Double Layer Patch Resonator

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***Abstract* — The theory and numeric results of the Double Layer Patch Resonator (DLPR) about resonant frequency, efficiency and bandwidth are presented. The full wave Transversal Transmission Line (TTL) method has been used for analysis. Starting from the Maxwell equations, a system that represents the electromagnetic fields are obtained, according with this concise and efficient procedures, to calculate the complex resonant frequency. This resonant frequency is calculated through double spectral variables. The new results are good and are in accordance with that results presented in the specialized literature.**

I. INTRODUCTION

The double layer patch resonator consists of one rectangular patch, with length *l* and width w, and three dielectrics regions, as shown in the Fig. 1. The analysis in the structure has been analyzed with the concise full wave transverse transmission line (TTL) method. This method is obtained of the electromagnetic fields general equations. In this work was obtained a form to calculate the resonant frequency of the double layer patch resonator.

This method was presented last year for single layer patch resonator [1-4]. The results were obtained in agreement with results researched in the literature. Computational simulations have been accomplished to analyze the behavior of resonators of double layer patch resonator. In the first part of this work, is used substrates with the same constant dielectric, and the results obtained are identical those with the single layer patch resonator. In the second part, the dielectric constant is changed and are presented new values of efficiency and bandwidth for variations of the resonant frequency.

An advantage in the use of double layers, for example, is to obtain one better flexibility and control of the calculated parameters, being a great option to be utilized in applications to smart and adaptative antenna array and others structures with different resonant patch [5]-[7].

II. FIELDS IN THE STRUCTURE

Due the double layer patch resonator to have length *l* and width w, the equations used for the analysis in the spectral domain, are the directions " x " and " z ". Therefore it is applied to the field equations the double Fourier transformed domain defined as:

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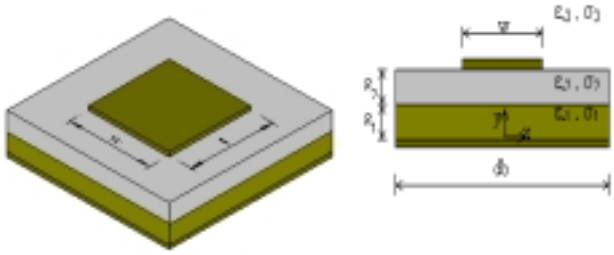
$$\tilde{f}(\alpha_n, y, \beta_k) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y,z)\cdot e^{j\alpha_n x}\cdot e^{j\beta_k z}\,dxdz \quad (1)$$



(a) (b)

Fig. 1 – (a) Spatial view of the double layer patch resonator, and (b) It transverse section.

After using the equations of Maxwell, the general equations of the fields in the TTL method, are obtained as:

$$\tilde{E}_{xi} = \frac{1}{\gamma_i^2 + k_i^2}\left[-j\alpha_n \frac{\partial}{\partial y}\tilde{E}_{yi} + \omega\mu\beta_k \tilde{H}_{yi}\right] \quad (2.1)$$

$$\tilde{E}_{zi} = \frac{1}{\gamma_i^2 + k_i^2}\left[-j\beta_k \frac{\partial}{\partial y}\tilde{E}_{yi} - \omega\mu\alpha_n \tilde{H}_{yi}\right] \quad (2.2)$$

$$\tilde{H}_{xi} = \frac{1}{\gamma_i^2 + k_i^2}\left[-j\alpha_n \frac{\partial}{\partial y}\tilde{H}_{yi} - \omega\varepsilon\beta_k \tilde{E}_{yi}\right] \quad (2.3)$$

$$\tilde{H}_{zi} = \frac{1}{\gamma_i^2 + k_i^2}\left[-j\beta_k \frac{\partial}{\partial y}\tilde{H}_{yi} + \omega\varepsilon\alpha_n \tilde{E}_{yi}\right] \quad (2.4)$$

where,

i = 1, 2, 3, represent the three dielectrics regions of the structure;

$$\gamma_i^2 = \alpha_n^2 + \beta_k^2 - k_i^2 \quad (2.5)$$

is the propagation constant in y direction; α_n is the spectral variable in "x" direction and β_k is the spectral variable in "z" direction.

$k_i^2 = \omega^2\mu\varepsilon = k_0^2\varepsilon_{ri}^*$ is the wave number of the i[th] dielectric region;

$\varepsilon_{ri}^* = \varepsilon_{ri} - j\dfrac{\sigma_i}{\omega\varepsilon_0}$ is the relative dielectric constant of the material with losses; $\omega = \omega_r + j\omega_i$ is the complex angular frequency;

$\varepsilon_i = \varepsilon_{ri}^* \cdot \varepsilon_0$ is the dielectric constant of the material;
ε_0 is the free space dielectric constant;
σ_i is the conductivity and μ is the permeability.

The equations above are applied to the resonator being calculated, the fields E_y and H_y through the solution of the wave equations of Helmoltz in the spectral domain [2]-[3]:

$$\left(\frac{\partial^2}{\partial y^2} - \gamma^2\right)\tilde{\tilde{E}}_y = 0 \quad (3.1)$$

$$\left(\frac{\partial^2}{\partial y^2} - \gamma^2\right)\tilde{\tilde{H}}_y = 0 \quad (3.2)$$

The solutions of those equations for the three regions of the structure are given for:

region 1:

$$\tilde{\tilde{E}}_{y1} = A_{1e} \cdot \cosh \gamma_1 y \quad (4.1)$$

$$\tilde{\tilde{H}}_{y1} = A_{1h} \cdot sinh\gamma_1 y \quad (4.2)$$

region 2:

$$\tilde{\tilde{E}}_{y2} = A_{2e} \cdot \operatorname{senh} \gamma_2 y + B_{2e} \cdot \cosh \gamma_2 y \quad (4.3)$$

$$\tilde{\tilde{H}}_{y2} = A_{2h} \cdot \operatorname{senh} \gamma_2 y + B_{2h} \cdot \cosh \gamma_2 y \quad (4.4)$$

region 3:

$$\tilde{\tilde{E}}_{y3} = A_{3e} \cdot e^{-\gamma_3 y} \quad (4.5)$$

$$\tilde{\tilde{H}}_{y3} = A_{3h} \cdot e^{-\gamma_3 y} \quad (4.6)$$

Substituting these solutions in the equations of the fields (2.1) to (2.4), as functions of the unknown constants A_{ie}, A_{ih}, B_{2e} and B_{2h} is obtained, for example for the region 2:

$$\tilde{\tilde{E}}_{x2} = \frac{-j}{\gamma_2^2 + k_2^2}[(\alpha_n \gamma_2 A_{2e} + j\beta_k \omega\mu B_{2h})\cosh\gamma_2 y + (\alpha_n \gamma_2 B_{2e} + j\beta_k \omega\mu A_{2h})\operatorname{senh}\gamma_2 y] \quad (4.7)$$

$$\tilde{\tilde{H}}_{x2} = \frac{j}{\gamma_2^2 + k_2^2}[(-\alpha_n \gamma_2 A_{2h} + j\beta_k \omega\varepsilon_2 B_{2e})\cosh\gamma_2 y + (-\alpha_n \gamma_2 B_{2h} + j\beta_k \omega\varepsilon_2 A_{2e})\operatorname{senh}\gamma_2 y] \quad (4.8)$$

For the determination of the unknown constants, are applied the boundary conditions for the regions 1, 2 and 3:

For the regions 1 and 2: y = g1

$$\tilde{\tilde{E}}_{x1} = \tilde{\tilde{E}}_{x2} \quad (5.1)$$

$$\tilde{\tilde{E}}_{z1} = \tilde{\tilde{E}}_{z2} \quad (5.2)$$

$$\tilde{\tilde{H}}_{x1} = \tilde{\tilde{H}}_{x2} \quad (5.3)$$

$$\tilde{\tilde{H}}_{z1} = \tilde{\tilde{H}}_{z2} \quad (5.4)$$

For the regions 2 and 3: y = g ; (g=g1+g2)

$$\tilde{\tilde{E}}_{x2} = \tilde{\tilde{E}}_{x3} = \tilde{\tilde{E}}_{xg} \quad (5.5)$$

$$\tilde{\tilde{E}}_{z2} = \tilde{\tilde{E}}_{z3} = \tilde{\tilde{E}}_{zg} \quad (5.6)$$

After several calculations the unknown constants "A" are obtained, for example:

$$A_{2e} = \frac{j(\alpha_n \tilde{\tilde{E}}_{xg} + \beta_k \tilde{\tilde{E}}_{zg})\operatorname{senh}\gamma_2 g_1 - (\alpha_n \tilde{\tilde{E}}_{xg} + \beta_k \tilde{\tilde{E}}_{zg})\operatorname{senh}\gamma_2 g_1}{\gamma_2 \operatorname{senh}\gamma_2 g_2} \quad (6.1)$$

$$A_{2h} = \frac{(\beta_k \tilde{\tilde{E}}_{xg} - \alpha_n \tilde{\tilde{E}}_{zg})\cosh\gamma_2 g - (\beta_k \tilde{\tilde{E}}_{xg} - \alpha_n \tilde{\tilde{E}}_{zg})\cosh\gamma_2 g_1}{\omega\mu \operatorname{senh}\gamma_2 g_2} \quad (6.2)$$

Then the general equations of the electromagnetic fields in the resonator structure as function of $\tilde{\tilde{E}}_{xg}$ and $\tilde{\tilde{E}}_{zg}$ are obtained.

III. ADMITANCE MATRIX

Applying the boundary conditions the following equations that relate the current densities ($\tilde{J}_{xg}$ and $\tilde{J}_{zg}$) and the magnetic fields in the interface y = g, are obtained:

$$\tilde{\tilde{H}}_{x2} - \tilde{\tilde{H}}_{x3} = \tilde{\tilde{J}}_{zg} \quad (7.1)$$

$$\tilde{\tilde{H}}_{z2} - \tilde{\tilde{H}}_{z3} = -\tilde{\tilde{J}}_{xg} \quad (7.2)$$

Being made the substitutions of the equations of the magnetic fields, given as function of ($\tilde{\tilde{E}}_{xg}$ and $\tilde{\tilde{E}}_{zg}$), and after some calculations it is obtained,

$$Y_{xx}\tilde{\tilde{E}}_{xg} + Y_{xz}\tilde{\tilde{E}}_{zg} = \tilde{\tilde{J}}_{zg} \quad (8.1)$$

$$Y_{zx}\tilde{\tilde{E}}_{xg} + Y_{zz}\tilde{\tilde{E}}_{zg} = \tilde{\tilde{J}}_{xg} \quad (8.2)$$

at the matrix form yield:

$$\begin{bmatrix} Y_{xx} & Y_{xz} \\ Y_{zx} & Y_{zz} \end{bmatrix}\begin{bmatrix} \tilde{\tilde{E}}_{xg} \\ \tilde{\tilde{E}}_{zg} \end{bmatrix} = \begin{bmatrix} \tilde{\tilde{J}}_{zg} \\ \tilde{\tilde{J}}_{xg} \end{bmatrix} \quad (9)$$

the " Y " admittances functions are the dyadic Green functions of the resonator and they are given, for example, for Y_{xx} e Y_{zz}:

$$Y_{xx} = \frac{j}{\omega\mu(\gamma_2^2 + k_2^2)}[(\frac{-\beta_k^2 \gamma_2 d_1}{B} + \frac{k_2^2 \alpha_n^2 e_1}{A})\cosh\gamma_2 g + (\frac{-\beta_k^2 \gamma_2 f_1}{B} + \frac{k_2^2 \alpha_n c_1}{A})\operatorname{senh}\gamma_2 g] - \frac{j(k_3^2 - \beta_k^2)}{\omega\mu\gamma_3} \quad (9.1)$$

$$Y_{zz} = \frac{j}{\omega\mu(\gamma_2^2 + k_2^2)}[(\frac{\alpha_n^2\gamma_2 d_1}{B} - \frac{\beta_k^2 k_2^2 e_1}{A})\cosh\gamma_2 g + \\ + (\frac{\gamma_2 f_1}{B} - \frac{\beta_k^2 k_2^2 c_1}{A})\operatorname{senh}\gamma_2 g] - \frac{j(k_3^2 + \alpha_n^2)}{\omega\mu\gamma_3} \quad (9.2)$$

where A and B are:

$$A = \gamma_1 \operatorname{senh}\gamma_1 g_1 \cdot \cosh\gamma_2 g_2 + \gamma_2 \frac{\varepsilon_1}{\varepsilon_2}\cosh\gamma_1 g_1 \cdot \operatorname{senh}\gamma_2 g_2 \quad (10.1)$$

$$B = \operatorname{senh}\gamma_1 g_1 \cdot \cosh\gamma_2 g_2 + \frac{\gamma_1}{\gamma_2}\cosh\gamma_1 g_1 \cdot \operatorname{senh}\gamma_2 g_2 \quad (10.2)$$

$$c_1 = \frac{\gamma_1}{\gamma_2}\operatorname{senh}\gamma_1 g_1 \quad (10.3)$$

$$d_1 = \frac{\gamma_1}{\gamma_2}\cosh\gamma_1 g_1 \quad (10.4)$$

$$e_1 = \frac{\varepsilon_1}{\varepsilon_2}\cosh\gamma_1 g_1 \quad (10.5)$$

$$f_1 = \operatorname{senh}\gamma_1 g_1 \quad (10.6)$$

The current densities are expanded in terms of known base functions as [3], [8]:

$$\tilde{J}_{xg} = \sum_{i=1}^{n} a_{xi} \cdot \tilde{f}_{xi}(\alpha_n, \beta_k) \quad (11.1)$$

$$\tilde{J}_{zg} = \sum_{j=1}^{m} a_{zj} \cdot \tilde{f}_{zj}(\alpha_n, \beta_k) \quad (11.2)$$

where a_{xi} and a_{zj} are constant unknown and the terms n and m are numbers integer and positive.

The base functions were chosen in the space domain, as example:

$$f_x(x,z) = f_x(x).f_x(z) \quad (12.1)$$

$$f_x(x) = \frac{1}{\sqrt{\left(\frac{w}{2}\right)^2 - x^2}} \quad (12.2)$$

$$f_x(z) = \cos\left(\frac{\pi z}{l}\right) \quad (12.3)$$

whose Fourier transformed are:

$$\tilde{f}_x(\alpha_n) = \pi \cdot J_o\left(\alpha_n \frac{w}{2}\right) \quad (13.1)$$

$$\tilde{f}_x(\beta_k) = \frac{2\pi l \cdot \cos\left(\frac{\beta_k l}{2}\right)}{\pi^2 - (\beta_k l)^2} \quad (13.2)$$

$$\tilde{f}_x(\alpha_n, \beta_k) = \frac{2\pi^2 l \cdot \cos\left(\frac{\beta_k l}{2}\right)}{\pi^2 - (\beta_k l)^2} \cdot J_o\left(\alpha_n \frac{w}{2}\right) \quad (13.3)$$

where J_0 is the Bessel function of first specie and zero order.

The Gallerkin method is applied to (9), to eliminate the electromagnetic fields. The new equation in matrix form is obtained,

$$\begin{bmatrix} K_{xx} & K_{xz} \\ K_{zx} & K_{zz} \end{bmatrix} \cdot \begin{bmatrix} a_x \\ a_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (14)$$

where,

$$K_{xx} = \sum_{-\infty}^{\infty} \tilde{f}_x \cdot Z_{xx} \cdot \tilde{f}_x^* \quad (14.1)$$

$$K_{xz} = \sum_{-\infty}^{\infty} \tilde{f}_z \cdot Z_{xz} \cdot \tilde{f}_x^* \quad (14.2)$$

$$K_{zx} = \sum_{-\infty}^{\infty} \tilde{f}_x \cdot Z_{zx} \cdot \tilde{f}_z^* \quad (14.3)$$

$$K_{zz} = \sum_{-\infty}^{\infty} \tilde{f}_z \cdot Z_{zz} \cdot \tilde{f}_z^* \quad (14.4)$$

The solution of the characteristic equation of the determinant of (14) supplies the resonant frequency [9]-[11].

IV. RESULTS

Computational programs were elaborated in the FORTRAN POWER STATION and in the MATLAB languages, to obtain the numeric results in the figures presented.

In the figs. 2 and 3, the results were obtained using substrates with dielectric constant values equal to 2.22 and 9.8.

The graphic of the fig. 2 represents the efficiency at function of the resonant frequency for values of w=10mm and w=15mm. As shown, the value of efficiency increases with the increase of resonant frequency.

The graphic of the fig. 3 represents the bandwidth as function of the resonant frequency of the patch, for values of w=10mm and w=15mm. In this figure, we can observe a decrease of the bandwidth with the increase of the frequency.

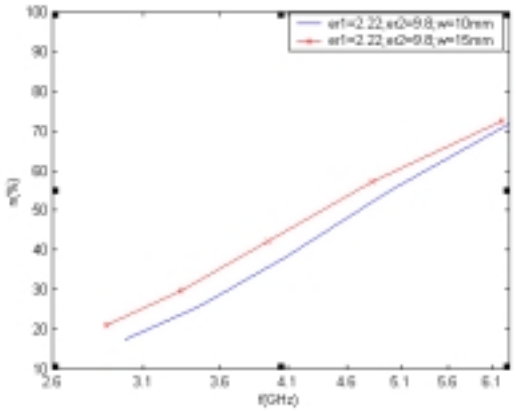


Fig. 2 – Curves of efficiency as function of the resonant frequency.

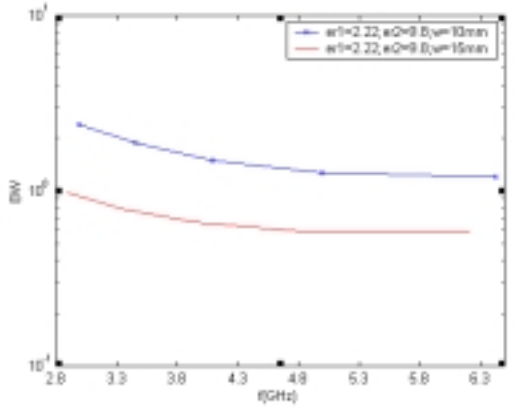


Fig.3 – Curves of bandwidth as function of the resonant frequency

The fig. 4 represents the graphic of the resonant frequency at function of the length of the patch for dielectric constant values of 2.22 and 9.8, to w=10mm and w=15mm. This graphic show that the resonant frequency decreases with the increase of the length *l*.

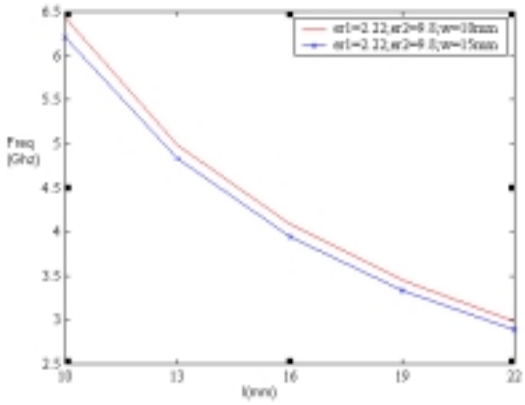


Fig.4 – Curves of resonant frequency as function of the length.

V. CONCLUSION

The full wave method transverse transmission line (TTL) was used in the analysis for obtaining the numeric results resonant frequency, bandwidth and efficiency of the double layer patch resonator. The TTL method is an efficient and accurate method applied to the analysis, for example, for design and applications in antenna array. It is very versatile to work with lossless or semiconductor substrate. Programs computational were elaborated in the FORTRAN POWER STATION and in MATLAB languages, for obtaining of the numeric results. This work received financial support by CNPq.

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