Analysis of Antenna Arrays Configurations with Random Parameters

Wamberto J. L. Queiroz, Marcelo S. Alencar and Fabricio G. S. Silva
Universidade Federal da Paraíba, Campina Grande PB, Brazil

Abstract—**This paper presents a performance analysis of two linear arrays structures considering the effect of excitation coefficients and random separation between elements. The results obtained show that these configurations can produce good radiation patterns in terms of side lobes level and directivity. Depending on the chosen parameters the irradiation pattern can be optimized as compared to the one obtained using binomial coefficients and as good as the patterns obtained using Dolph-Tschebyscheff. This method can be used in order to get a smoother irradiation pattern and can be useful to control de irradiation pattern in response to user mobility. Another advantage is related to the numerical processing involved in the computation of excitation coefficients. They are chosen randomly from a convenient set, while in other cases they are computed in real time and depend on antenna dimensions, a task which can be computer intensive.**

Keywords—**Array antennas, random coefficients, random spacing, array factor.**

I. INTRODUCTION

ALTHOUGH it is now a consolidated area, the antenna array theory has given researchers the opportunity of studying applications involving signal processing methods, in order to get controllable antenna patterns in accordance with information exchanged between the users of a wireless communication system and the radio base station. These antennas are called smart antennas and require that all numerical processing at the radio base station be fast and efficient. So, it is necessary to develop good algorithms in order to shape the irradiated pattern in a desired direction in response to a mobile user requirement.

An approach to control the irradiation pattern is to adjust certain parameters, like the spacing d_n between array elements, the excitation coefficients amplitude a_n and the transmitted phase θ_n. Therefore, in this work one proposes another method of allocating the sensors and controlling the amplitude of the excitation coefficients a_n. Uniformly distributed random parameters are used. Such parameter arrangement presents good advantages in relation to classical linear configurations and can be a practical solution to wireless local loop systems.

II. DEVELOPMENT

In the following the viability of three types of array configuration are analyzed, considering the irradiation pattern shape regarding the excitation coefficients values and displacement of the antenna elements. First, consider an array with an even number $2M$ of isotropic elements (sensors), symmetrically placed along the z-axis, as shown in Figure 1(a). The z axis is used for convenience, without loss of generality. Considering the spacing between elements constant, that amplitude of the excitation coefficients is symmetrically distributed around the z-axis origin and the irradiated field observations are taken at a point far

The authors are thankful with the Laboratório de Comunicações, Departamento de Engenharia Elétrica, Universidade Federal da Paraíba, Campina Grande - PB, Brazil, 58.109-970, Phone: +55 83 310 1410 Fax: +55 83 310 1049. E-mails: {malencar,wamberto,fabricio}@dee.ufpb.br.

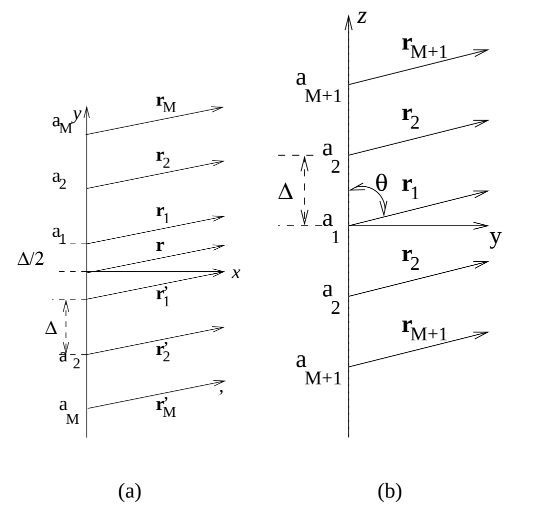

Fig. 1. Linear array with non-uniform amplitude excitation for (a) an even or (b) odd number of sensors distributed along the z axis.

away from array, The array factor can be written as

$$
\begin{aligned}
(AF)_{2M} &= a_1 e^{j\frac{kd}{2}\cos(\theta)} + a_2 e^{j\frac{3kd}{2}\cos(\theta)} \\
&+ \cdots + a_n e^{j\frac{(2M-1)kd}{2}\cos(\theta)} \\
&+ a_1 e^{-j\frac{kd}{2}\cos(\theta)} + a_2 e^{-j\frac{3kd}{2}\cos(\theta)} \\
&+ \cdots + a_n e^{-j\frac{(2M-1)kd}{2}\cos(\theta)}, \qquad (1)
\end{aligned}
$$

and can also be written in a normalized form as

$$(AF_{2M}) = \sum_{n=1}^{M} a_n \cos\left[\left(\frac{2n-1}{2}\right) kd\cos(\theta)\right], \qquad (2)$$

where a_n are the array excitation coefficients, $k = \frac{2\pi}{\lambda}$, d is the distance between the array elements, λ represents the wavelength and θ is the horizontal angle used to observe the irradiated field.

If the number of isotropic elements is odd, $2M+1$, as shown in Figure 1(b), the array factor array can be written as

$$
\begin{aligned}
(AF)_{2M+1} &= 2a_1 + a_2 e^{jkd\cos(\theta)} + a_3 e^{j2kd\cos(\theta))} \\
&+ \cdots + a_{M+1} e^{j2Mkd\cos(\theta))} \\
&+ a_2 e^{-jkd\cos(\theta)} + a_3 e^{-j2kd\cos(\theta)} \\
&+ \cdots + a_n e^{-j2Mkd\cos(\theta)}, \qquad (3)
\end{aligned}
$$

or in normalized form

$$(AF_{2M+1}) = \sum_{n=1}^{M+1} a_n \cos\left[(n-1)kd\cos(\theta)\right]. \qquad (4)$$

The two usual methods to obtain the excitation coefficients are the binomial expansion, using coefficients of the function $f(x) = (1+x)^{m-1}$, and the Dolph-Tchebyscheff's polynomial coefficients. In the first case, the positive coefficients of the series expansion for different values of m form the known Pascal's triangle. Using m as the number of array elements, then the expansion coefficients represent the relative amplitudes of the elements.

In the second case, considering $kd\cos(\theta) = u$, the terms $\cos(nu)$ in Equation 4 can be expanded in a series of cosine functions with u as argument. For example, for $m = 9$,

$$\begin{aligned} \cos(mu) &= 256\cos(u)^9 - 576\cos(u)^7 + 432\cos(u)^5 \\ &- 120\cos(u)^3 + 9\cos(u), \end{aligned} \quad (5)$$

form a Tchebyscheff polynomial of order 9, or using another notation,

$$T_9(z) = 256z^9 - 576z^7 + 432z^5 - 120z^3 + 9z. \quad (6)$$

In this work, instead of using a deterministic method to find the excitation coefficients, their values are assumed independent and uniformly distributed in the interval $[a_l, a_r]$, where a_l and a_r are related to the number of elements in the array. In this case, it is interesting to consider an average irradiation pattern. Figure 2, show normalized irradiation patterns of the proposed method in order to compare with the conventional methods.

In addition to the the use of random excitation coefficients, the distance d between elements can also be uniformly distributed in the interval $[d_l, d_r]$, which gives the pattern shown in Figure 6.

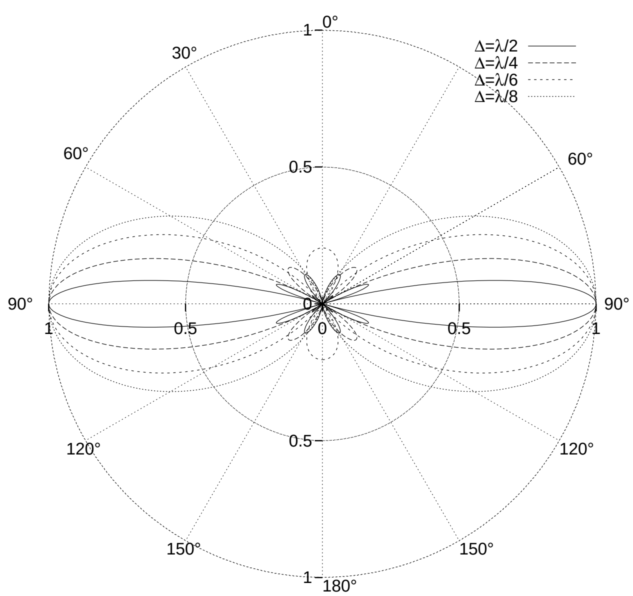

Fig. 2. Average array factor for an array with 8 elements distributed around the origin of the z axis with the amplitude of the excitation coefficients uniformly distributed in the interval $[8, 16]$. The spacing d between the elements are made equal to $\lambda/2$, $\lambda/4$, $\lambda/6$ and $\lambda/8$.

The average array factor ($\overline{AF}$), considering 2M isotropic elements and $a_n \sim U[a_l, a_r]$, can be written as

$$\begin{aligned} (\overline{AF})_{2M} &= E\left[\sum_{n=1}^{M} a_n \cos\left[\frac{2n-1}{2}kd\cos(\theta)\right]\right] \\ &= \frac{(a_l + a_r)}{2}\sum_{n=1}^{M}\cos\left[\frac{2n-1}{2}kd\cos(\theta)\right] \end{aligned} \quad (7)$$

and the array factor when $d_n \sim U[d_l, d_r]$, can be written as

$$\begin{aligned} (\overline{AF})_{2M} &= E\left[\sum_{n=1}^{M} a_n \cos\left[\frac{2n-1}{2}kd_n\cos(\theta)\right]\right] \\ &= \frac{d_r}{(d_r - d_l)}\sum_{n=1}^{M} a_n Sa((2n-1)u_r) - \\ &- \frac{d_l}{(d_r - d_l)}\sum_{n=1}^{M} a_n Sa((2n-1)u_l), \end{aligned} \quad (8)$$

where $u_r = \frac{kd_r\cos(\theta)}{2}$, $u_l = \frac{kd_l\cos(\theta)}{2}$ and $Sa(z) = \frac{\sin(z)}{z}$.

In (8) the excitation coefficients could be obtained using the binomial expansion or Tchebyscheff's polynomial and in this case it presents the irradiation pattern shown in Figure 3 normalized with respect to the binomial expansion. The normalizing factor was $(AF_{2M})\text{max} = 2^{2(M-1)}$.

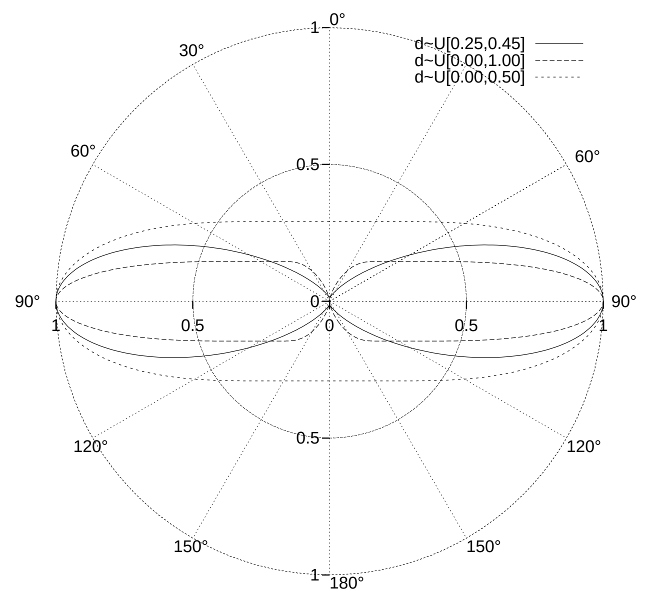

Fig. 3. Average array factor for an array with 8 elements distributed around the origin of the z axis, with the amplitude of the excitation coefficients given by the binomial expansion and the spacing d between the elements uniformly distributed in $[0.25, 0.45]\lambda$, $[0.00, 1.00]\lambda$ and $[0.0.0, 0.50]\lambda$.

As can be seen, its possible to obtain a good irradiation pattern control just controlling the spacing between the array elements. In the same way, the irradiation pattern when Tchebyscheff's polynomial is used, is shown in Figure 4.

As can be seen, the array factor produced the elimination of the side lobes when $d \sim U[0.25, 0.45]\lambda$ and an interesting behavior when $d \sim U[0.0, 0.5]\lambda$. Comparing Figure 4 with the Figure 5 one can see that the random sensors allocation provides an improvement over the conventional Tchebyscheff's method.

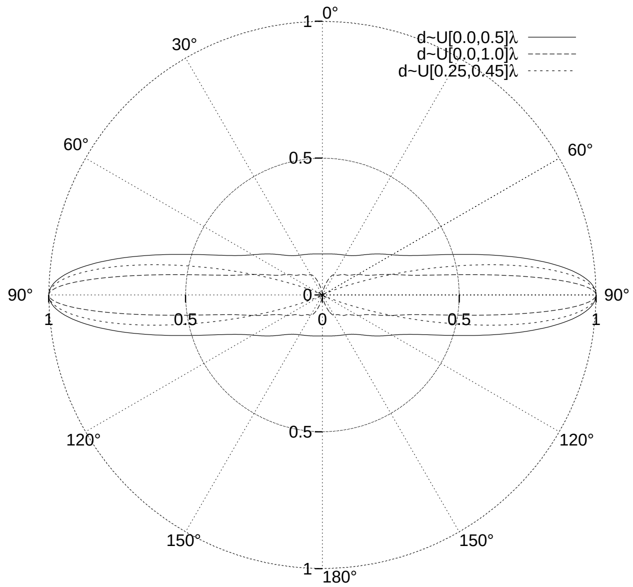

Fig. 4. Average array factor for an array with 10 elements, distributed around the origin of the z axis with the amplitude of the excitation coefficients given by Tchebyscheff's polynomial and the spacing d between the elements uniformly distributed in $[0.25, 0.45]\lambda$, $[0.00, 1.00]\lambda$ and $[0.0.0, 0.50]\lambda$.

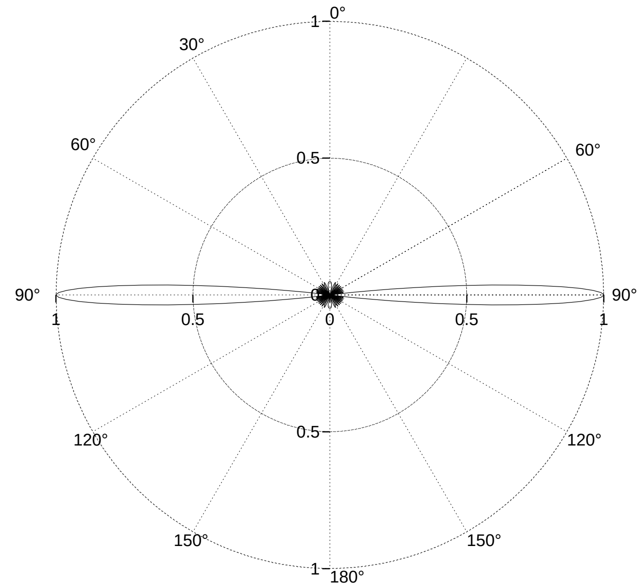

Fig. 5. Average array factor for an array with 10 elements distributed around the origin of the z axis with the amplitude of the excitation coefficients given by Tchebyscheff's polynomial and the spacing d between the elements given by $\lambda/2$.

The antenna length is given by the addition of all individuals lengths. For example, if the distances between the elements were $[\ 0.25\lambda \quad 0.32\lambda \quad 0.15\lambda \quad 0.45\lambda \quad 0.50\lambda\ ]$ so, the antenna length would be 1.67λ.

When both a_n and d_n are random, the average array factor is given by

$$(\overline{AF})_{2M} = E\left[\sum_{n=1}^{M} a_n \cos\left[\frac{2n-1}{2} k d_n \cos(\theta)\right]\right]. \qquad (9)$$

Assuming that a_n and d_n are completely independent, we have

$$\begin{aligned}(\overline{AF})_{2M} &= \frac{a_r + a_l}{2(d_r - d_l)}\left[\sum_{n=1}^{M} d_r Sa((2n-1)u_r)\right. \\ &- \left.\sum_{n=1}^{M} d_l Sa((2n-1)u_l)\right]. \qquad (10)\end{aligned}$$

When both d_n and a_n are uniformly distributed in appropriate intervals, the average irradiation patterns are as shown in Figure 6.

When compared to the irradiation pattern obtained with binomial expansion, as shown in Figure 7, there are some undesired side lobes that can be reduced with an appropriate parameter choice. Figure 7 shows a normalized irradiation pattern with 11 elements and excitation amplitudes given by the binomial expansion.

Although the binomial expansion can completely eliminate side lobes when $d = \lambda/2$ and $d = \lambda/4$, there is a disadvantage in using such method because the coefficients present a large variance, the compromises the practical use of the antenna and its efficiency be reduced.

An important parameter that can be obtained is the directivity. This parameter is defined as the ratio between the radiation intensity U in a given direction to the average radiation intensity over all directions and is given by

$$D = \frac{U_{\text{max}}}{U_0}, \qquad (11)$$

where U_0 is given by

$$U_0 = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} U(\theta)\text{sen}\theta d\phi d\theta \qquad (12)$$

and $U_{\text{max}} = \left[(AF)_{2M}^2\right]\big|_{\theta=90}$.

For an array with $2M$ elements positioned on the z axis, U_0 can be written as

$$\begin{aligned}U_o &= \sum_{n=1}^{M} {a_n}^2[1 + Sa((2n-1)kd)] \\ &+ 2\sum_{n=1}^{M-1}\sum_{m=n+1}^{M} a_n a_m Sa((n+m-1)kd) \\ &- 2\sum_{n=1}^{M-1}\sum_{m=n+1}^{M} a_n a_m Sa((n-m)kd) \qquad (13)\end{aligned}$$

and the maximum radiation intensity U_{max} is given by

$$U_{\text{max}} = \sum_{n=1}^{M} a_n^2 + 2\sum_{n=1}^{M-1}\sum_{m=n+1}^{M} a_n a_m. \qquad (14)$$

When $d = \lambda/2$, the directivity expression is reduced to

$$D = \frac{U_{\text{max}}}{U_o} = \frac{(\sum_{n=1}^{M} a_n)^2}{\sum_{n=1}^{M} {a_n}^2}. \qquad (15)$$

When the excitation coefficients are random the directivity D is a random variable and its average value is given by

$$\overline{D_{\text{max}}} = E\left[\frac{U_{\text{max}}}{U_o}\right]. \qquad (16)$$

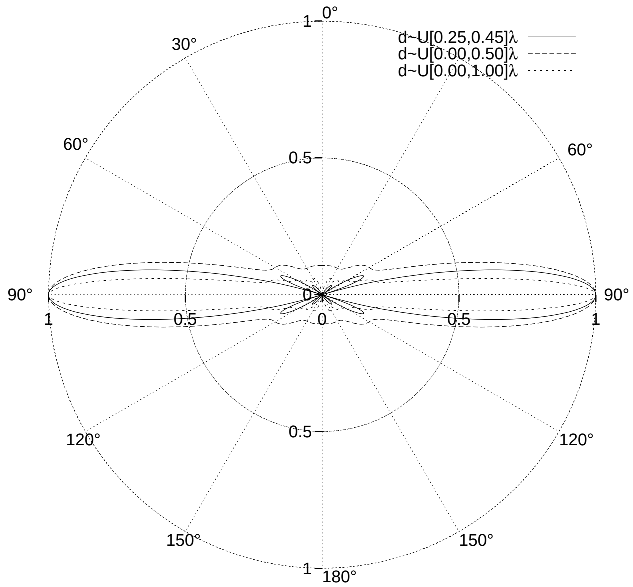

Fig. 6. Average array factor for an array with 10 elements distributed around the origin of the z axis with the amplitude of the excitation coefficients uniformly distributed in $[8.0, 16.0]$ and the spacing d between the elements uniformly distributed in $[0.25, 0.45]\lambda$, $[0.0, 0.50]\lambda$ and $[0.0, 1.0]\lambda$.

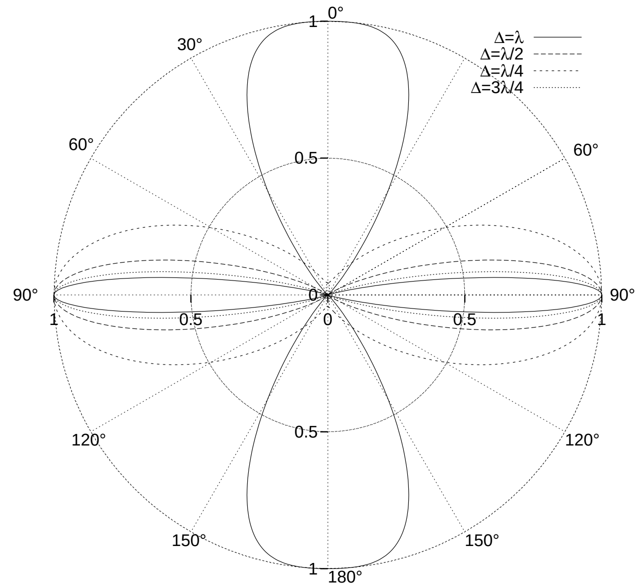

Fig. 7. Average array factor for an array with 11 elements distributed around the origin of the z axis with the amplitude of the excitation coefficients given by the binomial expansion and the distance d between the elements given by λ, $\lambda/2$, $\lambda/4$ e $3\lambda/4$.

Three values of directivity, for the Binomial expansion, Tchebyscheff's coefficients and random coefficients were obtained, respectively $D_1 = 7.31$dB, $D_2 = 9.48$dB and $D_3 = 9.23$dB. These values were obtained using an array with 10 elements and a spacing $d = \lambda/2$. The value D_3 was obtained using random excitation coefficients and, as can be seen, this method offer good results as compared to the classical methods.

Conclusion

This paper proposes another method for controlling the irradiation pattern of an array antenna with sensors displaced symmetrically around the z axis. The results shown here were obtained considering far field observations and show good results compared to classical methods, including binomial expansion and Tchebyscheff's polynomial. Its presents a practical and fast way of adjusting the irradiation pattern. Those characteristics are important when wireless local loop and mobile cellular systems are deployed.

Acknowledgments

The authors would like to express their thanks to Brazilian Council for Scientific and Technological Development (CNPq) for the financial support to this research.

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