MODULATION ARCHITECTURE $\{2m\text{-}QAM\}^2$ OVER RINGS OF INTEGERS FOR TCM

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Abstract- Based on the spectral overlapping of two conventional m-QAM modulations, an $\{m\text{-}QAM\}^2$ modulation technique is performed. This architecture has shown to have better bit error performance than its equivalent M-QAM modulation. The object of this paper is to present a multilevel coding method based on a ring of integer modulo-q over an expanded trellis coded $\{2m\text{-}QAM\}^2$ modulation, to collect associated gain in terms of E_b/N_0. Performance curves obtained by computer simulation with spectral efficiency for coded and uncoded schemes of 5 and 6 information bit/s/Hz for three codes combination are shown.

1 - INTRODUCTION

The multilevel convolutional coding method based on rings of integers modulo-q (q generally a nonprime integer) which is suitable for coded q-PSK modulation scheme is as well valid for convolutional codes by making a simple adaptation [1][2]. Another paper was developed using trellis coded modulation over rings of integers modulo-4 by QAM constellations [3].

A modulation architecture called $\{m\text{-}QAM\}^2$ is based on the partial spectral overlapping of two m-QAM schemes [4][5]. At the receiver, the two m-QAM signals are detached by means of a suitable hardware which may produce gains, in terms of E_b/N_0[dB] (bit energy to spectral noise density ratio), over an equivalent QAM modulation, looking for the same spectral efficiency and bit error rate. By doubling the number of symbols of each m-QAM and by using a trellis encoder for each modulation set [6][7][8], it is possible to built a $\{2m\text{-}QAM\}^2$ trellis encoder which sums the gain of the $\{m\text{-}QAM\}^2$ modulation scheme to the gain produced by the trellis encoding process [9].

This paper presents a coded modulation method based on a multilevel codes over finite ring of integer modulo-q on an expanded trellis coded $\{2m\text{-}QAM\}^2$ modulation. Tables of systematic convolutional codes and performance curves obtained by computer simulation are also shown.

2 - TCM OVER RINGS OF INTEGERS

The conception of the schemes that utilize coded modulation is to transmit k information bits per channel symbol by using a modulator with $q = 2^{k+1}$ waveform and then exploit the redundancy produced by using a suitable set of channel symbols. Ungerboeck [6][7][8] showed how it is possible to relate monotonically the Hamming distance involved in the binary convolutional coding with the Euclidean distance between channel symbols, by the named "mapping by set partitioning". Fig. 1 shows one alternative coding method, based on ring of integers modulo-q which is suitable for coded modulation schemes.

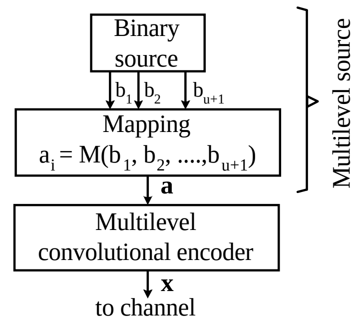

Fig.1 Encoder structure.

The u+1 parallel information bits (b_1, b_2, ..., b_{u+1}) from the binary source are mapped (Gray mapping was used) into one of $q = 2^{u+1}$ channel symbols a_i belonging to the expanded modulation signal set Z_q . The set $Z_q = \{ 0, 1, 2, ..., q\text{-}1 \}$ is defined as a ring of integer modulo-q. The input of the multilevel encoder is an information source $\mathbf{a} = (a_1, a_2, ..., a_k)$ with elements belonging to the ring Z_Q. The multilevel convolutional encoder then sends to the channel a coded sequence $\mathbf{x} = (x_1, x_2, ..., x_{k+1})$ the elements of which belong to the same ring Z_Q. The rate $R_c = k/k+1$ multilevel convolucional codes can be represented by the generator matrix G(D), where D is a delay element (memory) and the entries of the matrix are polynomials with coefficients belonging to the ring of integer modulo-q. The codes are assumed to be systematic to avoid catastrophicity and reduce the set of possible candidate codes which makes computer search algorithms less time consuming. Therefore, G(D) can be expressed by

$$G(D) = \left[\begin{array}{c|c} & g^{(1)}(D)/f(D) \\ & g^{(2)}(D)/f(D) \\ I_k & \vdots \\ & g^{(k)}(D)/f(D) \end{array} \right] \qquad (1)$$

where I_k is a k by k identity matrix and $g^{(i)}(D) = g_s^{(i)} D^s + ... + g_2^{(i)} D^2 + g_1^{(i)} D + g_0^{(i)}$ and $f(D) = f_s D^s + ... + f_2 D^2 + f_1 D + 1$ (with S=D+1) are the polynomials responsible for the feed-forward and feed-back connections, respectively, as shown in Fig. 2.

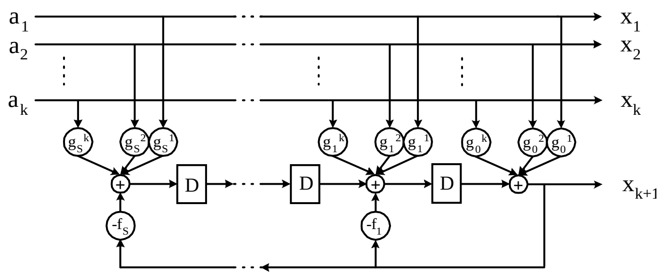

Fig.2. General structure of the encoder.

Notice that the problem is to find the matrix G(D) which maximizes the minimum free Euclidean distance d_{free} between all pairs of codewords. The main advantage of the alternative scheme proposed over Ungerboeck's scheme is the possibility to transmit fraction bits per modulation interval without the need to use constellations with more than two dimensions.

The most suitable modulation constellations for multilevel coding of sequences with elements belonging to the ring of integer modulo-q are those which have their modulation symbols distributed on a circumference, like q-PSK modulation. This distribution does not happen in m-QAM constellations with m > 4. Therefore, partitions to group the modulation symbols are needed, in order to allocate them over a circumference. For the 32-QAM constellation, using a partition of the signal space similar to that developed by Ungerboeck together with codes over Z_2 and Z_4 , it is possible to obtain significant asymptotic coding gains over uncoded 16-QAM.

Fig. 3 shows the signal space partition for 32-QAM. Each point of the signal space of 32-QAM is assigned by three symbols, one in Z_2(binary) and the others in Z_4(quaternary). The binary symbol selects one of the two subsets 16-QAM (protected by the code A). One quaternary symbol selects one of the four subsets 4-QAM produced by the partition (protected by the code B) and the other quaternary symbol selects a symbol in a subset (protected by the code C). Therefore it is possible to use three codes with different error correction capabilities, where A is the most powerful code in order to protect the nearest signal space, and code C, the least. Notice that both codes B and C over Z_4 protect symbols distributed on circumferences.

3 - The 32-QAM System

The spectral efficiency of the 16-QAM system is R = 4 bit/s/Hz. When coded modulation is applied over this system, its constellation is doubled and one code with rate $R_C = 4/5$ is inserted to keep the same spectral efficiency, therefore $R = R_C \log_2(2m)$. Then the trellis coded modulation over rings of integer modulo-q means doing q = 2m = 32.

As one encoder over Z_{32} will be tricky, we use the signal space 32-QAM partition to reduce the constellation and therefore the encoder´s complexity.

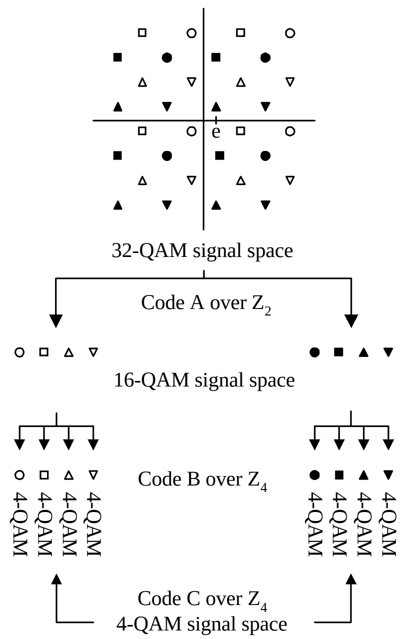

Fig.3. Signal space of 32-QAM partitioned into eight subsets.

Fig. 3 shows this mechanism. One possible encoder used in this work is shown in Fig. 4, with rate

$$R_C = \frac{a_A + 2a_B + 2a_C}{x_A + 2x_B + 2x_C} = \frac{4}{5} \tag{2}$$

Notice that for this encoder, code C does not exists.

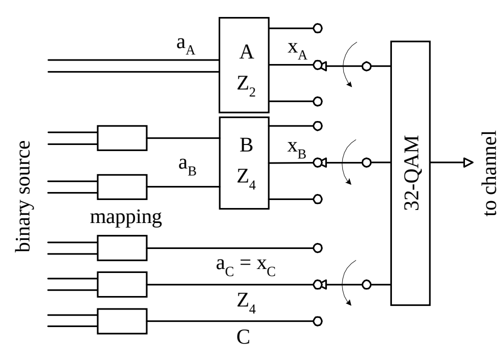

Fig.4. Encoder for RTCM(32-QAM) system.

4 - The $\{m\text{-QAM}\}^2$ System

A modulated wave $\{m\text{-QAM}\}^2$ can be represented as a narrowband signal, which is the summation of two m-QAM modulated waves separated in frequency by $\Delta f = |f_1 - f_2|$ [Hz]:

$$s(t) = \sum_{i=1}^{2} A_i \cos[2\pi f_i t + \theta_i] \tag{3}$$

where A_i , f_i and θ_i are the amplitude of the carrier, the carrier frequency and the phase of each m-QAM signal s_i . Either of the m-QAM modulation occupies a bandwidth equal to r_s , where $r_s = r_b/2 \log_2 m$ [symbol/s] is the transmitted symbol rate of each m-QAM with r_b the bit information rate. Then the total bandwidth of the signal $s(t)$ is $W = \Delta f + r_s = r_s(1+\Delta f/r_s)$ [Hz], and therefore the spectral efficiency R of the {m-QAM}^{2} system is given by

$$R = \frac{r_b}{W} = \frac{2\log_2 m}{(1+\Delta f/r_s)} \quad \text{[bit/s/Hz]} \tag{4}$$

The AWGN noise, at the receptor, is considered narrowband over the signal $s(t)$, and can be represented in the form,

$$n(t) = n_I(t)\cos(2\pi f_0 t) + n_Q(t)\,\mathrm{sen}(2\pi f_0 t) \tag{5}$$

where $n_I(t)$ and $n_Q(t)$ are the in-phase and quadrature noise components, respectively. Each noise component occupies a bandwidth of W [Hz]. The average power of each component is N_0 W, where N_0 [Watt/Hz] is the noise spectral density. The frequency f_0 is the mean between the two carrier frequencies f_1 and f_2.

The authors have shown [4][5] that the effect of the noise in the input of each decision/decoding device can be considered independent from each other. Taking this consideration into account, for high values of E_b/N_0 , the BER (bit error rate) for the {m-QAM}^{2} system can be evaluated by

$$BER \cong \frac{4\left(1-\frac{1}{\sqrt{m}}\right)}{\log_2 m}\,\mathrm{erfc}\left\{\sqrt{\frac{3\log_2 m}{2(m-1)(1+\Delta f/r_s)\,\delta(\Delta f/r_s)}\frac{E_b}{N_0}}\right\} \tag{6}$$

where $\delta(\Delta f/r_s)$ is a real valued function of $\Delta f/r_s$. The function $\delta(\Delta f/r_s)$ depends on the structure of the receiver, i. e., it depends on the correlators and linear transformation used to separate the information signals at the receiver [4][5]. Table 1 shows some values of $\delta(\Delta f/r_s)$ for given values of $\Delta f/r_s$.

TABLE 1:

VALUES OF $\delta(\Delta f/r_s)$ AS FUNCTION OF $\Delta f/r_s$

$\Delta f/r_s$	1	5/7	3/5	1/2	1/3	1/5	1/7
$\delta(\Delta f/r_s)$	0.43	0.48	0.53	0.63	1.02	2.19	3.85

The {m-QAM}^{2} scheme is compared with an equivalent M-QAM system, taking into account the same spectral efficiency. Them m and M are related by

$$M = 2^i \;;\; i = \sqrt{m}, \ldots\ldots, 2\sqrt{m}-1 \tag{7}$$

and associated with $\Delta f/r_s$, by

$$\Delta f/r_s = 2\frac{\log_2 m}{\log_2 M} - 1 \tag{8}$$

For instance, the {4-QAM}^{2} may be compared by 4 and 8-QAM, once the {16-QAM}^{2} will be with the traditional 16, 32, 64 and 128-QAM.

5 - THE RTCM{2m-QAM}^{2} SYSTEM

The RTCM{2m-QAM}^{2} scheme was built by trellis coded modulation over the {m-QAM}^{2} scheme in a context of coding from rings of integers modulo-q. Fig. 5 presents, in block diagram, the proposed system where $r_b = r_{b1} + r_{b2}$ is the total bit rate.

The system of interest is RTCM{32-QAM}^{2} , where 2m = 32, that results in an spectral efficiency of

$$R[\text{bit/s/Hz}] = \frac{8}{1+\Delta f/r_s} \tag{9}$$

In this work, we will compare the RTCM{32-QAM}^{2} system with a scheme 32-QAM (for R = 5 bit/s/Hz , $\Delta f/r_s$ = 3/5) and with a scheme 64-QAM (for R = 6 bit/s/Hz , $\Delta f/r_s$ = 1/3).

Table 2 shows codes over Z_2 and Z_4 used in the RTCM{32-QAM}^{2} system, that were found by computational search. It shows the polynomials $g^1(D)$, $g^2(D)$ and f(D) in hexadecimal notation for codes in Z_2 and Z_4 , E represents the number of states of the convolutional code, d^2_{free}/e^2 is the rate of the minimum square free Euclidean distance of the code and the square distance of the constellation 16-QAM (4-QAM for the code in Z_4), tp informs the amount of parallel transitions in the trellis of these codes and τ as the number of windows considered in the trellis.

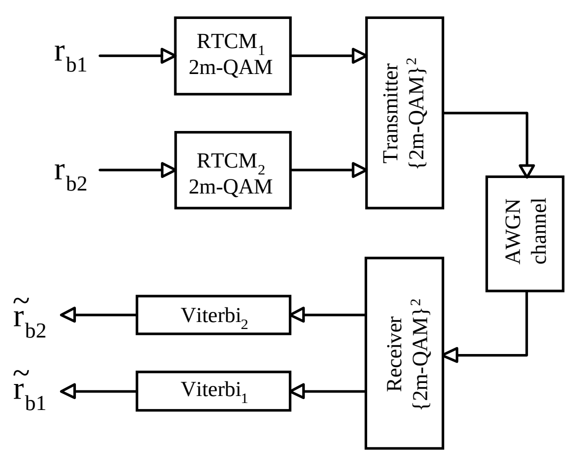

Fig. 5. Block diagram of the RTCM{2m-QAM}^{2} system.

For the system's simulation, the Monte Carlo technique was used, based on the C language.

Figures 6 and 7 show performances in terms of BER versus E_b/N_0[dB] for the RTCM{32-QAM}^{2}, {16-QAM}^{2} and QAM schemes, with the same spectral efficiency for all of them and using code B in combination with three different codes. In the simulations it was set for their constellations with equal mean power.

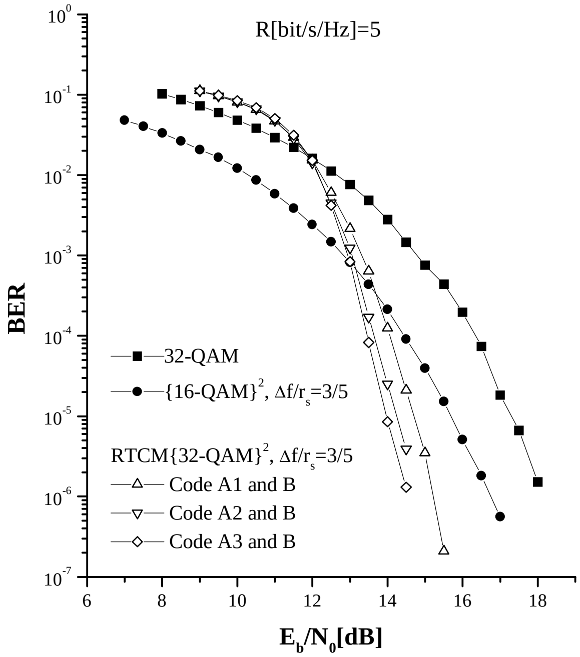

Fig. 6. BER performance for RTCM{32-QAM}^{2} with $\Delta f/r_s$= 3/5 ,versus {16-QAM}^{2} and QAM systems.

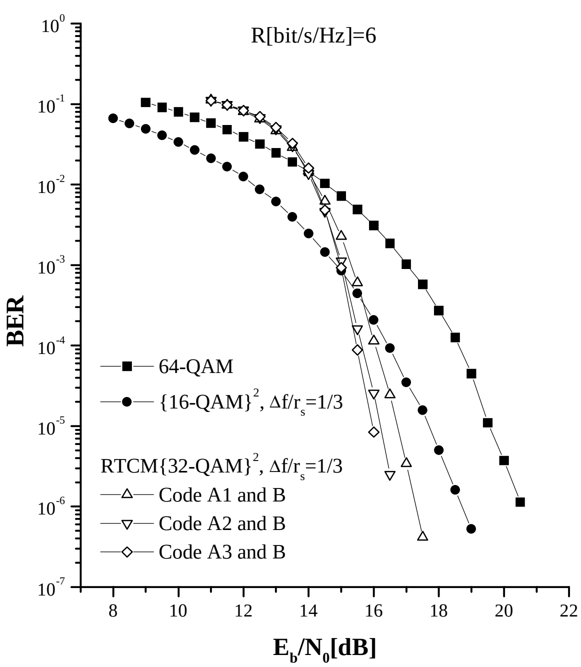

Fig.7. BER performance for RTCM{32-QAM}^{2} with $\Delta f/r_s$= 1/3, versus {16-QAM}^{2} and QAM systems.

So, the 32-QAM system is allocated at the rightmost side of the set of curves, then the {16-QAM}^{2} scheme with $\Delta f/r_s$= 3/5 shows better performance than the 32-QAM. Now, for approximately BER > 10^{-2} the RTCM{32-QAM}^{2} do not have advantages in terms of performance towards 32-QAM and {16-QAM}^{2} systems. But for BER < 10^{-4} the RTCM{32-QAM}^{2} with $\Delta f/r_s$= 3/5 needs less E_b/N_0 than the other systems to reach the same BER. Note when the combination of codes Ai (i = 1, 2, 3) and B is better in terms of free distance, the performance of the RTCM{32-QAM}^{2} is better too.

Is good to remark that the spectral efficiency of coded and uncoded schemes is 5 information bit/s/Hz.

The total asymptotic coding gain is the asymptotic gain produced by the uncoded {16-QAM}^{2} with $\Delta f/r_s$=3/5 over the conventional uncoded 32-QAM plus the asymptotic coding gain of the RTC{32-QAM}^{2} with $\Delta f/r_s$=3/5 over the uncoded {16-QAM}^{2}.

Similar results are obtained for the RTCM{32-QAM}^{2} with a superposition degree of the 32-QAM constellations by $\Delta f/r_s$= 1/3, as shown in Fig.7 but now for spectral efficiency of 6 bit/s/Hz.

TABLE II

CODES OVER Z_2 AND Z_4 FOR THE RTCM{32-QAM}^{2} SYSTEM

Codes Z_2	**$g^1(D)/g^2(D)/f(D)$**	*D*	*E*	**d^2_{free}/e^2**	**t_p**	**τ**
A1	11/13/15	3	8	16	1	15
A2	45/36/57	5	32	20	1	25
A3	161/045/147	6	64	24	1	30

Codes Z_4	**$g^1(D)/g^2(D)/f(D)$**	*D*	*E*	**d^2_{free}/e^2**	**t_p**	**τ**
B	23/12/31	1	4	8	4	5

6 - SUMMARY

A multilevel coding method based on a ring of integer modulo-q over an expanded trellis coded {2m-QAM}^{2} modulation was presented.

Figures show performance curves obtained by computer simulation with spectral efficiency for coded and uncoded schemes of 5 and 6 information bit/s/Hz , using code B in combination with three different codes.

The RTCM{32-QAM}^{2} scheme responds with a similar form to the binary systems and in terms of BER versus E_b/N_0 it shows abrupt falls more than those systems working over the binary algebra.

Other constellation, as the binary PSK, was researched using spectral superposition and submit to SBrT'01 [10].

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