## Analysis of Adaptive Multiuser Receivers for DS-CDMA

# Using Recurrent Neural Networks

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Abstract— In this paper we investigate adaptive multiuser receivers for DS-CDMA systems using recurrent neural networks (RNN). A comparative analysis of multiuser detection (MUD) schemes employing linear and non-linear structures is carried out. Adaptive minimum mean squared error (MMSE) linear MUD receivers are examined with the LMS algorithm and compared with MMSE neural MUD receivers operating with the real time recurrent learning (RTRL) algorithm. Computer simulation experiments including different communication channels and a varying number of users show that the neural MUD receiver operating with the RTRL algorithm outperforms linear MUD receivers with the LMS and the conventional single user detector (SUD).

#### I. Introduction

Neural networks have recently been used in the design of DS-CDMA multiuser receivers [1-3]. Neural receivers employing the minimum mean squared error (MMSE) [1-3 criterion usually show good performance and have simple adaptive implementation, at the expense of a higher computational complexity. The deployment of non-linear structures, such as neural networks, can mitigate more effectively intersymbol interference (ISI), caused by the multipath effect of radio signals, and multiple access interference(MAI), which arise due to the non-orthogonality between user signals. In the last few years, different artificial neural networks structures have been used in the design of multiuser detectors (MUDs): multilayer perceptrons (MLP) [1], radial-basis functions (RBF) [2], and recurrent neural networks (RNN) [3]. These neural systems make use of non-linear functions to create decision boundaries to detect transmitted symbols, whilst conventional MUDs employ linear functions to form such decision regions. In this work, we investigate adaptive multiuser receivers using dynamically driven recurrent neural networks, which are different from those employed in [3] and to the best knowledge of the authors have not been examined in multiuser receivers so far. Adaptive MMSE linear MUD receivers are examined with the LMS algorithm and compared to MMSE neural MUD receivers operating with the real time recurrent learning (RTRL) algorithm. Computer simulation experiments including AWGN, time-invariant frequency selective, flat fast Rayleigh fading, frequency selective slow Rayleigh fading communication channels and a varying number of users show that the neural MUD receiver operating with the RTRL algorithm outperforms linear MUD receivers with the LMS and the conventional single user detector (SUD).

This paper is organised as follows. Section II briefly describes the DS-CDMA system model and the adaptive MMSE linear multiuser receiver. The RNN receiver struc-

ture and the RTRL adaptive algorithm are detailed in Section III. Section IV presents and discusses the simulation results and Section V gives the concluding remarks of this work

#### II. System Model

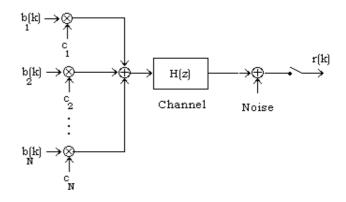


Fig. 1. Model of synchronous DS-CDMA system.

A synchronous DS-CDMA system with N users and PG chips per bit is depicted in Fig. 1, where  $b_i(k) \in \{\pm 1\}$  denotes the k-th bit of user i, the signature sequence for user i  $\mathbf{c}_i = [\mathbf{c}_{i,1} \dots \mathbf{c}_{i,PG}]^T$  is normalized to have a unit length, and the channel impulse response is given by

$$H(z) = \sum_{i=0}^{n_h - 1} h(i)z^{-i} \tag{1}$$

where the operator  $z^{-1}$  introduces a delay of one chip time in the transmitted signal.

The received signal after filtering by a chip-pulse matched filter and sampled at chip rate is described by

$$\mathbf{r}(k) = \mathbf{H} \begin{bmatrix} \mathbf{CA} & \mathbf{0} & \dots \mathbf{0} \\ \mathbf{0} & \mathbf{CA} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{CA} \end{bmatrix}$$
$$\begin{bmatrix} \mathbf{b}(\mathbf{k}) \\ \mathbf{b}(\mathbf{k} - \mathbf{1}) \\ \vdots \\ \mathbf{b}(\mathbf{k} - \mathbf{L} + \mathbf{1}) \end{bmatrix} + \mathbf{n}(k) \tag{2}$$

where the Gaussian noise vector  $\mathbf{n}(k) = [n_1(k) \dots n_{PG}(k)]^T$ with  $E[\mathbf{n}(k)\mathbf{n}^T(k)] = \sigma_n^2\mathbf{I}$ , the user bit vector is given by  $\mathbf{b}(k) = [b1(k) \dots b_N(k)]^T$ , the user signature sequence matrix is described by  $\mathbf{C} = [\mathbf{c}_1 \dots \mathbf{c}_N]$ , the diagonal user signal amplitude matrix is represented by  $\mathbf{A} = diag\{\mathbf{A}_1 \dots \mathbf{A}_N\}$ , and the  $PG \times (L \times PG)$  matrix H is expressed by

$$\mathbf{H} = \begin{bmatrix} h_0 & h_1 & \dots & h_{n_h-1} \\ & h_0 & h_1 & \dots & h_{n_h-1} \\ & & \ddots & \ddots & & \ddots \\ & & & h_0 & h_1 & \dots h_{n_h-1} \end{bmatrix}$$
(3)

The multiple access interference (MAI) is originated from the non-orthogonality between the user signature sequences. The intersymbol interference (ISI) span L depends on the length of the channel response and the length of the chip sequence. For  $n_h=1,\ L=1$  (no ISI), for  $1< n_h \leq PG, L=2$ , for  $PG< n_h \leq 2PG, L=3$  and so on.

Consider a one shot linear MUD (the receiver observes and detects only one symbol at each time instant), whose observation vector  $\mathbf{u}(k)$ , where  $\mathbf{u}(k) = \mathbf{C}^T \mathbf{r}(k)$ , is formed from the outputs of a bank of matched filters and is represented by:

$$\mathbf{u}(k) = [u_1 \dots u_N]^T \tag{4}$$

The detected symbols for this one shot multiuser receiver are given by the following expression:

$$\hat{b}_i(k) = sgn(\mathbf{w}_i^T(k)\mathbf{u}(k)) \tag{5}$$

where  $\mathbf{w}_i(k) = [w_1 \dots w_N]^T$  is the receiver weight vector for user i for the k - th bit in a system with N users.

The minimum mean squared error solution for this multiuser receiver can be obtained via the LMS algorithm [4], which uses the error signal  $e_i(k) = b_i(k) - \mathbf{w}_i^T(k)\mathbf{u}(k)$ , and is described by:

$$\mathbf{w}_i(k+1) = \mathbf{w}_i(k) + \mu e_i(k)\mathbf{u}(k) \tag{6}$$

where  $b_i(k)$  is the desired signal for the *i*-th user taken from the training sequence,  $\mathbf{u}(k)$  is the observation vector for the linear MUD and  $\mu$  is the algorithm step size.

## III. RECURRENT NEURAL NETWORKS

Recurrent neural networks (RNN) have one or more feedback connections, where each artificial neuron is connected to the others, as shown in Fig. 2. RNN structures are suitable to channel equalisation and multiuser detection applications, since they are able to cope with channel transfer functions that exhibit deep spectral nulls, forming optimal decision boundaries and are less computationally demanding than MLP networks [5]. To describe RNN systems we use a state-space approach, where the  $N \times 1$  vector  $\mathbf{x}_i(k)$  corresponds to the N states of the artificial neural network for user i, the  $N \times 1$  vector  $\mathbf{u}(k)$  to the channel N user symbols output observation vector and the output of the neural multiuser receiver  $\hat{b}_i(k)$  is given by:

$$\xi_i(k) = \left[\mathbf{x}_i^T(k-1) \ \mathbf{u}^T(k)\right]^T \tag{7}$$

$$\mathbf{x}_{i}(k) = tanh(\mathbf{w'}_{i}^{T}(k)\xi_{i}(k)) \tag{8}$$

$$\hat{b}_i(k) = sgn(\mathbf{D}\mathbf{x}_i(k)) \tag{9}$$

where the  $2N \times N$  matrix  $\mathbf{w'}_i(k)$  contains the coefficients of the RNN receiver for user i,  $\mathbf{D} = \begin{bmatrix} 1 & 0 & \dots & 0 \end{bmatrix}$  is the  $1 \times N$  matrix that defines the number of outputs of the network. Note that, in this work, we have only one output  $\hat{b}_i(k)$  per observation vector  $\mathbf{u}(k)$ , which corresponds to the one shot approach.

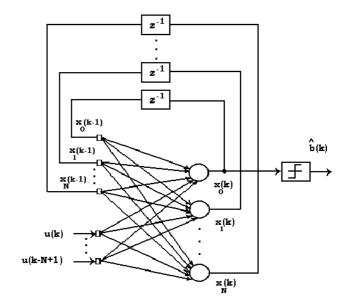


Fig. 2. Adaptive multiuser receiver structure based on a recurrent neural network.

To train the neural multiuser receiver parameters, we employ a stochastic gradient based adaptive technique called real time recurrent learning (RTRL) [2,5] algorithm. The RTRL algorithm employs the minimum mean squared error criterion (MMSE) formed by instantaneous values of the error  $e_i(k) = d_i(k) - \mathbf{Dx}_i(k)$  and is expressed by:

$$\mathbf{\Phi}_i(k) = diag(sech^2(\mathbf{w}_i^T(k)\xi_i(k))) \tag{10}$$

$$\mathbf{U}_{i,j}(k) = \begin{bmatrix} \mathbf{0}_u^T \ \xi_i^T(k) \ \mathbf{0}_l^T \end{bmatrix}$$
 (11)

$$\mathbf{\Lambda}_{i,j}(k+1) = \mathbf{\Phi}(k) \left[ \mathbf{w} \mathbf{p}'_{i}(k) \mathbf{\Lambda}_{i,j}(k) + \mathbf{U}_{i,j}(k) \right]$$
(12)

$$\Delta \mathbf{w}_{i,i}(k) = \mu \mathbf{\Lambda}_{i,i}^T(k) \mathbf{D}^T e_i(k) \tag{13}$$

$$\mathbf{w'}_{i}(k+1) = \mathbf{w'}_{i}(k) + \Delta \mathbf{w}_{i}(k) \tag{14}$$

where the index j varies from 1 to N, the state dimensionality of the neural structure and the index i corresponds to the desired user. The matrices  $\Phi_i(k)$ ,  $\mathbf{U}_{i,j}(k)$ ,  $\mathbf{\Lambda}_{i,j}(k)$ ,  $\mathbf{wp'}_i(k)$  (a partition of  $\mathbf{w'}_i(k)$  defined in [2]) and  $\Delta \mathbf{w'}_{i,j}(k)$  have dimensions  $N \times N$ ,  $N \times 2N$ ,  $N \times 2N$ ,  $N \times N$  and  $N \times N$ , respectively. Note that  $\mathbf{0}_u$  and  $\mathbf{0}_l$  are zero valued matrices with variable size that depend on  $N \times N$ , respectively [2].

#### IV. SIMULATION EXPERIMENTS

In this section, we conduct simulation experiments to assess and compare the BER performance of MUD linear receivers operating with the LMS, MUD neural receivers operating with the RTRL algorithm, the single user conventional detector (SUD) and the single user bound (SUBound), which corresponds to the SUD in a system with a single user and no MAI. We consider non-orthogonal random generated spreading sequences, processing gain PG=8, a step size  $\mu=0.01$  for the adaptive algorithms and assume perfect power control in the DS-CDMA system in all simulations. The algorithms are adjusted with 200 training data symbols during the training period and no adaptation occurs in data mode in all experiments. Note that the BER performance shown in the results refers to the average BER amongst the N users.

#### A. AWGN channel performance

To analyse the BER performance of the adaptive receivers in an AWGN channel, we have conducted simulations where the receivers process  $10^4$  data symbols, averaged over 100 independent experiments. The BER performance versus  $E_b/N_0$  for N=3 users is shown in Fig. 3.

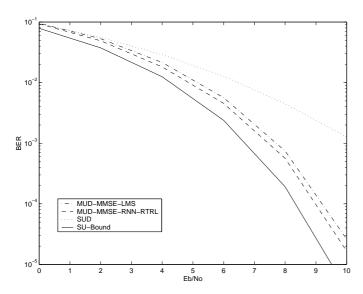


Fig. 3. BER performance of the receivers in AWGN channel with N=3 users

The curves plotted in Fig. 3 show that the MMSE neural receiver is superior to the MMSE linear MUD, saving up to 0.4 dB for the same BER performance.

In another situation, the same receivers were evaluated at  $E_b/N_0=6dB$  with a different number of users, as depicted in Fig. 4. The MMSE neural MUD achieves the best BER performance with a varying number of users, outperforming the MMSE linear receiver and the SUD. The results show that the use of neural receivers can increase the capacity of DS-CDMA systems in comparison with the MMSE linear receiver and the SUD.

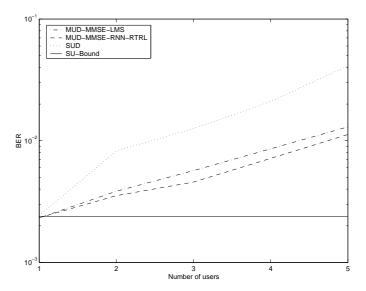


Fig. 4. BER performance of the receivers versus number of users in AWGN channel at  $E_b/N_0=6\,dB$  .

### B. Time-invariant frequency selective channel performance

To evaluate the BER performance of the adaptive receivers in a time-invariant frequency selective channel with AWGN, we have selected a channel with transfer function  $H(z) = 1 - 0.25z^{-1} + 0.4z^{-2}$ . We carried out simulations where the receivers processed  $10^4$  data symbols, averaged over 100 independent experiments. The BER performance versus  $E_b/N_0$  for N=3 users is shown in Fig. 5.

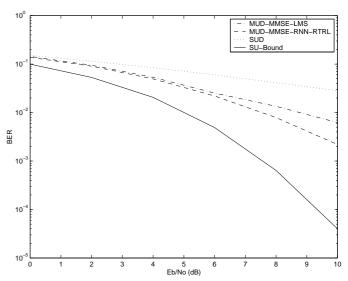


Fig. 5. BER performance of the receivers in a time invariant frequency selective channel with transfer function  $H(z)=1-0.25z^{-1}+0.4z^{-2}$  and AWGN with N=3 users.

The curves plotted in Fig. 5 show that the MMSE neural receiver is superior to the MMSE linear MUD, saving up to 1.5 dB for the same BER performance.

In another situation, the same receivers were evaluated at  $E_b/N_0=8dB$  and the number of users was varied, as depicted in Fig. 6. The MMSE neural MUD achieves the

best BER performance with a varying number of users, outperforming the MMSE linear receiver and the SUD. The results indicate that the MMSE neural MUD can support more users than the MMSE linear receiver and the SUD in a DS-CDMA system.

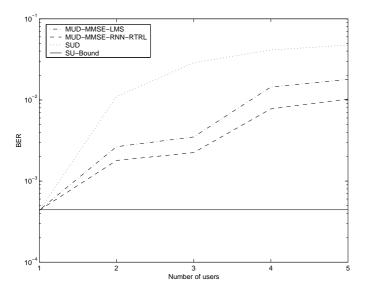


Fig. 6. BER performance of the receivers versus number of users in a time invariant frequency selective channel with transfer function  $H(z)=1-0.25z^{-1}+0.4z^{-2}$  and AWGN at  $E_h/N_0=8dB$ .

## C. Flat fast Rayleigh fading channel performance

This time, the BER performance of the receivers was evaluated in a flat fast Rayleigh fading channel with AWGN, that changes its characteristic at each transmitted symbol. All the receivers process  $10^3$  data symbols, averaged over 100 independent experiments. The BER performance versus  $E_b/N_0$  for 3 users is shown in Fig. 7.

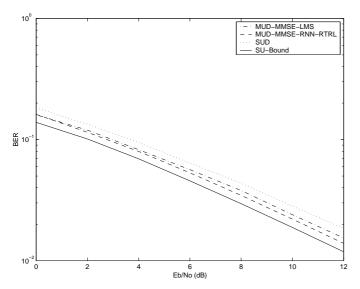


Fig. 7. BER performance of the receivers in a flat fast Rayleigh fading channel with AWGN for 3 users.

The curves plotted in Fig. 7 indicate that the MMSE

neural receiver is superior to the MMSE linear receiver and the SUD. The neural receiver can save up to 0.4 dB, for the same BER performance, when compared to the linear MUD.

In a situation with a varying number of users, the receivers were evaluated at  $E_b/N_0=10dB$ , as depicted in Fig. 8. The results show that the MMSE neural MUD achieves the best BER performance, outperforming the MMSE linear receiver and the SUD. In fact, the neural receiver can increase the capacity of the system in comparison with the MMSE linear receiver and the SUD.

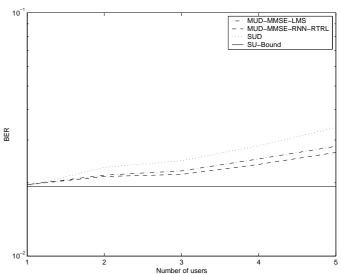


Fig. 8. BER performance of the receivers versus number of users in a flat fast Rayleigh fading channel with AWGN at  $E_b/N_0 = 10 dB$ .

## D. Frequency selective slow Rayleigh fading channel performance

In this experiment, the BER performance of the receivers has been assessed in a frequency selective slow Rayleigh fading channel with AWGN. The channel is modeled as a three-path slow Rayleigh fading one with a discrete power delay profile, whose taps are modeled as independent, zero-mean and complex random variables. The envelope of this channel input response has a Rayleigh probability distribution. The channel is composed of 3 paths, spaced by the chip period, with a multipath intensity profile given by  $[0.5\ 0.3\ 0.2]$ . The receivers process  $10^3$  data symbols, for 100 different channel realisations, averaged over 10 independent experiments. The BER performance versus  $E_b/N_0$  for 3 users is shown in Fig. 9.

The results shown in Fig. 9 for a three-path slow Rayleigh fading channel with AWGN indicate that the neural MUD is superior to the linear MUD, saving up to 0.5 dB, for the same BER performance.

In another situation, the receivers were evaluated at  $E_b/N_0 = 10dB$  with a varying number of users, as depicted in Fig. 10. Again, the MMSE neural MUD has outperformed the MMSE linear receiver and the SUD. Indeed, the MMSE neural receiver can accommodate more users in

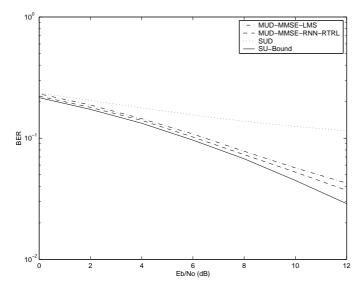


Fig. 9. BER performance of the receivers in a frequency selective slow Rayleigh fading channel with AWGN for 3 users.

a DS-CDMA system than the MMSE linear and the SUD receivers.

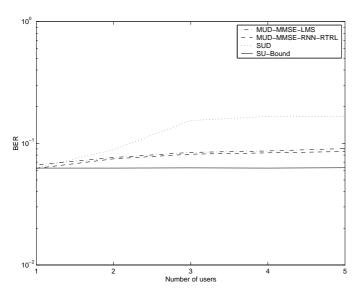


Fig. 10. BER performance of the receivers versus number of users in a frequency selective slow Rayleigh fading channel with AWGN at  $E_b/N_0=10dB$ .

## V. CONCLUDING REMARKS

In this paper we investigated adaptive multiuser receivers for DS-CDMA systems using recurrent neural networks. A comparative analysis of multiuser detection schemes employing linear and non-linear structures has been carried out. Adaptive MMSE linear MUD receivers have been examined with the LMS algorithm and compared with adaptive MMSE neural MUD receivers operating with the real time recurrent learning algorithm. Computer simulation experiments were carried out for AWGN, time-invariant frequency selective, flat fast Rayleigh fading, frequency se-

lective slow Rayleigh fading communication channels and a varying number of users. The BER performance results have shown that the neural MUD receiver operating with the RTRL algorithm outperforms linear MUD receivers with the LMS algorithm and the SUD.

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