Waveguide Components Design in Presence of Complementary Modes

G. Fontgalland
Centro Federal de Educação Tecnológica, CEFET/MA-LMCD, São Luis, MA, Brazil

T.P. Vuong
ESISAR, INPG, Valence, France

H. Baudrand
Laboratoire d'Electronique, ENSEEIHT/LEN7-GRE, Toulouse, France

***Abstract* - This paper presents a procedure to design passive waveguide components taking into account the presence of complementary modes in the formulation. The algorithm, based on the Boundary Element Method (BEM), solves the wave equation for each discontinuity plane using two different formulations (TE and TM). The complete waveguide structure is obtained by cascading all discontinuities. The scattering parameters for a rectangular waveguide transformer are presented and it is shown to be computationally affordable.**

***Index Terms* – Complementary modes, Ridge waveguide, BEM, Cutoff wavenumber, Integral Equation Method.**

I. INTRODUCTION

Ridge waveguide plays an important role in telecommunication applications. They are elements used in many devices as filters, transformers and polarizers. When complex components are required the numerical effort involved by electromagnetic models may easily become unaffordable. Moreover, these components are part of complex waveguides sub-systems that requires an accurate software tool, to perform, in short time, the analysis of the structures involved. In theses algorithms the complete set of the modes, which is the main part of the design, represents a significant computational effort. It takes over 80% [1-3] of the total CPU time.

The commonly approaches of electromagnetic (EM) problems analysis can be divided into three groups: the volume method, mode matching method and integral method (MoM). The modes solutions in these methods are obtained from the roots of the characteristic equation of the problem. These roots are the wavenumbers (eigenvalues). Once the boundary conditions have been specified, we can find accountable numbers of roots. Elsewhere, not all of them result in a physical mode solution [3].

G. Fontgalland is with the LMCD, Departamento de Eletroeletônica, CEFET-MA, São Luis, MA, Brazil, Tel +55 98 218-9040, Fax +55 98 218-9019, fontgalland@dee.cefet-ma.br, T.P. Vuong is with the ESISAR, INPG, 50, rue de Barthélémy de Laffemas, 26902, Valence, France, Tel +33 5 61628248, tan-phu_vuong@esisar.inpg.fr; H. Baudrand is with the ENSEEIHT/LEN7-GRE, 2 rue Charles Camichel, 31071, baudranh@len7.enseeiht.fr, Tel +33 5 61628272, Fax +33 5 61628277.

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The set of numerical solutions is composed by physical, nonphysical (spurious) and complementary solutions. This last one is intrinsic to the numerical solution of boundary methods, like as BEM. In this case, the surface domain of the problem is replaced by a contour one. Then, all information about the real direction of the outward normal vector $\vec{n}$ to the surface is lost (Fig. 1). By the way, others methods currently present cutoff wavenumbers (solutions) in the vicinity of the complementary solutions. It is pointed out that even the solutions of the hollow waveguide can be very closed to the modes of the ridge one [4].

In this paper, we use a particular case of the method of moments (MoM), the Galerkin method, to solve the homogenous system in matrix form **AX=0**. This matrix obtained from the system of partial differential equations is a non-linear eigenvalues problem. The solutions are determined using a monotonous functions approach [5]. The modes orthogonality and the inner product are automatically verified.

In order to illustrate the efficiency of the algorithm in the selection of the numerical solutions we design a metallic rectangular waveguide transformer (Fig. 2). For the design of these structures, some important aspects need to be taken into account, as the influence of the finite thickness, higher order mode coupling effects and the optimisation possibilities in stepped design (height and width). To study these components it is important to determine with accuracy; firstly, the different types of transitions used in the complete structure (Fig. 2); and secondly, the interaction between subsequent discontinuities by considering it in cascade. The contour integral formulation is used to calculate the coupling coefficients and to determine the transition parameters. The functions are cascaded by taking into account the numbers of coupled modes, in conjunction with an impedance matrix association technique [6]. The scattering parameters are calculated and they are in good agreement with the published data.

Of course, the analysis presented here is not limited to the shape of waveguide neither the complexity along the propagation axis *z*. The transformer is homogeneous and uniform.

II. FORMULATION

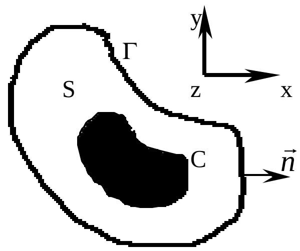

Fig.1 Cross-section of an arbitrary homogenous waveguide with perfect metallic wall.

The modeling method is divided in several successive steps. Its begin with the resolution of Helmholtz's equation, through a Green's function satisfying the boundary conditions, and finish in a matrix form by projection of the operators deduced from the segmentation method with Galerkin's procedure.

By using a 2D method (BEM) the domain of study can be reduced to the cross section presented in Fig.1. It represents the cross section of an arbitrary homogenous waveguide, uniform along the propagation axis *z*. In this waveguide, the electric and magnetic fields satisfy Helmholtz's equation (1).

$$(\nabla_T^2 + k_c^2)y(\vec{r}) = 0 \qquad (1)$$

where $y(\vec{r})$ represents the TE or TM modes.

To model the electromagnetic fields in a waveguide we use the scalar Green's function that satisfies (2).

$$(\nabla_T^2 + k_c^2)G(\vec{r},\vec{r}\,') = -d(\vec{r},\vec{r}\,') \qquad (2)$$

The TE and TM modes are trated independently with its appropriate boundary conditions. The matrix size is reduced with the help of a judicious choice of the Green function. In this way, for a rectangular contour Γ (Fig. 2) the basis functions $j(\vec{r})$ are developed in a series of sines and cossines trigonometric functions.

A. TM modes

Using the second Green's identity we can relate the fields and the scalar Green function.

$$y(\vec{r}) = \oint_\Gamma G(\vec{r},\vec{r}\,')\partial_n y(\vec{r}\,')dr' - \oint_\Gamma \partial_n G(\vec{r},\vec{r}\,')y(\vec{r}\,')dr' \qquad (3)$$

The Dirichlet conditions for TM modes $y(\vec{r}) = E_z = 0$, leads the follow final system on the contour Γ

$$\hat{G}\partial_n E_z(\vec{r}) = 0 \qquad (4)$$

Where $\hat{G}$ is a monotonous integral operator as defined in [5]. The unknowns $\partial_n E_z$ are developed in weighting functions.

B. TE modes

In this case we use the vector Green's function. The transverse electric field, solution of the wave equation, can be written as a complete set of basis functions with free curl, f^H, and free divergence, f^E.

$$E_T(x,y) = \nabla f_T^E + \nabla f_T^H \times \vec{n} \qquad (5)$$

where T denotes transverse coordinates and $\vec{n}$ is the normal unit vector.

The appropriate boundary conditions to theses basis leads the follow final system of equations to be solve

$$\hat{G}J_T(\vec{r}) = 0 \qquad (6)$$

where $\hat{G}$ is the monotonous integral operator as defined in [3] and the unknowns J_T are developed in weighting functions.

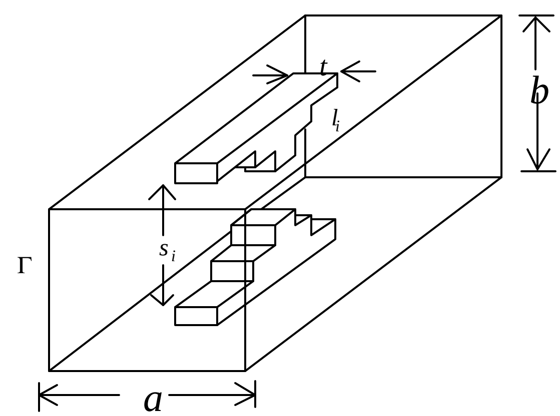

Fig.2: A metallic rectangular waveguide transformer.

C. Impedance matrix

The final systems of homogeneous equations ((4) and (6)) are represented in the matrix form using the Galerkin procedure and BEM, over the contour *C*. It is written as

$$[A][X] = 0 \quad (7)$$

A indicates all matrix elements of the system and the unknowns are placed in Nx1 column matrix, [X]. [A] is a NxN square matrix and N is the number of segments over discretized contour *C*.

The zeros of the matrix determinant are solved by a systematic procedure [3]. Once the coupling coefficients have been obtained, using the integral contour technique, the matrix impedance of each the discontinuity can then be evaluated by the Multimodal Variational method [7]. The **S**-parameters for the complete structure are deduced by cascading the **Z**-matrix of each discontinuity plane.

III. COMPLEMENTARY MODES

In the boundary elements method (BEM), the surface domain of the problem is replaced by a contour one. Then, all information about the real direction of the outward normal vector $\vec{n}$, when the fields unknowns are numerical represented on the contour C, are lost.

The set of cutoff wavenumbers calculated are then put in the modes expressions and they should satisfy the orthogonality properties.

$$\left\langle j_p(k_c)/j_q(k_c)\right\rangle = d_{pq} \quad (8)$$

where d_{pq} represents the Kronecker delta and j_p is the *p-th* mode (TE or TM).

We numerically identified the values of (8) that are greater than 1.001 as nonphysical modes and those that are less than 0.94 as complementary one (inside the hollow ridge). If the scalar products are between these two values, we state that the functions represent modes of ridge waveguide. This rule is a purely empirical one. It was verified in some cases of ridged rectangular and circular waveguides.

IV. NUMERICAL RESULTS

In Fig. (3), we can see the cross section of the transformer mounted in a WR-140. This is the geometry of each discontinuity with different values of S_i.

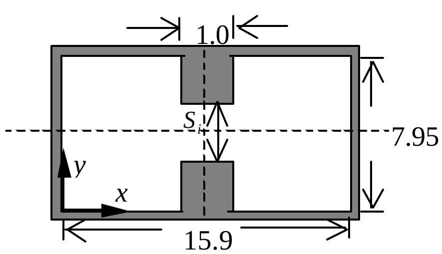

Fig.3: Cross-section of the rectangular waveguide transformer.

The dimensions presented in Fig. 3 are given in millimeters, as we can see in Table I. Where l_i is the length of *i-th* ridge and WG*i* is *i-th* ridge waveguide.

TABLE I

DIMENSIONS OF THE DOUBLE RIDGE WAVEGUIDE
(S_i *and* l_i *in millimeters*)

	G	G	G	G	G	G
S_i	6.67	5.32	4.29	3.34	2.44	2.00
l_i	5.98	6.00	7.37	6.90	5.62	6.02

The transformer has symmetry along the propagation axis, i.e., the first six ridges (WG*i*) are repeated to form the complete structure. The results for the cutoff wavenumbers that satisfy equation (8) for four TE and TM modes are shown in Table II.

TABLE II

FOUR CUTOFF WAVENUMBERS OF THE TRANSFORMER
(K_c *in rad/m*)

		K_c			
G	TE	192,61	578,40	812,16	952,15
	TM	844,08	1017,69	1283,58	1630,27
G	TE	182,32	549,91	807,46	904,06
	TM	874,82	1080,03	1332,72	1646,54
G	TE	171,50	525,24	804,99	882,63
	TM	887,50	1124,61	1408,81	1673,43
G	TE	159,00	503,25	804,16	871,31
	TM	893,07	1146,13	1468,15	1751,67
G	TE	144,32	483,77	803,65	864,24
	TM	895,26	1154,00	1487,79	1785,38
G	TE	135,80	474,75	803,21	861,59
	TM	895,73	1155,55	1491,01	1789,37

Once the modes orthogonality assured, we can obtain the matrix impedance of each discontinuity and so cascade all of them to compute the scattering parameter. It should be pointed out here that the cutoff wavenumbers presents in Table II are not the first ones detected by algorithm. They are the first ones that satisfy equation (8).

Fig. 4 presents the results for the module of the reflection coefficient ($|S_{11}|$ in *dB*) of the transformer. The frequencies are given in GHz. The results are very closed to the reference [10] even thought we can see a little displacement in the second resonance frequency. It can be attributed to the differences in the input data. In this case this difference is not so important, in fact, this power is -60 *dB*. Validated our results to experimental and theoretical one published we checked it with a commercial software based on finite elements method (FEM), HFSS. In a PC with 400 MHz and 64Mo, 35000 tetrahedral are used to discretize the entire structure.

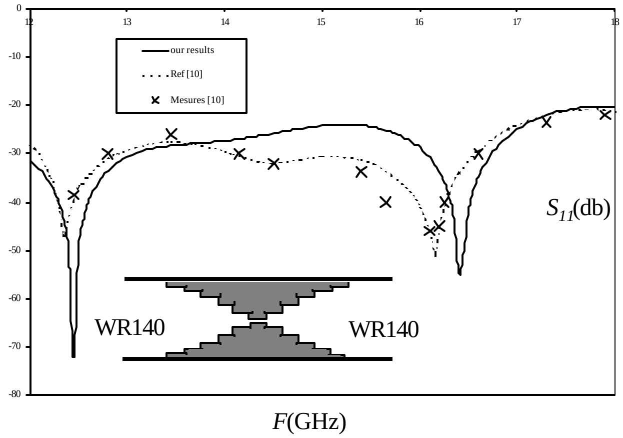

Fig.4: The module of the reflection parameter of the transformer as a function of the frequency (GHz).

The PC characteristics put the HFSS software under constraints limitations. The results obtained after 264 hours in CPU time were not good, even though the slope curve give us a single idea of the reflection coefficient behavior. By the way, our algorithm took 3,5 hours only to arrive in the above results (Fig. 4). A 65,5% of this time was reserved to search the cutoff wavenumbers of all ridge waveguide. In each step of this procedure the symmetry of the structure was considered.

V. CONCLUSIONS

In this paper we shown that the criterion use to check physical modes in ridge waveguide can be used without increase the CPU time. By the way, the algorithms based on the BEM should to apply any kid of selection technique to build a complete set of modes. The complementary modes are more pronounced when the dimensions of the ridge are in same order of the guide house. The results are accurate and the curves for the module of the reflection coefficient of a transformer with twelve discontinuities planes have been shown. In the analysis of the table II we can see the well behaved of the cutoff wavenumbers and it can be taken as intuitive information.

VI. ACKNOWLEDGMENTS

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