QPSK Detection Schemes for Rayleigh Fading Channels

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Abstract—**This paper presents a performance analysis of two QPSK detection schemes for flat Rayleigh fading channels. Closed-form expressions for the bit error probability of both schemes are obtained. It is shown that they present the same error probability and a complexity analysis is carried out to determine the best scheme in terms of the number of operations (multiplications, divisions, subtractions and comparisons) executed by each detector.**

Keywords— **Wireless communications, Rayleigh fading channel, detection schemes, system complexity.**

I. Introduction

Wireless communications systems are subject to severe multipath fading that can seriously degrade their performance. The use of coding and diversity techniques to mitigate the effects of fading has been explored by several authors [1–7]. However, many of those techniques cause an increase in the system complexity, mainly at the receiver. As a consequence, the delay introduced in the system is increased since the processing time also increases with the system complexity. Considering power-limited mobile handsets, the large number of operations leads to a higher power consumption and, consequently, to shorter operation times. In this way, the search for low-complexity (and faster) techniques is a challenging task which has attracted the attention of many researchers [8].

The amplitude and phase fluctuations inherent in wireless channels significantly degrade the bit-error rate performance of QPSK systems, where the receiver must scale the received signal to normalize the channel gain, so that its decision regions correspond to the transmitted signal constellation. Another approach is to scale the constellation symbols instead of the received signal.

In this work, two QPSK detection schemes (corresponding to the previous mentioned approaches) for flat Rayleigh fading channels are presented and their performance is evaluated in terms of the bit error probability and complexity. Closed-form expressions for the bit error probability of both schemes are obtained. It is shown that they present the same error probability and a complexity analysis is carried out to determine the best scheme in terms of the number of operations (multiplications, divisions, subtractions and comparisons) executed by each detector. To the best of the authors' knowledge, this type of performance comparison is new.

This paper is organized as follows. Section II presents the system model and the detection schemes used in this work. The performance of those detectors are obtained in terms of bit error probability and number of operations in Section III. Simulations results are presented and discussed in Section IV. Finally, Section V presents the conclusions.

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II. System Model

Consider the wireless system depicted in Figure 1 where the transmitter uses QPSK modulation.

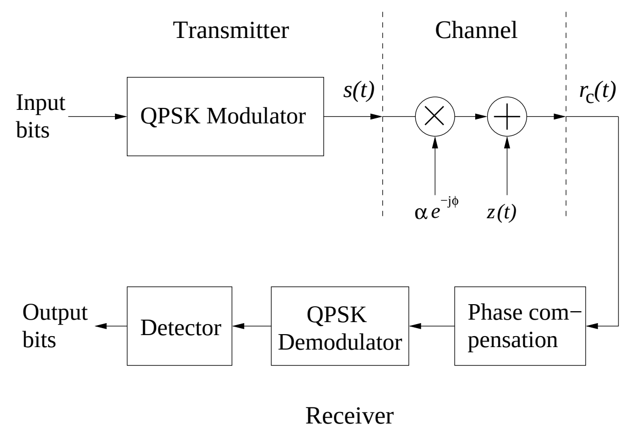

Fig. 1. The system model.

Assuming a frequency-nonselective, slowly fading channel, the received signal $\boldsymbol{r}_c(t)$ can be expressed as

$$\boldsymbol{r}_c(t) = \alpha e^{-j\phi}\boldsymbol{s}(t) + \boldsymbol{z}(t), \quad 0 \le t \le T, \tag{1}$$

where $\boldsymbol{s}(t)$ represents the transmitted signal, α is the fading amplitude, ϕ is the phase shift due to the channel, $z(t)$ denotes the additive Gaussian white noise (AWGN), and T is the signaling interval. In the lowpass representation, $\boldsymbol{r}_c(t)$, $\boldsymbol{s}(t)$ and $\boldsymbol{z}(t)$ are complex-valued random variables. Furthermore, the condition that the channel fades slowly implies that the multiplicative parameter $\alpha e^{-j\phi}$ may be regarded as a constant during at least one signaling interval.

The fading amplitude α is modeled as a Rayleigh[1] random variable (r.v.) whose probability density function (pdf) is expressed as

$$p_A(\alpha) = 2\alpha e^{-\alpha^2}u(\alpha), \tag{2}$$

where $u(\cdot)$ is the step-unit function. The additive noise $z(t)$ is modeled as a two-dimensional Gaussian r.v. having zero mean and variance $N_0/2$ by dimension. Consider, without loss of generality, a normalized fading power, that is, $E[\alpha^2] = 1$, where $E[\cdot]$ is the expected value operator.

Assuming that the channel fading is sufficiently slow so that the phase shift ϕ can be estimated from de received signal without error (that is, ideal phase channel information), the receiver

[1] The Rayleigh distribution is frequently used to model multipath fading with no direct line-of-sight (LOS) path.

can perform the phase compensation (multiplication of $\boldsymbol{r}_c(t)$ by $e^{j\phi}$). Then, the resulting received signal $\boldsymbol{r}(t)$ can be expressed as

$$\begin{aligned}\boldsymbol{r}(t) &= \boldsymbol{r}_c(t) \cdot e^{j\phi} = \alpha \boldsymbol{s}(t) + \boldsymbol{z}(t) \cdot e^{j\phi} \\ &= \alpha \boldsymbol{s}(t) + \boldsymbol{\eta}(t).\end{aligned} \tag{3}$$

It is important to note that the additive noise $\boldsymbol{\eta}(t) = \boldsymbol{z}(t) \cdot e^{j\phi}$ is also a two-dimensional Gaussian r.v. having zero mean and variance $N_0/2$ per dimension. This follows from the fact that the noise $\boldsymbol{z}(t)$ is additive and its probability density function $p_N(\eta)$ is independent of the transmitted signal. Moreover, the error probability is unaffected by a rotation, since $p_N(\eta)$ is spherically symmetric [9, pp. 247].

The *maximum a posteriori* criterion establishes that the optimum detector, on observing $\boldsymbol{r}(t)$, sets $\tilde{\boldsymbol{s}}(t) = \boldsymbol{s}_k(t)$ as the received symbol, whenever the decision function

$$P(\boldsymbol{s}_i) p_r(\boldsymbol{r}|\boldsymbol{s} = \boldsymbol{s}_i), \qquad i = 0, 1, 2, 3,$$

is maximum for $i = k$ [10].

Based on the *maximum a posteriori* criterion and considering equiprobable constellation symbols, two different strategies can be used for determining the most probable transmitted QPSK symbol from the noisy observation $\boldsymbol{r}(t)$. According to these strategies, two detectors can be defined:

- Detector I (DI): Compare $\boldsymbol{r}(t)$ with all the constellation QPSK symbols (multiplied by α) and choose as the received symbol the closest one to $\boldsymbol{r}(t)$, that is, the one that minimizes the metric $|\boldsymbol{r}(t) - \alpha \boldsymbol{s}_i(t)|$;
- Detector II (DII): Compare $\boldsymbol{r}(t)/\alpha$ with all the constellation symbols and choose as the received symbol the closest one to $\boldsymbol{r}(t)/\alpha$, that is, choose as the received symbol the one that minimizes the metric $|\boldsymbol{r}(t)/\alpha - \boldsymbol{s}_i(t)|$.

III. Performance Comparison

In this section, a performance comparison between the two detectors described in the previous section is done in terms of bit error probability and number of operations performed by each detector.

A. Probability of Error

A.1 Detector DI

As described in the previous section, the decision rule for detector DI is

$$\tilde{\boldsymbol{s}}(t) = \arg\min_{\boldsymbol{s}_i(t)} |\boldsymbol{r}(t) - \alpha \boldsymbol{s}_i(t)| \quad i = 0,\ 1,\ 2,\ 3. \tag{4}$$

Assuming that the receiver is able to estimate the actual value of the fading coefficient α, the bit error probability can be obtained by determining the error probability for a given α and then averaging this result with respect to the probability density function of α. Fortunately, there is a closed-form expression for the bit error probability for QPSK transmission over additive white Gaussian noise channels (AWGN) which is given by [10]

$$P(E) = \frac{1}{2} \mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right), \tag{5}$$

where E_b/N_0 is the signal energy per bit to one-sided noise spectral density and erfc$(\cdot)$ is the well-known complementary error function given by

$$\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt. \tag{6}$$

After the fading effect, the received signal-to-noise ratio will be modified by α^2. Then, the bit error probability, for a given α, can be expressed as

$$P(E|\alpha) = \frac{1}{2} \mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0} \cdot \alpha^2}\right). \tag{7}$$

The exact bit error probability for detector DI can be obtained averaging Equation (7) with respect to the pdf of α, that is,

$$\begin{aligned}P_{\mathrm{DI}} &= \int_{-\infty}^{\infty} P(E|\alpha) \cdot p_A(\alpha)\ d\alpha \\ &= \int_0^\infty \frac{1}{2} \mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}\alpha^2}\right) \cdot 2\alpha e^{-\alpha^2} d\alpha\end{aligned} \tag{8}$$

Taking into account that erfc$(x) = 1 - erf(x)$, the previous expression becomes

$$P_{\mathrm{DI}} = \int_0^\infty \alpha e^{-\alpha^2} \left[1 - \mathrm{erf}\left(\sqrt{\frac{E_b}{N_0}\alpha^2}\right)\right] d\alpha. \tag{9}$$

Using the substitution $\alpha^2 = u$, Equation (9) can be expressed as

$$P_{\mathrm{DI}} = \frac{1}{2} \int_0^\infty e^{-u} \left[1 - \mathrm{erf}\left(\sqrt{\frac{E_b}{N_0} u}\right)\right] du. \tag{10}$$

Taking into account that [11, Eq. 6.283]

$$\int_0^\infty e^{\beta x} [1 - \mathrm{erf}(\sqrt{\gamma x})] dx = \frac{1}{\beta}\left[\frac{\sqrt{\gamma}}{\sqrt{\gamma - \beta}} - 1\right], \tag{11}$$

for $\Re[\gamma] > 0$ and $\Re[\beta] < \Re[\gamma]$, the bit error probability for detector DI is finally obtained as

$$P_{\mathrm{DI}} = \frac{1}{2}\left[1 - \frac{\sqrt{E_b/N_0}}{\sqrt{E_b/N_0 + 1}}\right]. \tag{12}$$

A.2 Detector DII

The decision rule for detector DII is

$$\tilde{\boldsymbol{s}}(t) = \arg\min_{\boldsymbol{s}_i(t)} \left|\frac{\boldsymbol{r}(t)}{\alpha} - \boldsymbol{s}_i(t)\right| \quad i = 0,\ 1,\ 2,\ 3. \tag{13}$$

In this scheme, after fading compensation (division of $r(t)$ by α), the channel works as an additive noise channel because

$$\begin{aligned}\tilde{\boldsymbol{s}}(t) &= \arg\min_{\boldsymbol{s}_i(t)} \left|\frac{\alpha \boldsymbol{s}(t) + \boldsymbol{\eta}(t)}{\alpha} - \boldsymbol{s}_i(t)\right| \\ &= \arg\min_{\boldsymbol{s}_i(t)} \left|\boldsymbol{s}(t) + \frac{\boldsymbol{\eta}(t)}{\alpha} - \boldsymbol{s}_i(t)\right| \\ &= \arg\min_{\boldsymbol{s}_i(t)} |\boldsymbol{s}(t) + \boldsymbol{m}(t) - \boldsymbol{s}_i(t)|,\end{aligned} \tag{14}$$

where $\boldsymbol{m}(t) = \boldsymbol{\eta}(t)/\alpha$ is the additive noise obtained from the ratio between a Gaussian random variable and a Rayleigh random variable.

From the Appendix, the one-dimensional pdf of $\boldsymbol{m}(t) = m_i(t) + jm_q(t)$ is given by

$$\begin{aligned} p_M(m) &= p_{M_q}(m_q) = p_{M_i}(m_i) \\ &= \frac{1}{2}\frac{1}{\sqrt{N_0}(m^2/N_0+1)^{3/2}} \end{aligned} \tag{15}$$

and its cumulative density function (CDF) is

$$\begin{aligned} P_M(m) &= P_{M_q}(m_q) = P_{M_i}(m_i) \\ &= \frac{1}{2}\left(\frac{m}{\sqrt{m^2+N_0}}+1\right). \end{aligned} \tag{16}$$

Under this framework and considering Figure 2 which presents the constellation symbols at the transmitter[2] which will be corrupted by the additive noise $\boldsymbol{m}(t)$, the bit error probability for detector DII can be expressed as[3]

$$\begin{aligned} P_{\text{DII}} =& 0.5 \times P(S_1|S_0) + 1.0 \times P(S_2|S_0) \\ &+ 0.5 \times P(S_3|S_0), \end{aligned} \tag{17}$$

where $P(S_i|S_j)$ denotes the probability of receiving symbol S_i given that symbol S_j was transmitted.

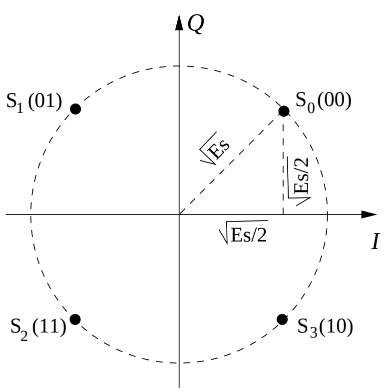

Fig. 2. Problem geometry for calculating the bit error probability for detector DII.

From the symmetric configuration $P(S_1|S_0) = P(S_3|S_0)$, thus P_{DII} can be expressed as

$$P_{\text{DII}} = P(S_1|S_0) + P(S_2|S_0). \tag{18}$$

From Figure 2, one can see that

$$P(S_1|S_0) = P(m_i < -\sqrt{E_S/2}; m_q > -\sqrt{E_S/2}), \tag{19}$$

where E_S is the symbol energy. Since m_i and m_q are independent variables, Equation (19) becomes

$$P(S_1|S_0) = P(m_i < -\sqrt{E_S/2}) \cdot P(m_q > -\sqrt{E_S/2}). \tag{20}$$

[2]Note that a Gray code was used to assign the bits to the constellations symbols.

[3]Without loss of generality consider that the symbol S_0 was transmitted.

The first factor on the right-hand of Equation (20) is

$$\begin{aligned} P(m_i < -\sqrt{E_S/2}) &= \int_{-\infty}^{-\sqrt{E_S/2}} p_M(m)dm \\ &= \frac{1}{2}\cdot\frac{m}{\sqrt{m^2+N_0}}\bigg|_{-\infty}^{-\sqrt{E_S/2}} \\ &= \frac{1}{2}\left(1-\frac{\sqrt{E_S/2}}{\sqrt{E_S/2+N_0}}\right) \end{aligned} \tag{21}$$

and the second factor is

$$\begin{aligned} P(m_q > -\sqrt{E_S/2}) &= \int_{-\sqrt{E_S/2}}^{\infty} p_M(m)dm \\ &= \frac{1}{2}\cdot\frac{m}{\sqrt{m^2+N_0}}\bigg|_{-\sqrt{E_S/2}}^{\infty} \\ &= \frac{1}{2}\left(1+\frac{\sqrt{E_S/2}}{\sqrt{E_S/2+N_0}}\right). \end{aligned} \tag{22}$$

Substituting Equations (21) and (22) into Equation (20), $P(S_1|S_0)$ can be expressed as

$$P(S_1|S_0) = \frac{1}{4}\left(1-\frac{E_S/2}{E_S/2+N_0}\right). \tag{23}$$

For calculating $P(S_2|S_0)$, one can see from Figure 2 that

$$\begin{aligned} P(S_2|S_0) &= P\left(m_i < -\sqrt{E_S/2}; m_q < -\sqrt{E_S/2}\right) \\ &= P\left(m_i < -\sqrt{E_S/2}\right)\cdot P\left(m_q < -\sqrt{E_S/2}\right) \\ &= \left[\frac{1}{2}\left(1-\frac{\sqrt{E_S/2}}{\sqrt{E_S/2+N_0}}\right)\right]^2 \\ &= \frac{1}{4}\left(1-2\frac{\sqrt{E_S/2}}{\sqrt{E_S/2+N_0}}+\frac{E_S/2}{E_S/2+N_0}\right). \end{aligned} \tag{24}$$

In this way, the overall bit error probability for detector DII is given by

$$\begin{aligned} P_{\text{DII}} &= P(S_1|S_0) + P(S_2|S_0) \\ &= \frac{1}{4}\left[2-2\cdot\frac{\sqrt{E_S/2}}{\sqrt{E_S/2+N_0}}\right] \end{aligned} \tag{25}$$

For QPSK modulation $E_b = E_S/2$. As a consequence Equation (25) reduces to

$$P_{\text{DII}} = \frac{1}{2}\left[1-\frac{\sqrt{E_b/N_0}}{\sqrt{E_b/N_0+1}}\right]. \tag{26}$$

Equations (12) and (26) show that the two detection schemes have the same bit error probability, even if one uses two distinct decision rules, when the receiver is able to estimate without error the fading coefficient. Next section presents a performance comparison between both detection schemes in terms of number of operations performed by each detector.

B. Number of Operations

B.1 Detector DI

After receiving $\boldsymbol{r}(t)$, the detector DI computes the metrics $|\boldsymbol{r}(t) - \alpha \boldsymbol{s}_i(t)|$ for $i = 0, 1, 2, 3$ and compares them to obtain the minimum value.

For each $\boldsymbol{s}_i(t)$ the detector performs:

- one real-complex multiplication: $\alpha \boldsymbol{s}_i(t) \rightarrow$ two real multiplications;
- one complex subtraction: $\boldsymbol{r}(t) - \alpha \boldsymbol{s}_i(t) \rightarrow$ two real subtraction;
- one modulus operation.

Since there are four possible symbols $\boldsymbol{s}_i$, the detector performs (4×2) eight multiplications, (4×2) eight subtractions and (4×1) four modulus operations. At this point, the detector has four values of metric. Then, three comparisons are performed to find the minimum value which corresponds to the most probable $\boldsymbol{s}_i(t)$. Table I summarizes the total number of operations required for detector DI.

TABLE I

NUMBER OF REAL OPERATIONS REQUIRED BY DETECTOR DI TO OBTAIN THE ESTIMATE $\tilde{\boldsymbol{s}}(t)$ OF THE TRANSMITTED SYMBOL $\boldsymbol{s}(t)$ BASED ON THE NOISY OBSERVATION $\boldsymbol{r}(t)$.

Operation	Number
multiplication	8
subtraction	8
modulus	4
comparison	3

B.2 Detector DII

After receiving $\boldsymbol{r}(t)$, the detector DII computes the metrics $|\boldsymbol{r}(t)/\alpha - \boldsymbol{s}_i(t)|$ for $i = 0, 1, 2, 3$ and compares them to obtain the minimum value. Its important to note that the detector calculates $\boldsymbol{r}(t)/\alpha$ only once. This corresponds to one complex-real division (two real divisions).

For each $\boldsymbol{s}_i(t)$ the detector performs:

- one complex subtraction $(\boldsymbol{r}(t)/\alpha - \boldsymbol{s}_i(t)) \rightarrow$ two real subtractions;
- one modulus operation.

At the end, the detector DII has four values of metric. Three comparisons are required to find the minimum between them, and consequently, the most probable $\boldsymbol{s}_i(t)$. Table II summarizes the total number of operations required for detector DII.

TABLE II

NUMBER OF REAL OPERATIONS REQUIRED BY DETECTOR DII TO OBTAIN THE ESTIMATE $\tilde{\boldsymbol{s}}(t)$ OF THE TRANSMITTED SYMBOL $\boldsymbol{s}(t)$ BASED ON THE NOISY OBSERVATION $\boldsymbol{r}(t)$.

Operation	Number
division	2
subtraction	8
modulus	4
comparison	3

B.3 Performance Comparison

Tables I and II show that both detection schemes perform the same number of subtractions, comparisons and moduli. The basic difference is that detector DI requires 8 multiplications while detector DII requires only 2 divisions.

According to Knuth [12], a multiplication or a division of two n-bit numbers can be accomplished in $O(n \log n \log \log n)$ steps. Current implementations of these operations can actually be done using the same number of clock cycles. Thus, it is reasonable to assign the same complexity for both multiplication and division. Based on these statements, one can conclude that detector DII has a lower complexity when compared to that of detector DI.

IV. SIMULATION RESULTS

In order to validate the expressions obtained for the bit error probability for the two detection schemes a set of simulations was performed. For each detection scheme a minimum of 10^4 channel realizations was done for each bit error probability investigated. The bit error probability was estimated from the ratio between the number of received bits in error and the total transmitted bits.

Figure 3 shows the performance of the two detection schemes as a function of the signal-to-noise ratio per bit (E_b/N_0), which ranges from zero to 30 dB. It is observed that they show the same performance in terms of the bit error probability. For comparison purposes, the curve obtained from the analytical expression for the bit error probability (Equations 12 and 26) is also plotted in the figure.

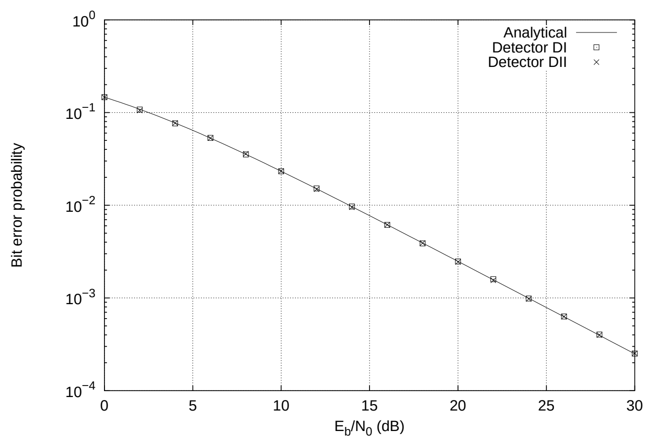

Fig. 3. Bit error probability of the two detection schemes as function of the signal-to-noise ratio (E_b/N_0). The analytical curve (from Equations (12) and (26)) is also plotted.

V. CONCLUSIONS

This paper presented a performance analysis of two QPSK detection schemes for Rayleigh fading channels under the assumption of ideal channel estimation. It was shown, analytically and by simulation, that both schemes lead to the same bit error probability. A complexity evaluation was done in terms of the number of basic operations performed by each detector.

As a future work, the authors will investigate the performance of the two detection schemes for frequency-selective fading channels and the influence of the channel estimation errors on the communications system.

APPENDIX
THE PDF AND CDF OF $\boldsymbol{m}(t)$

This appendix deals with the evaluation of the pdf and CDF of the random variable $\boldsymbol{m}(t) = \boldsymbol{\eta}(t)/\alpha$. It has been assumed that $\boldsymbol{\eta}(t) = \eta_i(t) + j\eta_q(t)$ is a complex Gaussian r.v. having zero mean and variance $N_0/2$ by dimension, that is,

$$p_N(\eta) = p_{N_i}(\eta_i) = p_{N_q}(\eta_q) = \frac{1}{\sqrt{\pi N_0}} e^{-\eta^2/N_0}. \quad (27)$$

On the other hand, α is a real-valued r.v. with Rayleigh distribution given by

$$p_A(\alpha) = 2\alpha e^{-\alpha^2} u(\alpha), \quad (28)$$

where $u(\cdot)$ is the step-unit function.

If $\boldsymbol{\eta}(t) = \eta_i(t) + j\eta_q(t)$, then

$$\boldsymbol{m}(t) = \frac{\eta_i(t)}{\alpha} + j\frac{\eta_q(t)}{\alpha} = m_i(t) + jm_q(t). \quad (29)$$

Since $\eta_i(t)$ and $\eta_q(t)$ are independent r.v's, it follows that $m_i(t)$ and $m_q(t)$ are also independent. Moreover, they have the same probability distribution given by

$$p_M(m) = p_{M_i}(m_i) = p_{M_q}(m_q). \quad (30)$$

In this case the r.v. $M = N/A$, where N is a Gaussian r.v. and A is a Rayleigh r.v. The fdp of M is given by [13]

$$p_M(m) = \int_{-\infty}^{\infty} |\alpha| p_{NA}(m\alpha, \alpha) d\alpha, \quad (31)$$

where $p_{NA}(\eta, \alpha)$ is the joint probability of N and α given by

$$p_{NA}(\eta, \alpha) = \frac{2}{\sqrt{\pi N_0}} \alpha e^{-(\alpha^2 + \eta^2/N_0)} u(\alpha). \quad (32)$$

Thus, the pdf of M is given by

$$\begin{aligned} p_M(m) &= \int_0^{\infty} \alpha \frac{2}{\sqrt{\pi N_0}} \alpha e^{-(\alpha^2 + m^2\alpha^2/N_0)} d\alpha \\ &= \frac{2}{\sqrt{\pi N_0}} \int_0^{\infty} \alpha^2 e^{-\alpha^2(1+m^2/N_0)} d\alpha. \end{aligned} \quad (33)$$

Using the fact that [14, pp. 1030]

$$\int_0^{\infty} x^2 e^{-x^2} dx = \frac{\sqrt{\pi}}{4}, \quad (34)$$

one can show that

$$\int_0^{\infty} x^2 e^{-\rho x^2} dx = \frac{1}{\rho^{3/2}} \cdot \frac{\sqrt{\pi}}{4}. \quad (35)$$

So, Equation (33) can be expressed as

$$\begin{aligned} p_M(m) &= \frac{2}{\sqrt{\pi N_0}} \cdot \frac{1}{(m^2/N_0 + 1)^{3/2}} \cdot \frac{\sqrt{\pi}}{4} \\ &= \frac{1}{2} \cdot \frac{N_0}{(m^2 + N_0)^{3/2}}. \end{aligned} \quad (36)$$

The CDF of M can be obtained by

$$P_M(m) = \int_{-\infty}^{m} p_M(x) dx. \quad (37)$$

Thus, one can show that

$$P_M(m) = \frac{1}{2} \left(\frac{m}{\sqrt{m^2 + N_0}} + 1 \right). \quad (38)$$

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