CONSTRUCTIVE THEORY OF ANTENNA SYNTHESIS ON THE BASE OF THE NEW CLASS OF ATOMIC-FRACTAL FUNCTIONS

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Abstract – This report is devoted to investigation of the new class of atomic-fractal functions and their applications in problems of antenna synthesis

1 - Introduction

The new class of fractal functions based on specific properties of atomic functions (AF) y_r and π_m [1–7] is proposed and justified in this work. For the first time, the new types of atomic-fractal functions (AFF) constructed on the base of combinations of Kravchenko AFF with classical nondifferentiable Boltzano (1830), Weierstrass (1872), Besicovitch (1922), Van-der-Waerden (1930), etc. functions are obtained. The work consists of two parts. First of them is devoted to foundations of AF and their fractal properties. Here, the construction of AFF is also described. In the second part the applications of AFF for solving some problems of synthesis of discrete and continuous fractal radiating structures are presented. Numerical experiments and their comparison with results obtained by D. Werner et. al. [8] prove the efficiency of the new class of AFF.

2 - The Family of AFF y_r and $\pi_m(x)$

The Fourier transforms of AFF are as follows. For the function $y_r(x)$:

$$\prod_{i=1}^{\infty}\frac{\operatorname{shc}\left(k2^{-1}+it2^{-n}\right)}{\operatorname{shc}(k/2)},\ (\operatorname{shc}(x)\equiv\operatorname{sh}(x)/x);$$

for the function $\pi_m(x)$:

$$\frac{\prod_{k=1}^{m}\left[\sin\left(\frac{(2m-1)t}{(2m)^k}\right)+\sum_{\nu=2}^{m}(-1)^{\nu}\sin\left(\frac{(2m-2\nu+1)t}{(2m)^k}\right)\right]}{(3m-2)t/(2m)^k}.$$

Taking into account ideas and results presented in [5] and according to [9], we can construct the new class of AFF by multiplying distributions $y_r(x)$, $\pi_m(x)$ by another fractal function (Weierstrass, Cantor, Besicovitch, Van-der-Waerden, etc.).

Let us consider one of the important AFF $\pi_m(x)$ (Fig.1). It has the following properties: 1. $\operatorname{supp}\pi_m(x)=[-1,1]$; 2. $\pi_m(-x)=\pi_m(x)$; 3. $\pi_m(x)\in C^{\infty}[-1,1]$; 4. $\int_{-\infty}^{\infty}\pi_m(x)dx=1$.

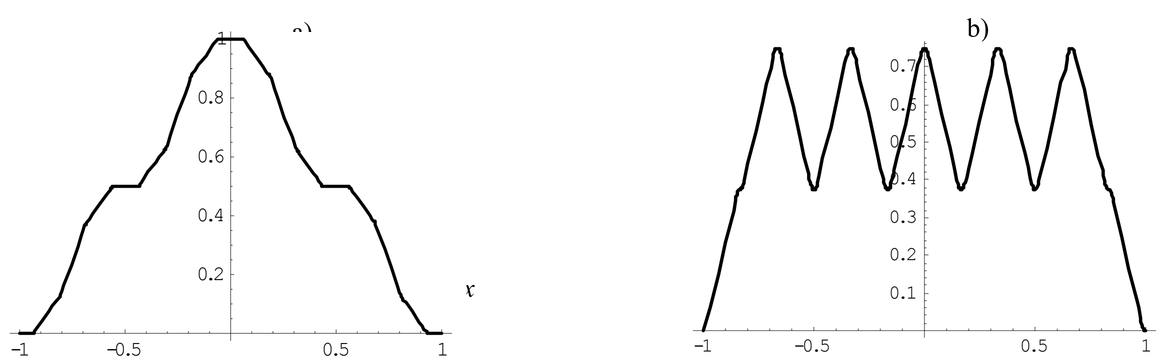

Fig. 1. Plots of functions $\pi_m(x)$ for m=2 (a) and 6 (b).

3 - Synthesis of Atomic-Fractal Radiation Patterns

A family of functions, called the generalized Weierstrass functions [5] are known to play a pivotal role in the theory of fractal radiation pattern synthesis. These functions are everywhere continuous but nowhere differentiable and exhibit fractal behavior at all scales. This class of functions can be represented by $f(x)=\sum_{n=1}^{\infty}\eta^{(D-2)n}g(\eta^n x)$, where $1<D<2$, g is a suitable bounded periodic function, and $\eta>1$. The array factor for the nonuniformly but symmetrically spaced linear array of 2*N* elements may be expressed in the form

$$f(\theta)=2\sum_{n=1}^{N}I_n\cos(k\,d_n\cos\theta+\alpha_n). \quad (1)$$

Here, $k=2\pi/\lambda$, I_n and α_n are amplitude and phase excitations, respectively, d_n represents the array element locations. Suppose the factor of an array with finite number of elements to be expressed by the generalized Weierstrass function with cosine function *g* and random phase α_n as

$$f(u)=2\sum_{n=1}^{N}\eta^{(D-2)n}\cos(a\eta^n u+\alpha_n),$$

where *a* is the constant and $u=\cos\theta$. The fractal radiation pattern (RP) defined by the latter expression possesses the self-similarity at the infinite range of scales. It represents the array factor for a nonuniform linear array of 2*N* elements. Thus, in this case the Weierstrass partial sum (1) may be classified as band-limited since the resulting RP exhibits fractal behavior over a finite range of scales with lower bound $2\pi/a\eta^N$. The range of scales may be controlled by the number of elements *N* in the array. Any fractal function is constructed by using recursive algorithms with an appropriate generating function. Let the RP of a linear radiator of infinite length be represented as the following band-limited generalized Weierstrass function:

$$F(u)=\sum_{n=0}^{N-1}\eta^{(D-2)n}g(\eta^n u) \quad (2)$$

with generating function *g*(*u*). Here, we assume that *g*(*u*) is periodic and even, i.e., *g*(*u*+2)=*g*(*u*), *g*(–*u*)=*g*(*u*).

4 - The New Class of Generating Functions Based on AFF

For the first time, the new family of generating functions was obtained:

a) $g_1(u)=\pi_m(u)$, $g_2(u)=y_r(u)$ (Kravchenko AFFs);

b) $g_3(u)=y_5(u)V(u)$ (the Kravchenko-Weierstrass AFF);

c) $g_4(u)=\pi_6(u)\cdot Cantor(u)$ (the Kravchenko-Cantor AFF);

d) $g_5(u)=\pi_{12}(u)\cdot Besicovitch(u)$ (the Kravchenko-Besicovitch AFF);

e) $g_6(u)=y_5(u)\cdot W(u)$ (the Kravchenko-Van-der-Waerden AFF). Any of these synthesized functions can be expressed by series

$$g(u)=\frac{a_0}{2}+\sum_{m=1}^{\infty}a_m\cos(m\pi u) \quad (3)$$

with Fourier coefficients $a_m=2\int_0^1 g(u)\cos(m\pi u)\,du$. Then, substituting (3) into (2), we get the following expression for the RP:

$$F(u)=\frac{a_0}{2}\frac{\eta^{(D-2)N}-1}{\eta^{(D-2)}-1}+\sum_{m=1}^{\infty}\sum_{n=0}^{N-1}a_m\eta^{(D-2)n}\cos(m\pi\eta^n u),$$

$$a_m=2\int_0^1 g(u-1)\cos(m\pi u)\,du, \quad (4)$$

where $\eta>1$ and $1<D<2$. Expression (4) represents the Fourier expansion of the fractal RP *F*(*u*) with respect to the basis of AFF in combination with the band-limited cosine Weierstrass function.

5 - Numerical Experiments

To illustrate the synthesis procedure for the linear radiation source, we considered some examples with the generating distribution. Using the properties of the AFF generating functions, let us present an example of the normalized RP for a linear source $F(u)$ with given values of D, η, and N (Fig. 2). The directivity of AFF arrays with the Kravchenko-Besicovitch generating function is defined as

$$G_{AFF}(u_0)=2f_N^2(u_0)\Big/\int_{-1}^{1}f_N^2(u)\,du,$$

where $f_{N\,AFF}(u)$ is determined from [3], and coefficients a_m are the expansion coefficients with respect to the basis of band-limited cosine Weierstrass functions.

As is seen from Fig. 2, the new Kravchenko-Besicovitch function possesses interesting physical properties, namely, with the increase of *D* the energy is redistributed uniformly from the main-lobe of the RP to its side lobes, and the directivity varies from 10 dB to –21 dB.

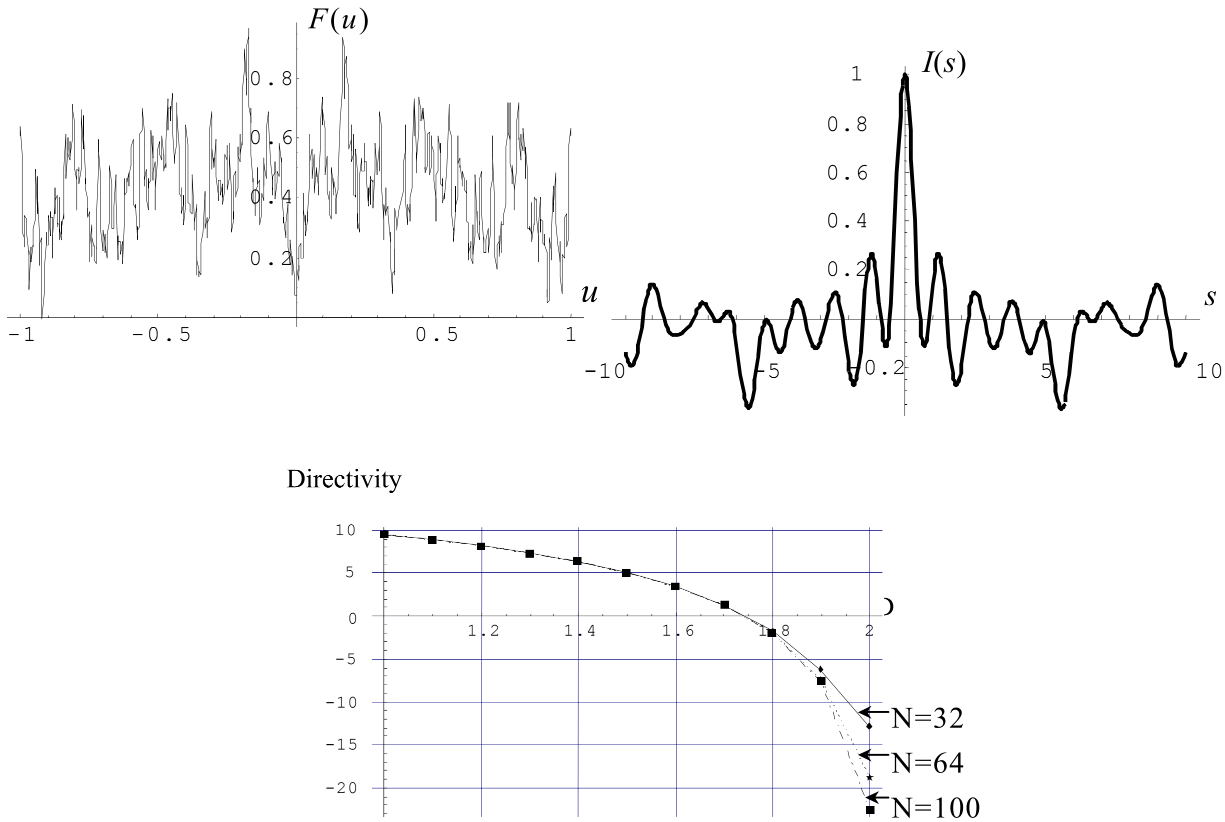

Fig. 2. The radiation pattern (a), current distribution (b), and dependency of directivity from fractal dimension D and scale factor $\eta > 1$ (c) for Kravchenko-Besicovitch generating function.

Conclusion

For the first time, in this work the new types of atomic-fractal functions (AFF) were constructed and justified. Results of investigations carried out show the effectiveness of the novel technique for solving problems of self-similar fractal radiator synthesis, and have a good coincidence with known data from works of D.N. and P.N. Werner [8]. The method proposed and justified in this work can be applied for synthesis of the wide class of both equally- and nonequally spaced (two- and three-dimensional) fractal antenna arrays.

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