RIGOROUS ANALYSIS OF THE FINLINE RESONATORS

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Abstract - **The direct and efficient Transverse Transmission Line (TTL) method is used to analyse the unilateral fin-line resonators. This method in conjunction with the Galerkin's procedure and Parseval's relation are applied to obtaining a homogeneous equation system with two variables. The determinant of this system gives the complex resonance frequency. The full wave TTL method gives precise and concise equations and the results bring good advantages. Computational programs are developed in Fortran Power Station and MATLAB 5.0, and the results for the unilateral fin line resonator are presented for different parameters including substrate thickness, length and width of the slot resonator, in 2 and 3D.**

I. INTRODUCTION

The resonator of unilateral finline consists of a closed rectangular waveguide (cavity) inside of which three regions dielectrics exists being the second region a substrate placed in the most central part and covered by a conductive sheet with a rectangular slot of width w and length *l*. A slot resonator in unilateral fin line is a circuit element that is extensively used as a building block in the design of fin-line filters, for example[1-2].

The cavity has length L of, at least, 15 times the length *l* to reduce the effects of its terminal walls on the fields in the slot resonator. In this resonator type lowest order resonance occurs when the electrical length of the resonator becomes equal one half wavelength of the fin line.

In the unilateral fin line rectangular slot resonator are considered the x and z electromagnetic field quantities in terms of fields in "y" direction in the Fourier transformed domain. This is basically the beginning of the TTL method, which express all the components of the electric and magnetic fields as function of the transverse components in the "y" direction [3-6].

In this TTL method in conjunction with the Galerkin's procedure and Parseval's relation, is obtained a homogeneous system of equations with two variables. The determinant of this system is the complex resonance frequency. The full wave method of the TTL given precise and concise equations and results, bring then good advantages.

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In the fig. 1 is the representation of the structure in study, showing all your dimensional parameters:

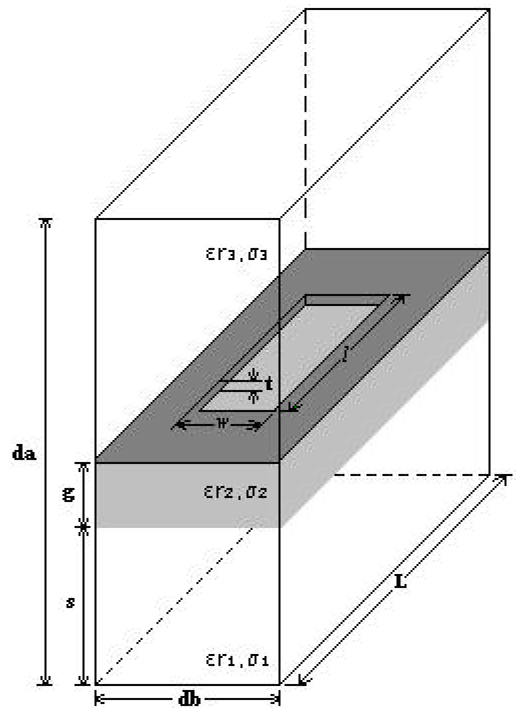

Fig. 1. Representation of the rectangular unilateral fin line resonator.

II. THEORY

The fin line resonator is limited in its length and the equations are obtained in the spectral domain in " x " and " z " directions as functions of the "y" direction fields. Therefore the field equations are applied for double Fourier transformed defined as[4-5]:

$$\tilde{f}(\alpha_n, y, \beta_k) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y,z) \cdot e^{j\alpha_n x} \cdot e^{j\beta_k z} dxdz \qquad (1)$$

Where α_n is the spectral variable in the " x " direction and β_k the spectral variable in the "z" direction.

Them, the electric and magnetic fields in the "x" and "z" directions are given as following:

$$\tilde{E}_{xi} = \frac{1}{\gamma_i^2 + k_i^2}\left[-j\alpha_n \frac{\partial}{\partial y}\tilde{E}_{yi} + \omega\mu\beta_k \tilde{H}_{yi} \right] \tag{2}$$

$$\tilde{E}_{zi} = \frac{1}{\gamma_i^2 + k_i^2}\left[-j\beta_k \frac{\partial}{\partial y}\tilde{E}_{yi} - \omega\mu\alpha_n \tilde{H}_{yi} \right] \tag{3}$$

$$\tilde{H}_{xi} = \frac{1}{\gamma_i^2 + k_i^2}\left[-j\alpha_n \frac{\partial}{\partial y}\tilde{H}_{yi} - \omega\varepsilon\beta_k \tilde{E}_{yi} \right] \tag{4}$$

$$\tilde{H}_{zi} = \frac{1}{\gamma_i^2 + k_i^2}\left[-j\beta_k \frac{\partial}{\partial y}\tilde{H}_{yi} + \omega\varepsilon\alpha_n \tilde{E}_{yi} \right] \tag{5}$$

Where:
i = 1, 2, 3, represents the three dielectric regions of the structure;
$\gamma_i^2 = \alpha_n^2 + \beta_k^2 - k_i^2$ is the propagation constant in the "y" direction;
α_n is the spectral variable in the "x" direction;
β_k is the spectral variable in the "z" direction;
$k_i^2 = \omega^2\mu\varepsilon = k_0^2\varepsilon_{ri}^*$ is the wave number of the i-th dielectric region;
$\varepsilon_{ri}^* = \varepsilon_{ri} - j\frac{\sigma_i}{\omega\varepsilon_0}$ is the relative electric permittivity of the material with losses;
$\omega = \omega_r + j\omega_i$ is the complex angular resonant frequency, and
$\varepsilon_i = \varepsilon_{ri}^* \cdot \varepsilon_0$, is the electric permittivity of the material.

A – The green admittance functions:

Using the relation among the current densities in the metal fins and the fields in the slot interface of the conductive ribbon, are obtained the following equations[5-7]:

$$Y_{xx}\tilde{E}_{xt} + Y_{xz}\tilde{E}_{zt} = \tilde{J}_{xt} \tag{6}$$

$$Y_{zx}\tilde{E}_{xt} + Y_{zz}\tilde{E}_{zt} = \tilde{J}_{zt} \tag{7}$$

Where the equations above can be represented in matrix form:

$$\begin{bmatrix} Y_{xx} & Y_{xz} \\ Y_{zx} & Y_{zz} \end{bmatrix}\begin{bmatrix} \tilde{E}_{xt} \\ \tilde{E}_{zt} \end{bmatrix} = \begin{bmatrix} \tilde{J}_{xt} \\ \tilde{J}_{zt} \end{bmatrix} \tag{8}$$

A.1 – Base functions:

The electric field distributions $\mathbf{E_x}$ (α_n, β) and $\mathbf{E_z}$ (α_n, β) in the slot can be expanded in terms of known base functions $\hat{\mathbf{e}}_{xm}$ (α_n, β) and $\hat{\mathbf{e}}_{zm}$ (α_n, β), respectively.

The component of the field in the z direction in a structure of unilateral finline is despicable being able to, therefore, to be ignored without damage of the results.

Using a component of the electric field ($\mathbf{E}_{xt}$), was chosen the base functions that is expressed in the space domain for[1]:

$$f_x(x,z) = f_x(x).f_x(z) \tag{9}$$

$$f_x(x) = \frac{1}{\sqrt{\left(\frac{w}{2}\right)^2 - x^2}} \tag{10}$$

$$f_x(z) = \cos\left(\frac{\pi z}{l}\right) \tag{11}$$

that in the spectral domain are obtained as:

$$\tilde{f}_x(\alpha_n, \beta_k) = \frac{2\pi^2 l \cdot \cos\left(\frac{\beta_k l}{2}\right)}{\pi^2 - (\beta_k l)^2} \cdot J_o\left(\alpha_n \frac{w}{2}\right) \tag{12}$$

where J_0 is the Bessel function of first specie and zero order.

Applying the Gallerkin's method, a particular case of the moments method to Eq. (8), and Parseval's relation, the current densities on the conducting fins are eliminated, because the Fourier transformed of the current densities on the conducting fins are related to the Fourier transform of the electric fields E_x and E_z in the slot region.

Then is obtained an homogeneous matrix system with two variables [8],

$$\begin{bmatrix} K_{xx} & K_{xz} \\ K_{zx} & K_{zz} \end{bmatrix} \cdot \begin{bmatrix} a_x \\ a_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{13}$$

The previous procedure are described for the most general cases with the use of the two components of the electric fields. The basic matrix equation is exact.

A numerical solution is obtained by introducing a known set of basis functions. Therefore, the accuracy of the solution depends on the accuracy with which the basis functions represent the electric field distribution in the resonant slot.

The Eq. (13) has a non-trivial complex solution, which determinant of the characteristic equation is made equal to zero. The characteristic equation is general and can be applied to slot resonators of arbitrary shape[8].

The solution of the characteristic equation is the complex resonance frequency.

III. RESULTS

Programs in Fortran Powerstation language are used for the numerical computation that uses this full wave TTL method. Graphics were obtained using Matlab.

The fig. 2, we have the variation of the resonance frequency when the dielectric substrate of the second region varies among ε_r=2.22 and ε_r =12.0. It is noticed that the measure that there an increase of the normalized width, this frequency tends to increase for this structures. The resonator has, da=7.112 mm, db=3.556 mm (WR-28), s=2.8 mm and $\boldsymbol{l}$=3.6 mm.

The figs. 3 and 4, shows respectively, a three-dimensional representation among the real and imaginary resonance frequency, of the normalized width of the waveguide and of the thickness of the dielectric substrate. It is observed that the real frequencies range is between 20.0 and 30.0 GHz. It is noticed although the resonance frequency presents a quite accentuated variation when it is had variations in the thickness of the dielectric substrate. In the same way, there is a considerable variation when the normalized width of the waveguide is increased. The resonator has, da=7.112 mm, db=3.556 mm, ε_{r2} =12.0, s=2.8 mm and $\boldsymbol{l}$=3.6 mm.

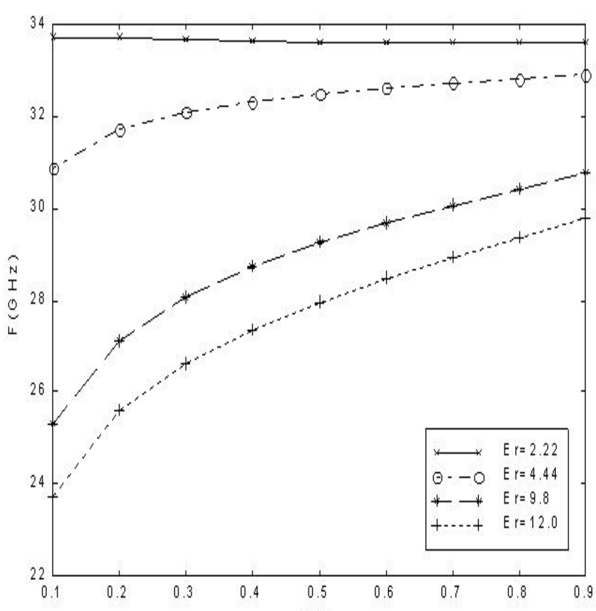

Figure 2. 2D results of the real resonance frequency as function of the normalized width of the waveguide for different dielectric substrates.

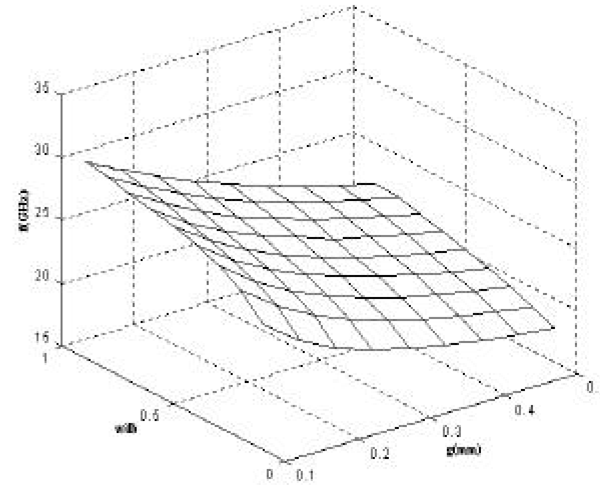

Figure 3. (a) 3D results of the real resonance frequency as function of the thickness of the dielectric substrate and of the normalized width of the waveguide.

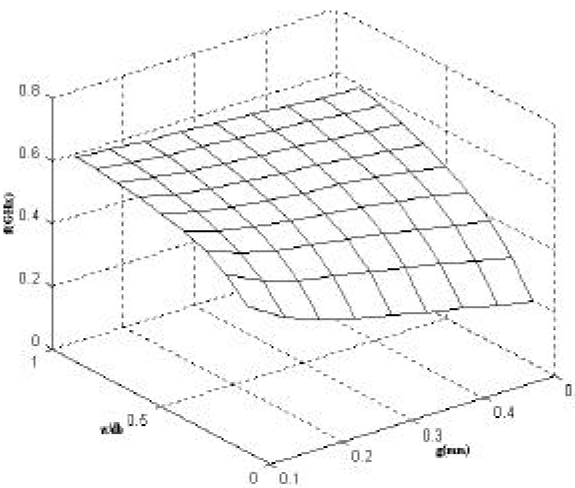

Figure 4. 3D results of the imaginary resonance frequency as function of the thickness of the dielectric substrate and of the normalized width of the waveguide.

IV. CONCLUSIONS

The unilateral fin line rectangular resonator was analyzed using the concise full wave TTL Method. Results for the complex resonance frequency confirm the exactness of the TTL method were presented. Graphics that show the variation of the resonance frequency with relation to the normalized width and thickness of dielectric substrate of the rectangular slot in the fin line resonator were presented in 2-D and 3-D.

In conclusion, The Transverse Transmission Line - TTL is an efficient and accurate method used to analysis and design of the rectangular resonator in fin line. This work received financial support from CNPQ.

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