER stochastic gradient update equation for the linear equaliser operating in the frequency domain in a system with multiple channels is given by:

$$\mathbf{w}_n(k+1) = \mathbf{w}_n(k) + \mu i_{d_n}(k)d_n(k)\mathbf{r}_n(k) \quad (12)$$

Note that the expression in (6) equal (12) if we replace $e_n(k)$ by $i_d(k)d_n(k)$. In practice, a modified error indicator function $i_{d_n}(k) = \frac{1}{2}(1 - sgn(d_n(k)y_n(k) - \tau))$ is employed, where the threshold τ is responsible for increasing the algorithm rate of convergence. This algorithm updates when an error is made and also when an error is almost made, becoming a smarter choice for updating the filter coefficients.

C. The LBER algorithm

Considering a binary signalling communication system, the equaliser BER depends on the distribution of the decision variable $y(k)$, which is a function of the weights of the equaliser. The sign-adjusted decision variable for the linear equaliser $y_s(k) = sgn(x(k-D))y(k)$ is drawn from a Gaussian mixture, described by:

$$y_s(k) = sgn(x(k-D))\left(\mathbf{w}^T\mathbf{H}\mathbf{x}(k) + \mathbf{w}^T\mathbf{n}(k)\right)$$

$$y_s(k) = sgn(x(k-D))y'(k) + n'(k) \quad (13)$$

where the first term of (13) is the noise free sign-adjusted equaliser output.

Consider that K samples of the transmitted symbols $x(k)$ and K samples of the received symbols $r(k)$ are available from the samples $d(k) = x(k-D)$ of a binary training sequence. A kernel density estimate [6] of the p.d.f. of y_s is given by:

$$p_{y_s}(y_s) = \frac{1}{K\sqrt{2\pi}\rho(\mathbf{w^T w})^{1/2}}$$

$$\sum_{k=1}^{K} exp\left(\frac{-(y_s - sgn(d(k))y(k))^2}{2\rho^2\mathbf{w^T w}}\right) \quad (14)$$

where ρ is the radius parameter of the kernel density estimate [6].

Substituting the expected value of the gradient with a single point estimate, we have:

$$\hat{p}_{y_s}(y_s(k)) = \frac{1}{K\sqrt{2\pi}\rho(\mathbf{w^T w})^{1/2}}$$

$$exp\left(\frac{-(y_s - sgn(d(k))y(k))^2}{2\rho^2 \mathbf{w^T w}}\right) \quad (15)$$

The probability of error is estimated by:

$$P_\epsilon = P(y_s < 0) = \int_{-\infty}^{0} \hat{p}_{y_s}(y_s)dy_s = Q\left(\frac{sgn(d(k))y(k)}{\rho(\mathbf{w^T w})^{1/2}}\right) \quad (16)$$

The gradient term of P_ϵ is:

$$\frac{\partial P_\epsilon}{\partial \mathbf{w_f}} = \frac{exp\left(\frac{-y(k)^2}{2\rho^2\mathbf{w^T w}}\right) sgn(d(k))}{\sqrt{2\pi}\rho}\left(\frac{-\mathbf{r}(k)}{(\mathbf{w^T w})^{1/2}} + \frac{\mathbf{w}y(k)}{(\mathbf{w^T w})^{3/2}}\right) \quad (17)$$

An algorithm similar to the LMS was devised in [6] by substituting the exact pdf by its instantaneous estimate and adjusting the receiver weights such that $\mathbf{w}^T(k)\mathbf{w}(k) = 1$:

$$\mathbf{w}(k+1) = \mathbf{w}(k) - \mu\left[\frac{\partial P_\epsilon}{\partial \mathbf{w}}\right]_k \quad (18)$$

The LBER algorithm can be used in a DMT system with M subchannels to equalise DMT tones in the frequency domain by using the following expression:

$$\mathbf{w}_n(k+1) = \mathbf{w}_n(k) + \mu\frac{1}{\sqrt{2\pi}\rho}exp\left(\frac{-(y(k))^2}{2\rho^2}\right) sgn(d(k))$$

$$\times(\mathbf{r}_n(k) - \mathbf{w}_n(k)y_n(k)) \quad (19)$$

where $\mathbf{w}(k) = [w_0 \ \ldots \ w_N]$ is the receiver coefficient vector, $d_n(k)$ is the desired signal taken from the binary training sequence, $y_n(k)$ is the decoded binary symbol in the DMT system for channel n, μ is the algorithm step size and ρ the radius parameter which is related to the noise standard deviation σ. Whilst in the AMBER, a non-zero τ defines a region boundary where the algorithm will continue to update, in the LBER, the effect of the distance from the decision boundary is controlled by an exponential term [6].

V. Simulations

In this section, we conduct simulation experiments to assess the BER performance of the linear equalisers operating with the algorithms described and perform a comparative analysis of them. To evaluate the receivers, we have simulated their operation under a typical telephone-type communication channel.

The simulation experiments, conducted to assess the BER performance of the different algorithms, employed 1000 training symbols averaged over 20 independent experiments. Note that during the training period, the system employs binary signalling since the equalisation is performed in the frequency domain and the algorithms were described for the binary signalling case for the sake of simplicity. The DMT system has $M = 256$ subchannels and process 10^5 data symbols, which have QAM constellations with 4 and 16 points. Power allocation is performed using Chow´s algorithm [1] with a rate adaptive loading criterion on the subchannels. The rate adaptive criterion maximises the number of bits per symbol subject to a given energy constraint [1]. This algorithm determines the modulation scheme (no transmission, 4-QAM or 16-QAM) to be used on each subchannel given the SNR, the channel gain and the subchannel gap Γ. Furthermore, we use a small fixed threshold $\tau = 0.1$ for the AMBER algorithm, $\rho = 8\sigma^2$ for the LBER method and subchannel gap $\Gamma = 1$. In addition, the linear equalisers have 1 tap in each channel, since they operate in the frequency domain, and the stochastic gradient algorithms operate with $\mu = 0.01$. We consider a linear channel with transfer function $H(z) = \frac{2.5(0,1-0,1z^{-2})}{1-1,5z^{-1}+0.54z^{-2}}$, whose frequency response is given in Fig. 3.

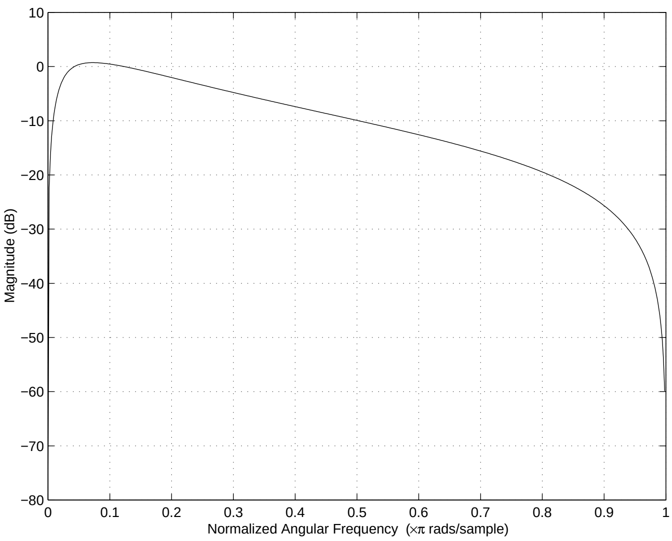

Fig. 3. Frequency response of the channel $H(z) = \frac{2.5(0,1-0,1z^{-2})}{1-1,5z^{-1}+0.54z^{-2}}$.

The BER performance of the different adaptive algorithms using the DMT system described is shown in Fig. 4. According to the curves the system operating with the LBER algorithm achieved the best BER performance, followed by the AMBER and the LMS algorithm.

The DMT system operating with the LBER algorithm can save up to 1 dB in comparison with the LMS approach, for the same SNR. When using the AMBER algorithm, the DMT system can save up to 0.5 dB in comparison with the LMS algorithm, whereas the LBER saves up to 0.5 dB in comparison with the AMBER technique, for the same SNR.

Note that the LBER algorithm performance is rather weak at low SNR values, whilst it shows good performance at high SNR values. On the other hand, the AMBER algorithm shows good performance at both high and low SNR, whilst requiring a lower computational complexity, since due to the presence of the error indicator function weight updating occurs less frequently.

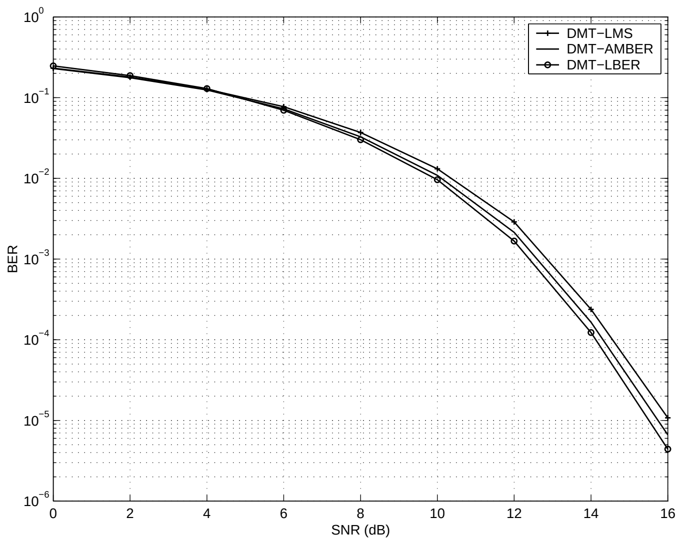

Fig. 4. BER performance of the DMT system operating with the LMS, the AMBER and the LBER algorithms.

VI. Concluding Remarks

We investigated the use of adaptive minimum bit error rate (MBER) algorithms for channel equalisation in DMT system applications. The algorithms approximate the bit error rate (BER) from training data using linear transversal equaliser structures operating in the frequency domain. A comparative analysis of frequency-domain linear equalisers in DMT systems, employing minimum mean squared error (MMSE) and MBER algorithms was carried out. Computer simulation experiments have shown that the LBER and the AMBER approaches outperform the LMS algorithm and can save transmitting power, whilst achieving the same BER performance.

References

[1] T. Starr, J. M. Cioffi, and P. J. Silverman, *Understanding Digital Subscriber Line Technology*, Prentice-Hall PTR, 1999.

[2] D. J. Rauschmayer, *ADSL/VDSL Principles: A Practical and Precise Study of Asymetric Digital Subscriber Lines and Very High Speed Digital Subscriber Lines*, Macmillan Technology Series, 1999.

[3] J. S. Chow, J. M. Cioffi, and J. A. Bingham, “Equalizer training algorithms for multicarrier modulation systems,” *Proc. IEEE Int. Conference on Communications*, vol. 2, Geneva, Switzerland, pp. 761-765, May 1993.

[4] N. Al-Dhahir and J. M. Cioffi, “Optimum finite-length equalization for multicarrier transceivers,” *IEEE Trans. on Communication*, vol. 44, pp. 56-63, January 1996.

[5] C. Yeh and J. R. Barry, “Approximate minimum bit-error rate equalization for binary signaling”, *IEEE Proc International Conference on Communications*, vol. 1, 1998, pp. 16-20.

[6] B. Mulgrew and S. Chen, “Stochastic Gradient Minimum BER Decision Feedback Equalisers,” *Proc. IEEE Adaptive Systems for Signal Processing, Communications, and Control Symposium.*, Canada, October 2000, pp. 93-98.

[7] S. Haykin, *Adaptive Filter Theory*, 3rd edition, Prentice-Hall, Englewood Cliffs, NJ, 1996.