Adaptive Minimum BER Decision Feedback Multiuser Receivers for DS-CDMA

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Abstract— **In this paper we investigate the use of adaptive minimum bit error rate (MBER) decision feedback (DFE) multiuser receivers (MUD) for DS-CDMA systems. We examine stochastic gradient adaptive algorithms for approximating the bit error rate (BER) from training data. Computer simulation experiments show that the DFE-MUD structure employing adaptive MBER algorithms outperforms linear MUDs with these algorithms and the DFE-MUD with the minimum mean square error (MMSE) criterion.**

I. Introduction

Linear multiuser receivers employing the minimum mean squared error (MMSE) [1-4] criterion have become rather successful, since they usually show good performance and have simple adaptive implementation. However, it is well known that the MSE cost function is not optimal in digital communications applications, and the most appropriate cost function is the bit error rate (MBER). The approximate minimum bit error rate (AMBER) [5] and the least bit error rate (LBER) [6] are two of the most successful and suitable algorithms for adaptive implementation, provided the application can handle a long training sequence. The AMBER is a stochastic gradient algorithm which is similar to the signed error LMS algorithm except for the fact that in the vicinity of the decision boundary it continues to update the receiver weights. The algorithm is appealing due to its computational simplicity and has been investigated in linear multiuser receivers [5]. The LBER is also a stochastic gradient algorithm that makes use of kernel density estimation to approximate the BER as a function of the data and has been examined in linear multiuser receivers in [6]. The advantage of the LBER is that an error does not need to be observed to guarantee an estimate of the error rate and the smooth function is a convenient route to gradient algorithms. The use of non-linear MUD structures, such as decision feedback, can combat more effectively intersymbol interference and multiple access interference (MAI) [1,2], which arise due to the loss of orthogonality between user signals. In this work, we examine decision feedback multiuser receivers employing the AMBER and the LBER adaptive MBER algorithms and analyse their convergence and BER performance.

This paper is organised as follows. Section II briefly describes the DS-CDMA system model. The decision feedback multiuser receiver is presented in Section III. Section IV is dedicated to the AMBER algorithm and Section V to the LBER approach. Section VI presents the simulation results and Section VII the conclusions of this work.

II. DS-CDMA System Model

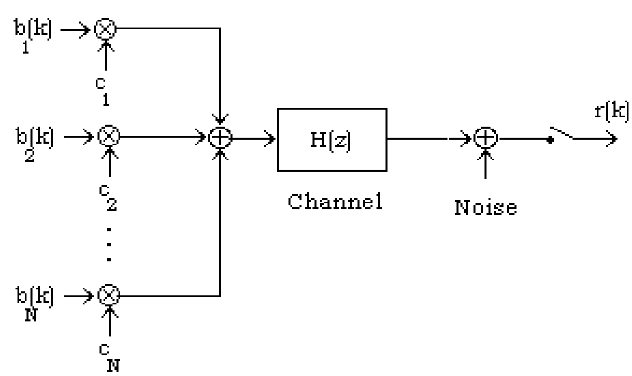

Fig. 1. Model of synchronous DS-CDMA system.

Following the description in [6], the synchronous DS-CDMA system with N users and PG chips per bit is depicted in Fig. 1, where $b_i(k) \in \{\pm 1\}$ denotes the $k-th$ bit of user i, the signature sequence for use i $c_i = [c_{i,1} \ldots c_{i,PG}]^T$ is normalized to have a unit length, and the channel impulse response is given by

$$H(z) = \sum_{i=0}^{n_h-1} h(i) z^{-i} \qquad (1)$$

where the operator z^{-1} introduces a delay of one chip time in the transmitted signal.

The received signal after filtering by a chip-pulse matched filter and sampled at chip rate is described by

$$\mathbf{r}(k) = \mathbf{H} \begin{bmatrix} \mathbf{CA} & \mathbf{0} & \ldots\mathbf{0} & \\ \mathbf{0} & \mathbf{CA} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \ldots & \mathbf{0} & \mathbf{CA} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{b(k)} \\ \mathbf{b(k-1)} \\ \vdots \\ \mathbf{b(k-L+1)} \end{bmatrix} + \mathbf{n}(k) = \mathbf{s}(k) + \mathbf{n}(k) \qquad (2)$$

where the Gaussian noise vector $\mathbf{n}(k) = [n_1(k) \ \ldots \ n_{PG}(k)]^T$ with $E[\mathbf{n}(k)\mathbf{n}^T(k)] = \sigma_n^2 \mathbf{I}$, $\mathbf{s}(k)$ is the noise-free signal vector, the user bit vector is given by $\mathbf{b}(k) = [b1(k) \ \ldots \ b_N(k)]^T$, the user signature sequence matrix is described by $\mathbf{C} = [c_1 \ \dots \ c_N]$, the diagonal user signal amplitude matrix is represented by $\mathbf{A} = diag\{A_1 \ \dots \ A_N\}$, and the $PG \ \times (L \times PG)$ matrix H is expressed by

$$\mathbf{H} = \begin{bmatrix} h_0 & h_1 & \dots & h_{n_h-1} & & & \\ & h_0 & h_1 & \dots & h_{n_h-1} & & \\ & & \ddots & \ddots & \dots & \ddots & \\ & & & h_0 & h_1 & \dots h_{n_h-1} \end{bmatrix} \quad (3)$$

The multiple access interference (MAI) is originated from the non-orthogonality between the user signature sequences. The intersymbol interference (ISI) span L depends on the length of the channel response, which is related to the length of the chip sequence. For $n_h = 1, \ L = 1$ (no ISI), for $1 < n_h \leq PG, L = 2$, for $PG < n_h \leq 2PG, L = 3$ and so on.

Consider a general linear MUD, whose observation vector $\mathbf{y}(k) = \left[\mathbf{r}^T(k) \dots \mathbf{r}^T(k-M+1)\right]^T$ is formed from the outputs of a chip rate sampler. The user i detected symbols for this multiuser receiver are given by the following expression:

$$\hat{b}_i(k) = sgn(\mathbf{w}_i^T(k)\mathbf{y}(k)) = sgn(x_i(k)) \quad (4)$$

where $sgn(.)$ is the sign function, $\mathbf{w}_i(k) = [w_1 \dots w_{PG\times M}]^T$ is the receiver weight vector and $x_i(k)$ is the estimated symbol for user i and symbol k in a system with N users.

Consider now a one-shot MUD, where $M = 1$ and whose observation vector is $\mathbf{y}(k) = \mathbf{r}(k)$. The detected symbols for this one shot receiver and user i are expressed by:

$$\hat{b}_i(k) = sgn(\mathbf{w_i}^T(k)\mathbf{r}(k)) = sgn(x_i(k)) \quad (5)$$

where $\mathbf{w}_i(k) = [w_1 \dots w_{PG}]^T$ is the receiver weight vector and $x_i(k)$ is the estimated symbol for user i and symbol k in a system with N users.

III. Decision Feedback MUD

The use of a decision feedback (DFE) section in a multiuser receiver improves its multiple access interference (MAI) and intersymbol interference (ISI) cancellation capabilities [1,2]. Indeed, the DFE structure minimises the effects of MAI as well as ISI by forcing zeros in the impulse responses of the interferers at the decision instants. Also, the DFE-based systems, as the one shown in Fig. 2, can reduce the noise enhancement effect, allowing the forward linear filter to have greater flexibility to mitigate ISI and MAI [1,2].

The output of the one-shot DFE multiuser receiver ($M = 1$, $\mathbf{y}(k) = \mathbf{r}(k)$) is described by:

$$x_i(k) = \mathbf{w}_i^T(k)\mathbf{r}(k) - \mathbf{f}_i^T(k)\hat{\mathbf{b}}(k) \quad (6)$$

where $\mathbf{r}(k)$ is the $PG \times 1$ received vector of chip-matched filters outputs corresponding to symbol k, and $\hat{\mathbf{b}}(k)$ is the $N \times 1$ vector of decisions at the output of the decision device. The feedforward matrix $\mathbf{w}(k)$ is $PG \times N$, and the feedback matrix $\mathbf{f}(k)$ is $N \times N$ and is constrained to have zeros along the diagonal to avoid cancelling the desired symbols. Note that in this work we employ a full matrix $\mathbf{f}(k)$, except for the diagonal, which corresponds to parallel decision feedback [2].

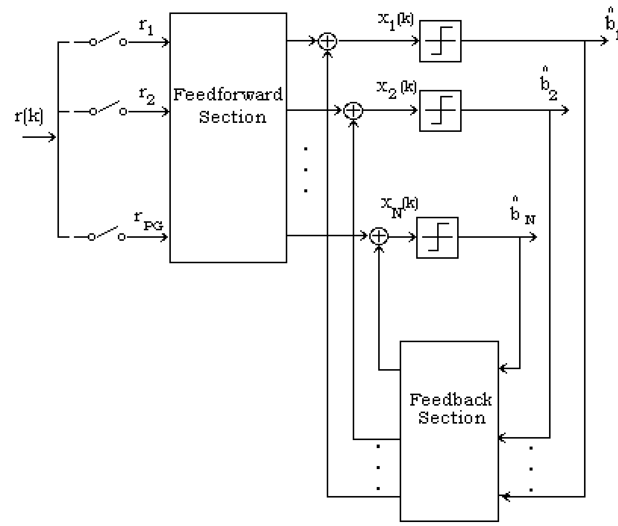

Fig. 2. Decision feedback MUD receiver.

The detected symbol for the DFE multiuser receiver is given by:

$$\hat{b}_i(k) = sgn(x_i(k)) \quad (7)$$

where $x_i(k)$ is the $k-th$ estimated symbol for user i and $sgn(.)$ is the sign function.

IV. The AMBER algorithm

Given a user i transmitted training sequence $\mathbf{d}_i$, the bit error probability $P(\epsilon|\mathbf{d}_i)$, for the linear and the DFE receivers, is expressed by:

$$P\big(\epsilon|\mathbf{d}_i\big) = P_{\epsilon_i} = P\big(d_i(k)sgn(x_i(k)) = -1\big)$$

$$P_{\epsilon_i} = P\big(sgn(d_i(k)x_i(k)) = -1\big) = P\big(d_i(k)x_i(k) < 0\big) \quad (8)$$

where $x_i(k)$ is given through (5), for the linear receiver, and expressed by (6), in the case of the DFE MUD and $d_i(k)$ is the desired symbol taken from the training sequence for user i and symbol k.

The AMBER is a stochastic gradient that attempts to approximate the exact MBER performance [5]. The algorithm is appealing due to its very low complexity, simplicity and straightforward extension to the complex signalling case. The MUD solution that minimises the BER criterion via the AMBER algorithm [5] employs the vector function $g(\mathbf{w}_i(k))$ [5] to approximate an expression for a coefficient vector $\mathbf{w}_i(k)$ that achieves a MBER performance with linear receiver structures, as described by:

$$g(\mathbf{w}_i(k)) = E\left[Q\left(\frac{d_i(k)\mathbf{w}_i^T(k)\mathbf{s}(k)}{\parallel \mathbf{w}_i(k) \parallel \sigma}\right)d_i(k)\mathbf{s}(k)\right] \quad (9)$$

where $d_i(k)$ is the desired transmitted symbol for user i, taken from the training sequence, $Q(.)$ is the Gaussian error function and $\mathbf{s}(k)$ are the received samples without noise taken from the outputs of chip-matched filters. A simple stochastic solution for $\mathbf{w}_i(k)$ can be derived by using $g(\mathbf{w}_i(k))$ and adjusting the receiver weights by:

$$\mathbf{w}_i(k+1) = \mathbf{w}_i(k) + \mu g(\mathbf{w}_i(k)) \tag{10}$$

Note that for linear receiver structures the quantity $Q\Big(\frac{d_i(k)\mathbf{w}_i^T(k)\mathbf{s}(k)}{\|\mathbf{w}_i(k)\|\sigma}\Big)$ inside the expected value operator in (9) corresponds to the conditional bit error probability given the product $d_i(k)\mathbf{s}(k)$. This quantity can be replaced in (9) by an error indicator function $i_{d_i}(k)$ given by:

$$i_{d_i}(k) = \frac{1}{2}(1 - sgn(d_i(k)x_i(k))) \tag{11}$$

where $x_i(k)$ is the estimated symbol and $d_i(k)$ is the desired signal provided by the training sequence.

The AMBER algorithm, as devised for linear MUDs [5], is described by the following equalities:

$$\mathbf{w}_i(k+1) = \mathbf{w}_i(k) + \mu E\left[Q\left(\frac{d_i(k)\mathbf{w}_i^T(k)\mathbf{s}(k)}{\parallel \mathbf{w}_i(k) \parallel \sigma}\right)d_i(k)\mathbf{s}(k)\right]$$

$$\mathbf{w}_i(k+1) = \mathbf{w}_i(k) + \mu E\Big[E\big[i_{d_i}(k) \mid d_i(k)\mathbf{s}(k)\big]d_i(k)\mathbf{s}(k)\Big]$$

$$\mathbf{w}_i(k+1) = \mathbf{w}_i(k) + \mu E\big[i_{d_i}(k)d_i(k)\mathbf{s}(k)\big]$$

Since $\mathbf{s}(k) = \mathbf{r}(k) - \mathbf{n}(k)$, and $i_{d_i}(k)$ and $d_i(k)$ are statistically independent, we have $E[i_{d_i}(k)d_i(k)\mathbf{n}(k)] = E[d_i(k)]E[i_{d_i}(k)\mathbf{n}(k)] = 0$ and thus:

$$\mathbf{w}_i(k+1) = \mathbf{w}_i(k) + \mu E\Big[i_{d_i}(k)d_i(k)\mathbf{r}(k)\Big] \tag{12}$$

The AMBER stochastic gradient update equation for the linear equaliser is given by:

$$\mathbf{w}_i(k+1) = \mathbf{w}_i(k) + \mu i_{d_i}(k)d_i(k)\mathbf{r}(k) \tag{13}$$

And the AMBER solution for the DFE equaliser is expressed by :

$$\mathbf{w}_i(k+1) = \mathbf{w}_i(k) + \mu i_{d_i}(k)d_i(k)\mathbf{r}(k)$$

$$\mathbf{f}_i(k+1) = \mathbf{f}_i(k) - \mu i_{d_i}(k)d_i(k)\mathbf{b}(k) \tag{14}$$

In practice, a modified error indicator function $i_{d_i}(k) = \frac{1}{2}(1 - sgn(d_i(k)x_i(k) - \tau))$ is employed, where the threshold τ is responsible for increasing the algorithm rate of convergence. This algorithm updates when an error is made and also when an error is almost made, becoming a smarter choice for updating the filter coefficients.

V. The LBER algorithm

The MUD BER depends on the distribution of the decision variable $x_i(k)$, which is a function of the weights of the receiver. The sign-adjusted decision variable for the DFE equaliser $x_{s_i}(k) = d_i(k)x_i(k)$ is drawn from a Gaussian mixture, described by:

$$x_{s_i}(k) = sgn(d_i(k))\left(\mathbf{w}_i^T\mathbf{s}(k) - \mathbf{f}_i^T\hat{\mathbf{b}}(k) + \mathbf{w}_i^T\mathbf{n}(k)\right)$$

$$x_{s_i}(k) = sgn(d_i(k))x_i'(k) + n'(k) \tag{15}$$

where the first term of (15) is the noise free sign-adjusted MUD output.

Consider that K samples of the transmitted symbols $b_i(k)$ and K samples of the estimated symbols $\mathbf{x}_i(k)$ are available from the samples $d_i(k) = b_i(k)$ of a training sequence. A kernel density estimate [6] is given by:

$$p_x(x_{s_i}) = \frac{1}{K\sqrt{2\pi}\rho\sqrt{\mathbf{w_i^T w_i}}}\sum_{k=1}^{K} exp\left(\frac{-(x_{s_i} - sgn(d_i(k))x_i(k))^2}{2\rho^2\mathbf{w_i^T w_i}}\right) \tag{16}$$

where ρ is the radius parameter of the kernel density estimate [6].

Substituting the expected value of the gradient with a single point estimate, we have:

$$\hat{p}_{x_{s_i}}(x_{s_i}) = \frac{1}{K\sqrt{2\pi}\rho\sqrt{\mathbf{w_i^T w_i}}} exp\left(\frac{-(x_{s_i} - sgn(d_i(k))x_i(k))^2}{2\rho^2\mathbf{w_i^T w_i}}\right) \tag{17}$$

The probability of error for user i is estimated by:

$$P_{\epsilon_i} = P(x_{s_i} < 0) = \int_{-\infty}^{0}\hat{p}_{x_{s_i}}(x_{s_i})dx_{s_i} = Q\left(\frac{sgn(d_i(k)x_i(k)}{\rho(\mathbf{w_i^T w_i})^{1/2}}\right) \tag{18}$$

The gradient terms of P_ϵ are:

$$\frac{\partial P_{\epsilon_i}}{\partial \mathbf{w}_i} = \frac{exp\left(\frac{-x_i(k)^2}{2\rho^2\mathbf{w_i^T w_i}}\right)sgn(d_i(k))}{\sqrt{2\pi}\rho}\left(\frac{-\mathbf{r}(k)}{(\mathbf{w_i^T w_i})^{1/2}} + \frac{\mathbf{w_i}x_i(k)}{(\mathbf{w_i^T w_i})^{3/2}}\right) \tag{19}$$

and

$$\frac{\partial P_{\epsilon_i}}{\partial \mathbf{f}_i} = \frac{1}{\sqrt{2\pi}\rho\sqrt{\mathbf{w_i^T w}_i}} exp\left(\frac{-x_i(k)^2}{2\rho^2\mathbf{w_i^T w}_i}\right)sgn(d_i(k))\hat{\mathbf{b}}(k) \tag{20}$$

An algorithm similar to the LMS was devised in [6] by substituting the exact pdf by its instantaneous estimate and adjusting the receiver weights $\mathbf{w}_i(k)$ such that $\mathbf{w}_i^T(k)\mathbf{w}_i(k) = 1$:

$$\mathbf{w}_i(k+1) = \mathbf{w}_i(k) - \mu\left[\frac{\partial P_{\epsilon_i}}{\partial \mathbf{w}_i}\right]_k \tag{21}$$

$$\mathbf{f}_i(k+1) = \mathbf{f}_i(k) - \mu\left[\frac{\partial P_{\epsilon_i}}{\partial \mathbf{f}_i}\right]_k \tag{22}$$

The LBER algorithm for the linear MUD is given by ($\mathbf{f}_i = 0$):

$$\mathbf{w}_i(k+1) = \mathbf{w}_i(k) + \mu\frac{1}{\sqrt{2\pi}\rho}exp\left(\frac{-(x_i(k))^2}{2\rho^2}\right)sgn(d_i(k))$$

$$\times(\mathbf{r}(k) - \mathbf{w}_i(k)x_i(k)) \tag{23}$$

The LBER algorithm for the DFE MUD is expressed by:

$$\mathbf{w}_i(k+1) = \mathbf{w}_i(k) + \mu\frac{1}{\sqrt{2\pi}\rho}exp\left(\frac{-(x_i(k))^2}{2\rho^2}\right)sgn(d_i(k))$$

$$\times(\mathbf{r}(k) - \mathbf{w}_i(k)x_i(k)) \tag{24}$$

$$\mathbf{f}_i(k+1) = \mathbf{f}_i(k) - \mu \frac{1}{\sqrt{2\pi}\rho} exp\left(\frac{-(x_i(k))^2}{2\rho^2}\right) sgn(d_i(k))\hat{\mathbf{b}}(k) \quad (25)$$

where $d_i(k) = b_i(k)$ is the desired signal taken from the training sequence, μ is the algorithm step size and ρ the radius parameter is related to the noise standard deviation σ. Whilst in the AMBER, a non-zero τ defines a region boundary where the algorithm will continue to update, in the LBER, the effect of the distance from the decision boundary is controlled by an exponential term [6]. Indeed, this can be viewed as a soft distance metric, the size of an update is a continuous and decreasing function of the distance from the boundary and both algorithms have a complexity of $O(M)$ with two parameters that require tuning.

VI. Simulation Results

In this section, we conduct simulation experiments to assess the convergence and the BER performance of the DFE-MUD operating with the LMS, the AMBER and the LBER algorithms and perform a comparative analysis with a linear MUD using the same adaptive techniques. We consider linear channels and PN Gold non-orthogonal user sequences in all simulations. In addition, we use a small fixed threshold $\tau = 0.2$ for the AMBER algorithm in order to increase its convergence rate, and $\rho_n = 2\sigma_n$ for the LBER algorithm, in all situations.

A. Convergence performance

To analyse the convergence of the adaptive receivers with the LMS, the AMBER and the LBER algorithms, we have conducted simulations to assess the BER at each iteration. All convergence curves were obtained with 2000 training bits averaged over 100 independent experiments. The receivers operate with processing gain $PG = 7$, $N = 4$ users and use a step size $\mu = 0.005$.

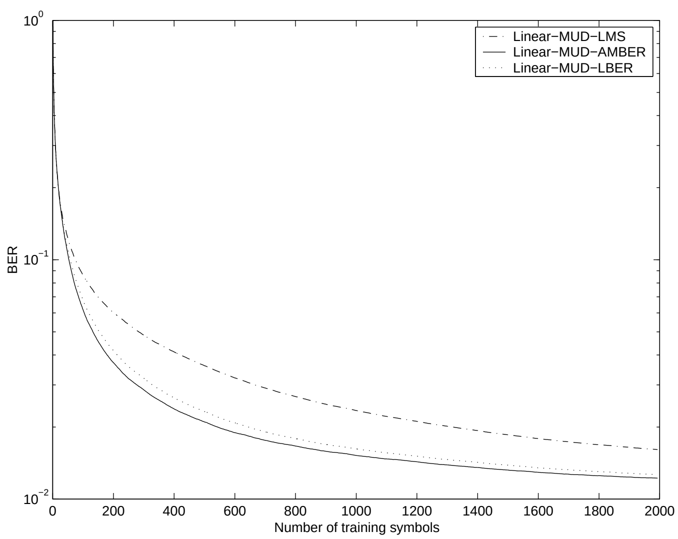

Fig. 3. Convergence of the algorithm with Linear-MUD receivers at $E_b/N_0 = 7dB$ for the channel with $H(z) = 1 + 0.25z^{-1} - 0.4z^{-2}$.

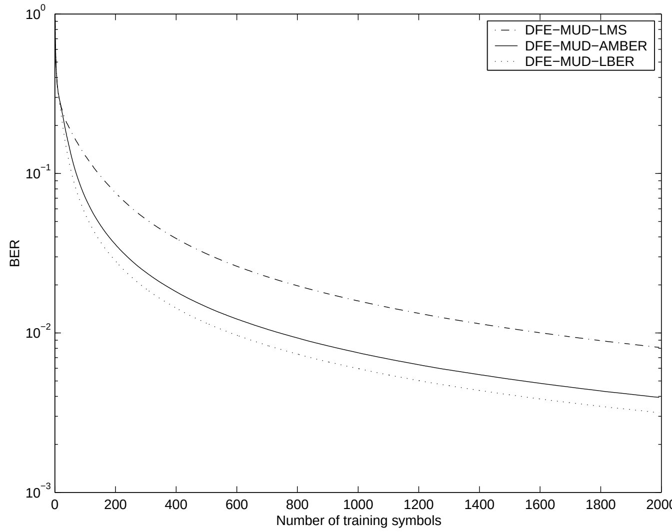

Fig. 4. Convergence of the algorithm with MUD-DFE receivers $E_b/N_0 = 10dB$ for the channel with $H(z) = 1.2+1.1z^{-1}-0.2z^{-2}$.

Figs. 4 and 5 show the convergence of the three adaptive algorithms for linear and DFE multiuser receivers, respectively. For the linear receivers at $E_b/N_0 = 7dB$ the AMBER algorithm was found to achieve the best convergence performance, followed by the LBER and the LMS algorithms. In the case of DFE receiver structures at $E_b/N_0 = 10dB$, the LBER algorithm was found to achieve the best convergence performance, followed by the AMBER and the LMS algorithms.

B. BER performance

All BER simulation results were obtained with 1000 training data bits and 10^4 data bits averaged over 100 independent experiments. All receivers operate with processing gain $PG = 7$, $N = 4$ users, use a step size $\mu = 0.0025$ during training and no adaptation occurs in data mode.

Fig. 5 shows the BER performance of linear MUD receivers and DFE ones employing the LMS, the AMBER and the LBER algorithms to adjust the filter parameters. The DFE-MUD structure employing adaptive MBER algorithms outperforms linear MUDs with these algorithms and the DFE-MUD with the minimum mean square error (MMSE) criterion. The LBER algorithm with the DFE MUD has outperformed the LMS and the AMBER algorithms with both receiver structures, at high E_b/N_0. At low E_b/N_0, the DFE-MUD with the AMBER technique has outperformed the LMS and the LBER algorithms. The LBER algorithm with the linear MUD has outperformed the LMS and the AMBER algorithms with linear receiver structures, at high E_b/N_0. At low E_b/N_0, the linear MUD with the AMBER technique has outperformed the LMS and the LBER algorithms. With the LMS approach, the DFE system can save up to 1 dB in comparison with the linear structure, for the same BER performance. When using the AMBER and the LBER algorithms, the DFE system can also save up to 1 dB in comparison with the linear structure, for the same BER performance.

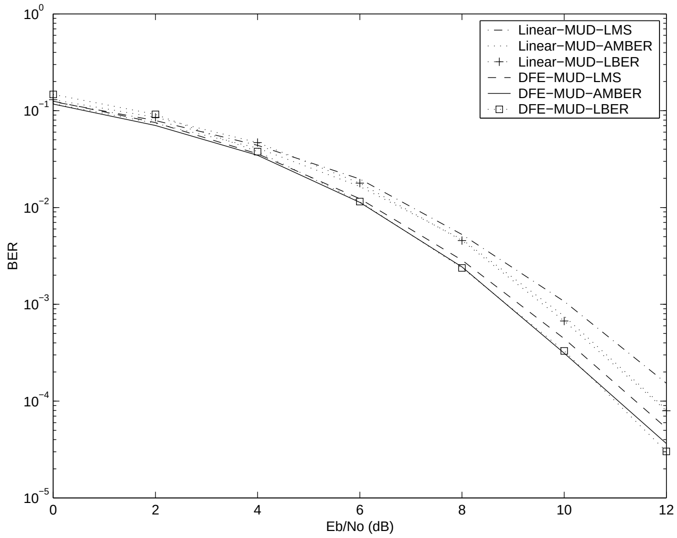

Fig. 5. BER Performance of MUD receivers for the channel with $H(z) = 1.2 + 1.1z^{-1} - 0.2z^{-2}$.

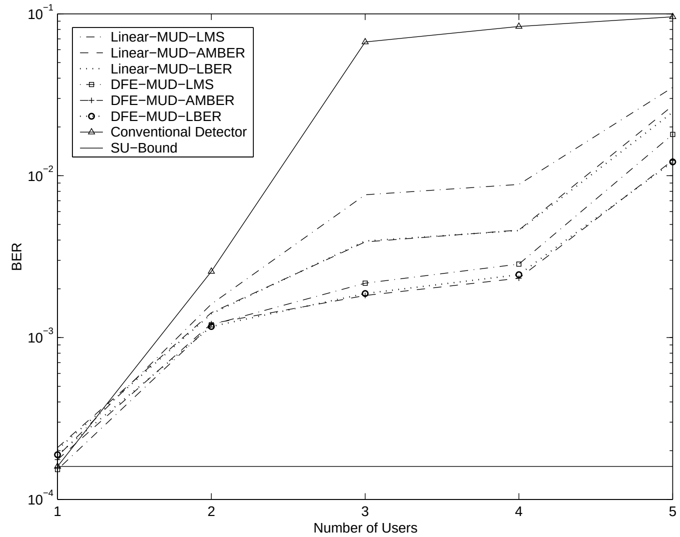

Fig. 6. BER Performance of MUD receivers with a varying number of users at $E_b/N_0 = 8dB$ for the channel with $H(z) = 1.2 + 1.1z^{-1} - 0.2z^{-2}$.

In another situation, the BER performance of linear and DFE MUDs, operating with the LMS, the AMBER and the LBER techniques, was assessed with a varying number of users at $E_b/N_0 = 8dB$. The results, depicted in Fig. 6, indicate that the DFE MUD with the LBER algorithms has achieved the best BER performance, followed by the DFE MUD with the AMBER and the LMS techniques, and the linear MUDs with the LBER, the AMBER and the LMS algorithms. Indeed, the deployment of MBER algorithms and a DFE structure can increase the capacity of a DS-CDMA system, for a given BER performance. Considering the BER performance of the conventional single-user detector, shown in Fig. 6, the capacity increase is rather significant for the DFE MUDs operating with MBER algorithms.

VII. Conclusions

We have examined the use of adaptive minimum bit error rate (MBER) algorithms with decision feedback multiuser receivers for DS-CDMA systems. Computer simulation experiments have demonstrated that the DFE-MUD structure employing adaptive MBER algorithms outperforms linear MUDs with these algorithms and the DFE-MUD with the LMS algorithm. The LBER algorithm with the DFE and the linear MUD receiver has outperformed the LMS and the AMBER algorithms at high E_b/N_0. The use of DFE structures can save up to 1 dB in comparison with linear receivers. With the AMBER and the LBER algorithms, the DFE receiver can save up to 0.4 dB when compared to the DFE operated with the LMS algorithm.

References

[1] S. Verdu, *Multiuser Detection*, Cambridge, 1998.
[2] M. Honig and M. K. Tsatsanis, “Adaptive techniques for multiuser CDMA receivers,” *IEEE Signal Processing Magazine*, pp. 49-61,May 2000.
[3] U. Madhow and M. L. Honig, “MMSE interference suppression for direct-sequence spread-spectrum CDMA,” *IEEE Transactions on Communications*, vol. 42, no. 12, December 1994.
[4] S. L. Miller, “An adaptive DS-CDMA Receiver for Multiuser Interference Rejection, ” *IEEE Transactions on Communications*, vol. 43, no. 2/3/4, February/March/April 1995.
[5] C. Yeh, R. R Lopes and J. R. Barry, “Approximate Minimum Bit-Error Rate Multiuser Detection”, *GLOBECOM´98*, Sydney, Australia, November 1998.
[6] S. Chen, A K. Samingan, B. Mulgrew and L. Hanzo, “Adaptive Minimum BER Linear Multiuser Detection,” *IEEE Proc. Int. Conf. Acoust. Speech and Sig. Proc.*, 2001.