Linear Phase Impulse Response Shortening for xDSL DMT modems

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Abstract – **The problem of shortening the effective channel impulse response in xDSL is addressed. A new approach is described based on linear phase FIR equaliser filters. The mathematical framework for the LPIRS method is developed and simulation results are presented for several VDSL loops.**

For a similar performance, the new method offers a design with considerably lower computational complexity and a simpler implementation, on either DSP or dedicated hardware solutions.

I – Introduction

Discrete Multitone Modulation is a powerful modulation scheme [1], adopted by the standardisation bodies [2], [3] in the development of recent xDSL (Digital Subscriber Line) standards. One of the main issues in the xDSL systems is the severe attenuation and distortion caused by the channel and other services on the same bundle. To overcome these channel effects on the transmitted signals, equalisation techniques are commonly used in time [4], [5], [6], [7], [8] and frequency [9] domain. The equalisation process can be simplified by periodically extending each symbol with a convenient number of prefix and suffix samples [1]. Hence, circular convolution can be used instead of linear convolution, thus the channel equalisation can then be easily done in the frequency domain. Unfortunately, this is only possible if the effective length of the channel impulse response is shorter than the guard period (cyclic prefix and suffix). In order to shorten the impulse response several methods have been proposed [4], [5], [6], [7], [8], based on FIR (Finite Impulse Response) equalisers.

In this paper we describe an impulse response shortening method based on linear phase FIR equalisers.

In the next section we review the original Minimum Shortening Signal-to-Noise Ratio (MSSNR) method [4] and the improvements introduced in [5]. In section III we develop the Linear Phase Impulse Response Shortening (LPIRS) method and the mathematical framework that leads to the optimal solution for every type of linear phase filter, based on full rank extension matrices.

Simulation results are presented in section IV which show that the achieved shortened signal noise ratio (SSNR) are similar to the other methods, with a considerably lower computational complexity.

Finally, in section V, we discuss the influence of some design parameters, such as, the filter type (symmetry) and length, and draw conclusions about the new method.

II – Maximum Shortening SNR Method

Unlike previously done [7], this method explicitly uses the length n of the cyclic prefix CP as the desired effective length of channel impulse response, thus minimising the channel's ISI (Inter Symbolic Interference). As Melsa et al. refer in [4], there is no straight relation between the achievable bit rate and this method, but this is surely a good criterion. The coefficients of the optimal shortening filter are obtained by an eigenvector decomposition of the energy function SSNR.

Consider the effective channel $\mathbf{h}_{\text{eff}}$ in matrix form as

$$\mathbf{h}_{\text{eff}} = \mathbf{H}\mathbf{w}, \tag{1}$$

where $\mathbf{H}$ is the convolution matrix of the channel impulse response and $\mathbf{w}$ is the equaliser filter of length t. Defining the vector $\mathbf{h}_{\text{win}}$ as a window of n+1 consecutive samples of $\mathbf{h}_{\text{eff}}$, starting at sample d, and the vector $\mathbf{h}_{\text{wall}}$ as the remaining samples of $\mathbf{h}_{\text{eff}}$, they are given by

$$\mathbf{h}_{win} = \mathbf{H}_{win}\mathbf{w} = \begin{bmatrix} h_{eff}(d) \\ h_{eff}(d+1) \\ \vdots \\ h_{eff}(d+n) \end{bmatrix} = \begin{bmatrix} h(d) & h(d-1) & \cdots & h(d-t+1) \\ h(d+1) & h(d) & \cdots & h(d-t+2) \\ \vdots & \vdots & \ddots & \vdots \\ h(d+n) & h(d+n+1) & \cdots & h(d+n-t+1) \end{bmatrix} \begin{bmatrix} w(0) \\ w(1) \\ \vdots \\ w(t-1) \end{bmatrix} \tag{2}$$

and

$$\mathbf{h}_{wall} = \mathbf{H}_{wall}\mathbf{w} = \begin{bmatrix} h_{eff}(0) \\ \vdots \\ h_{eff}(d-1) \\ h_{eff}(d+n+1) \\ \vdots \\ h_{eff}(M+t-2) \end{bmatrix}$$

$$= \begin{bmatrix} h(0) & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ h(d-1) & h(d-2) & \cdots & h(d-t) \\ h(d+n+1) & h(d+n) & \cdots & h(d+n-t+2) \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & h(M-1) \end{bmatrix} \begin{bmatrix} w(0) \\ w(1) \\ \vdots \\ w(t-1) \end{bmatrix} \quad (3)$$

The energy of both vectors can be expressed as

$$\mathbf{h}_{wall}^T\mathbf{h}_{wall} = \mathbf{w}^T\mathbf{H}_{wall}^T\mathbf{H}_{wall}\mathbf{w} = \mathbf{w}^T\mathbf{A}\mathbf{w} \quad (4)$$

$$\mathbf{h}_{win}^T\mathbf{h}_{win} = \mathbf{w}^T\mathbf{H}_{win}^T\mathbf{H}_{win}\mathbf{w} = \mathbf{w}^T\mathbf{B}\mathbf{w}, \quad (5)$$

where **A** and **B** are presumably positive definite matrices. The solution to the shortening problem can be found by minimising (4) or maximising (5).

Originally, in [4], the optimal solution was found by minimising the energy of $\mathbf{h}_{\text{wall}}$. To avoid an unbounded solution [6], an unit energy constraint was imposed on $\mathbf{h}_{\text{win}}$, given by

$$\|\mathbf{h}_{win}\|^2 = 1. \quad (6)$$

Assuming that matrix **B** is positive definite, we can use Cholesky decomposition to obtain

$$\mathbf{B} = \mathbf{QDQ}^T \left(\mathbf{Q}\sqrt{\mathbf{D}}\right)\left(\mathbf{Q}\sqrt{\mathbf{D}}\right)^T = \sqrt{\mathbf{B}}\sqrt{\mathbf{B}}^T, \quad (7)$$

where **D** is the diagonal matrix formed from the eigenvalues of **B** and the columns of **Q** are the orthonormal eigenvectors of **B**. Defining the following matrix

$$\mathbf{C} = \left(\mathbf{Q}\sqrt{\mathbf{D}}\right)^{-1}\mathbf{A}\left(\sqrt{\mathbf{D}}\mathbf{Q}^T\right)^{-1} = \left(\sqrt{\mathbf{B}}\right)^{-1}\mathbf{A}\left(\sqrt{\mathbf{B}}^T\right)^{-1}, \quad (8)$$

the optimal solution is given as

$$\mathbf{w}_{opt} = \left(\sqrt{\mathbf{B}}^T\right)^{-1}\mathbf{l}_{\min}, \quad (9)$$

where $\mathbf{l}_{\min}$ is the unit-length eigenvector corresponding to the minimum eigenvalue $l_{\min}$ of **C**.

The optimal shortening SNR is defined as

$$SSNR_{opt} = 10\log\left(\frac{\mathbf{w}_{opt}^T\mathbf{B}\mathbf{w}_{opt}}{\mathbf{w}_{opt}^T\mathbf{A}\mathbf{w}_{opt}}\right) = -10\log\left(l_{\min}\right), \quad (10)$$

which depends on the minimum eigenvalue $l_{\min}$ of the composite matrix **C**. This approach holds as long as **B** is non-singular and $\sqrt{\mathbf{B}}$ exists, which is true if the length of the equaliser t is shorter than n [5].

In [5], the solution to the shortening problem is found by maximising (5) and imposing an unit energy constraint on $\mathbf{h}_{\text{wall}}$,

$$\|\mathbf{h}_{wall}\|^2 = 1. \quad (11)$$

and assuming that matrix **A** is non-singular. So, **A** can be decomposed using Cholesky decomposition as

$$\mathbf{A} = \mathbf{QDQ}^T = \left(\mathbf{Q}\sqrt{\mathbf{D}}\right)\left(\mathbf{Q}\sqrt{\mathbf{D}}\right)^T = \sqrt{\mathbf{A}}\sqrt{\mathbf{A}}^T \quad (12)$$

and the new composite matrix **C** is defined as

$$\mathbf{C} = \left(\mathbf{Q}\sqrt{\mathbf{D}}\right)^{-1}\mathbf{B}\left(\sqrt{\mathbf{D}}\mathbf{Q}^T\right)^{-1} = \left(\sqrt{\mathbf{A}}\right)^{-1}\mathbf{B}\left(\sqrt{\mathbf{A}}^T\right)^{-1}. \quad (13)$$

The new solution is given in [5] as

$$\mathbf{w}_{opt} = \left(\sqrt{\mathbf{A}}^T\right)^{-1}\mathbf{l}_{\max}, \quad (14)$$

where $\mathbf{l}_{\max}$ is the unit-length eigenvector corresponding to the maximum eigenvalue $l_{\max}$ of **C**.

The new shortening SNR is then defined as

$$SSNR_{opt} = 10\log\left(\frac{\mathbf{w}_{opt}^T\mathbf{B}\mathbf{w}_{opt}}{\mathbf{w}_{opt}^T\mathbf{A}\mathbf{w}_{opt}}\right) = 10\log\left(l_{\max}\right). \quad (15)$$

This new solution holds for every choice of t and, as it was shown in [5], the performance is the same as the original [4].

A common and key issue is to guaranty the non-singularity of both **A** and **B** matrices. By ensuring that there is at least one non-zero sample of $\mathbf{h}_{\text{eff}}$ outside $\mathbf{h}_{\text{win}}$, guaranties, by definition, that **A** is positive definitive. The same notion can be applied to matrix **B**.

III – THE LPIRS METHOD

Linear phase is an important property in digital signal processing. It ensures the absence of phase distortion on the signal (only an integer delay) and a simpler implementation complexity, in regard of the number of required multipliers.

Having this in mind, we propose to constraint the equaliser filter **w** to have linear phase. This can be easily done as

$$\mathbf{w}_L = \mathbf{Lx}, \quad (16)$$

where $\mathbf{w}_{\text{L}}$ is the linear phase equaliser filter of length t, **L** is an extension matrix (see Table 1 and 2) and **x** is a generic vector.

TABLE 1: Extension matrices $\mathbf{L}_{(t\times t/2)}$ for even length vectors.

Symmetric	Anti-symmetric
$\mathbf{L} = \begin{bmatrix} 1 & 0 & 0 & \cdots & & 0 \\ 0 & 1 & 0 & & & \vdots \\ 0 & 0 & \ddots & & & \\ & & & 0 & 1 & 0 \\ \vdots & & & 0 & 0 & 1 \\ & & & 0 & 0 & 1 \\ & & & 0 & 1 & 0 \\ 0 & 0 & ⋰ & & & \\ 0 & 1 & 0 & & & \vdots \\ 1 & 0 & 0 & \cdots & & 0 \end{bmatrix}$	$\mathbf{L} = \begin{bmatrix} 1 & 0 & 0 & \cdots & & 0 \\ 0 & 1 & 0 & & & \vdots \\ 0 & 0 & \ddots & & & \\ & & & 0 & 1 & 0 \\ \vdots & & & 0 & 0 & 1 \\ & & & 0 & 0 & -1 \\ & & & 0 & -1 & 0 \\ 0 & 0 & ⋰ & & & \\ 0 & -1 & 0 & & & \vdots \\ -1 & 0 & 0 & \cdots & & 0 \end{bmatrix}$

TABLE 2: Extension matrices $\mathbf{L}_{(t\times(t-1)/2)}$ for odd length vectors.

Symmetric	Anti-symmetric
$\mathbf{L}=\begin{bmatrix} 1 & 0 & 0 & \cdots & & & 0 \\ 0 & 1 & 0 & & & & \vdots \\ 0 & 0 & & \ddots & & & \\ & & & & 0 & 1 & 0 \\ \vdots & & & & 0 & 0 & 1 \\ & & & & 0 & 1 & 0 \\ 0 & 0 & & \iddots & & & \\ 0 & 1 & 0 & & & & \vdots \\ 1 & 0 & 0 & \cdots & & & 0 \end{bmatrix}$	$\mathbf{L}=\begin{bmatrix} 1 & 0 & 0 & \cdots & & & 0 \\ 0 & 1 & 0 & & & & \vdots \\ 0 & 0 & & \ddots & & & \\ & & & & 0 & 1 & 0 \\ \vdots & & & & 0 & 0 & 0 \\ & & & & 0 & -1 & 0 \\ 0 & 0 & & \iddots & & & \\ 0 & -1 & 0 & & & & \vdots \\ -1 & 0 & 0 & \cdots & & & 0 \end{bmatrix}$

There are four possible $\mathbf{L}$ matrices depending on the length of vector $\mathbf{w}_L$ and the type of symmetry adopted.
As in [4] and [5], we rewrote (4) and (5) as

$$\mathbf{h}_{wall}^T\mathbf{h}_{wall} = \mathbf{w}_L{}^T\mathbf{H}_{wall}^T\mathbf{H}_{wall}\mathbf{w}_L = \mathbf{x}^T\mathbf{A}_L\mathbf{x} \qquad (17)$$

$$\mathbf{h}_{win}^T\mathbf{h}_{win} = \mathbf{w}_L{}^T\mathbf{H}_{win}^T\mathbf{H}_{win}\mathbf{w}_L = \mathbf{x}^T\mathbf{B}_L\mathbf{x}, \qquad (18)$$

where $\mathbf{A}_L$ and $\mathbf{B}_L$ are given by

$$\mathbf{A}_L = \mathbf{L}^T\mathbf{A}\mathbf{L}, \qquad (19)$$

$$\mathbf{B}_L = \mathbf{L}^T\mathbf{B}\mathbf{L}. \qquad (20)$$

In this work we adopted the solution proposed in [5], since it holds independently of the lengths of the *CP* or the equaliser.
Assuming that a proper delay is chosen so that $\mathbf{A}$ is positive definite, we can easily show that $\mathbf{A}_L$ is also positive definite as

$$\mathbf{w}_L{}^T\mathbf{A}\mathbf{w}_L = (\mathbf{L}\mathbf{x})^T\mathbf{A}(\mathbf{L}\mathbf{x}) = \mathbf{x}^T(\mathbf{L}^T\mathbf{A}\mathbf{L})\mathbf{x} = \mathbf{x}^T\mathbf{A}_L\mathbf{x} > 0, \qquad (21)$$

since the matrices $\mathbf{L}$ are full rank ($t/2$ or $(t-1)/2$, respectively, for even or odd length filters).
As in [5], Cholesky decomposition is used on matrix $\mathbf{A}_L$,

$$\mathbf{A}_L = \left(\mathbf{Q}\sqrt{\mathbf{D}}\right)\left(\mathbf{Q}\sqrt{\mathbf{D}}\right)^T = \sqrt{\mathbf{A}_L}\sqrt{\mathbf{A}_L}^T. \qquad (22)$$

Using a similar development as in [5], we define

$$\mathbf{C}_L = \left(\sqrt{\mathbf{A}_L}\right)^{-1}\mathbf{B}_L\left(\sqrt{\mathbf{A}_L}^T\right)^{-1} \qquad (23)$$

and the optimum solution for the equaliser $\mathbf{w}_L$ is given by

$$\mathbf{w}_{Lopt} = \mathbf{L}\left(\sqrt{\mathbf{A}_L}^T\right)^{-1}\mathbf{l}_{L\max}, \qquad (24)$$

where $\mathbf{l}_{L_{\max}}$ is the unit-length eigenvector corresponding to the maximum eigenvalue $l_{L\max}$ of $\mathbf{C}_L$.
Is very important to refer that the dimensions of matrices $\mathbf{A}_L$ and $\mathbf{B}_L$ are $t/2\times t/2$ or $(t-1)/2\times(t-1)/2$, which results in a substantial complexity reduction on the Choleski decomposition and eigen analysis, when compared to [4] and [5].

IV – SIMULATION RESULTS

To evaluate the performance of the proposed method, we have done several experiments on different VDSL test loops from [3] and [10].
In Fig. 1 we show the original impulse response of the FSAN Cabinet #3 test loop, one of the most difficult VDSL test loops. In Fig. 2 we show the impulse response of the same loop after MSSNR equalisation.
In all figures we have used 4096 carries, a target impulse response length of 64 samples and a equaliser $\mathbf{w}$ with a length $t = 10$.

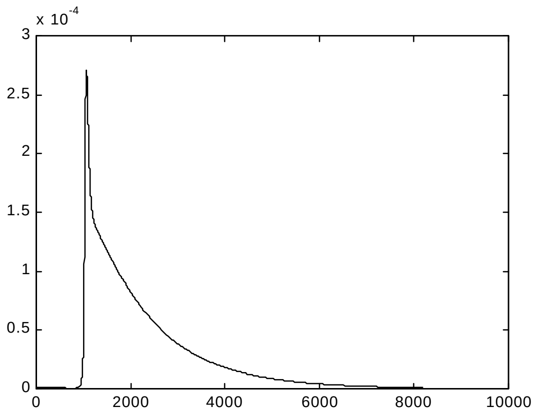

Fig. 1. FSAN Cabinet #3 impulse response.

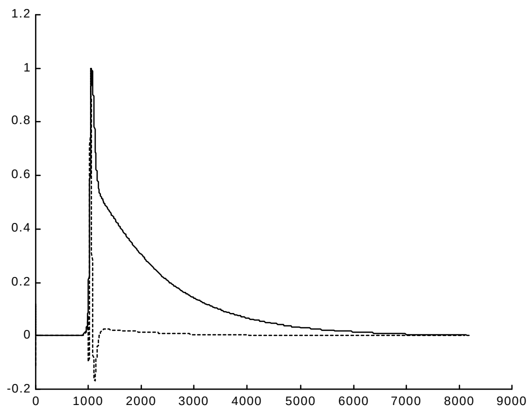

Fig. 2. FSAN Cabinet #3 original and MSSNR equalised impulse response (with dashed line).

In Fig. 3 is represented the impulse response of FSAN Cabinet #3 test loop, with MSSNR and LPIRS (symmetric) equalisation.

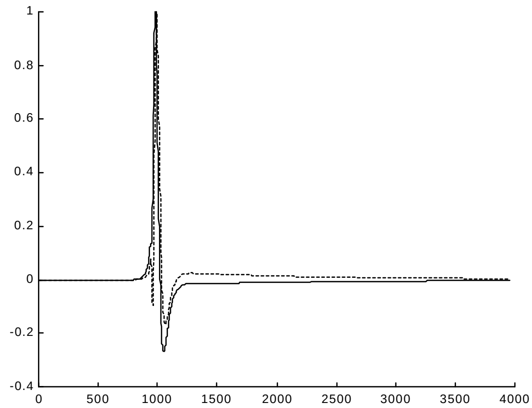

Fig. 3. FSAN Cabinet #3 MSSNR (with dashed line) and LPIRS equalised impulse response.

For performance comparison purpose we present several results in Table 3 and 4, considering the same target impulse response and number of carriers as in the previous figures. In Table 3 we have used an equaliser of length $t = 10$.

TABLE 3: Results in dB for an even length equaliser.

Test Loops	MSSNR	LPIRS	
		Symmetric	Anti-symmetric
ANSI VDSL Loop 3	70.00	69.94	69.90
ANSI VDSL Loop 4	38.61	38.38	36.96
ANSI VDSL Loop 7	47.18	39.17	44.79
FSAN FTTEx #3	73.65	68.36	67.47
FSAN Cab #3	11.79	7.94	8.01

In Table 4 we have used an equaliser of length $t = 19$.

TABLE 4: Results in dB for an odd length equaliser.

Test Loops	MSSNR	LPIRS	
		Symmetric	Anti-symmetric
ANSI VDSL Loop 3	71.72	71.72	71.62
ANSI VDSL Loop 4	40.33	39.92	39.45
ANSI VDSL Loop 7	52.90	47.93	48.23
FSAN FTTEx #3	81.53	78.11	77.18
FSAN Cab #3	12.05	8.17	9.06

Considering the short equaliser filter lengths (low complexity implementation), the results in Tables 3 and 4 show, from a practical point of view, a similar performance between the MSSNR and LPIRS methods, taking in consideration the high levels of obtained SSNR (with the exception of the highly demanding FSAN Cabinet #3 loop).

V – CONCLUSIONS

A new approach to the problem of shortening the effective channel impulse response in xDSL as been described using linear phase FIR equaliser filters. The use of the extension matrices is the key issue in this new method. The resulting objective functions are defined with half size of the original matrices, leading to notably complexity reduction. At the moment this is a crucial point as neither DSP nor dedicated hardware solutions are able to implement the full VDSL standards. As shown in section IV, the equaliser performance attained by the LPIRS method is very much the same as that of the reference method proposed by Melsa et al.

REFERENCES

[1] John A. C. Bingham, “ADSL, VDSL and Multicarrier Modulation, *Wiley Series in Telecommunications and Signal Processing*, 2000.

[2] ETSI TM, “VDSL – Part 2: Transceiver Specification”, TS 101 270-2 v.1.1.1, Feb 2001.

[3] T1E1.4, “VDSL Metallic Interface – Part 1: Functional Requirements and Common Specification”, Draft Trial-Use Standard, Feb. 2001.

[4] Peter Melsa, Richard Younce and Charles Rohrs, “Impulse response shortening for discrete multitone transceivers”, *IEEE Trans. on Communications*, vol. 44, nº 12, pp. 1662-1672, December 1996.

[5] Changchuan Yin and Guangxin Yue, “Optimal impulse response shortening for discrete multitone transceivers”, *Electronics Letters*, vol. 34, pp. 35-36, January 1998.

[6] D. Daly, C. Heneghan and A. Fagan, “A minimum mean-squared error interpretation of residual ISI channel shortening for discrete multitone transceivers”, *Proc. IEEE International Conference on Acoustic Speech and Signal Processing (ICASSP)*, 2001.

[7] Naofal Al-Dhahir, John Cioffi, “Optimum finite-length equalization for multicarrier transceivers”, *Proc. IEEE Transactions on Communications*, vol. 44, nº1, pp 56-64, January 1996.

[8] Romed Schur, Joachim Speidel, Ralf Angerbauer, *“Reduction of guard interval by impulse compression for DMT modulation on twisted pair cables”*, IEEE Global Telecommunications Conference, pp. 1632-1636, November 2000.

[9] Katleen Van Acker, Geert Leus, Marc Moonen, Olivier Van de Wiel and Thierry Pollet, “Per tone equalization for DMT-based systems”, *Proc. IEEE Trans. on Communications*, vol. 49, nº.1, pp. 109-119, January 2001.

[10] Full Service Access Networks group (FSAN), www.fsan.org.