

# Cluster finding algorithm using linear array antenna

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*Abstract - Fast advance of Array Antenna research ensures its role as a key technology for future cellular systems [1,2]. However, most Smart Antenna techniques are restricted by the assumption that the number of signals is smaller than the array, although cost limits them to small sizes and researches indicate that usual path-distributions do not satisfy such assumption [3]. Alternatively, a good model is that a user's multipaths arrive at the base Gaussianly and, according to the Central Limit Theorem, so is the distribution of their crowd. Clearly, a small DOA deviation results when the base station is near a publicly relevant facility where users tend to concentrate while a large one results when users are spread. In order to joint the benefits of code (CDMA) and spatial (SDMA) orthogonality among users, an algorithm that estimate the path-distribution using a small array is urged and is the contribution of this paper.*

*Keywords: Array Antenna, Smart Antenna, DOA Estimation, CDMA and SDMA.*

## I. INTRODUCTION

Different from current commercial Code Division Multiple Access (CDMA) systems, the standards for future CDMA [4,5] systems foresee that users have different Spreading Factors (SF). However, the optimum SFs assigned to a user in order to achieve a desired BER is still an open problem [6]. Since spreading is essentially a trade-off between bandwidth (throughput) for SINR the need for spreading gain is directly associated to the amount of Multiple Access Interference (MAI).

With a non-adaptive antenna, the amount of MAI relative to a user at the base-station is only dependent on the number of simultaneous users, their transmitting powers and the scattering structures of their channels. These parameters are hard to predict and control, being this fact the reason for the spreading gains in currently commercial CDMA systems to be non-adaptive and the same, usually determined by statistical network considerations.

Smart antennas, however, may be able to cancel large amounts of the MAI through their adjustable radiation patterns. Unfortunately, most of the smart antenna techniques are restricted by the unrealistic assumption that the number of incoming signals is smaller than the number of array elements, although research indicates this is not true in cellular applications [1,2].

Once it is virtually impossible to estimate the Directions of Arrival (DOA) of the large number of multipaths using a relatively small array, it is wise to consider another scenario where some knowledge of the power-angle distribution is available. A widely ac-

cepted model is that the multipath distribution of a user around the base is Gaussian, from the Central Limit Theorem, it is concluded that the envelope distribution of paths from a crowd of users (cluster) is likewise. This model can well represent both the case when a base station is near some facility of public relevance (such as a station, a mall etc.), around which users tend to concentrate (narrow Gaussian), and the case when users are roughly uniformly distributed around the base (large Gaussian).

In this paper, an algorithm to estimate a cluster's mean DOA and size is derived. It is shown via computer simulations that when the total number of users is large, the beam-pattern obtained with the estimates is nearly the same as the optimum pattern obtained with the full knowledge of all users' spatial signatures. As a result, it is demonstrated that the spreading factor necessary to achieve a desired BER can be predicted over such estimates nearly as well as it would be determined over the full knowledge of each user's spatial signature.

The paper is organized as follows: Section II introduces the channel model and basic assumptions, followed by the fundamentals of the proposed cluster finding algorithm in section III. Beam forming is briefly discussed in Section IV. Section V considers the issue of adaptive spreading based on the cluster's estimated parameters and Section VI displays simulation results. Finally, Section VII wraps up with conclusions

## II. CHANNEL MODEL

Common assumptions:

- The antenna array is linear with omni directional elements equally spaced and is perfectly calibrated.
- The source signals are narrowband
- The signal sources are uncorrelated and concentrated at certain geographic points (clusters) with number of concentrations less than number of antenna elements.
- The difference in time delay between incoming rays is small relative to the inverse signal bandwidth.
- Since scatters are next to the local sources may admit that spread angle ( $\Delta\theta$ ) is relatively small.
- The sensor noise  $n(t)$  is independent of the signal, zero-mean, temporally and spatially white, complex Gaussian.

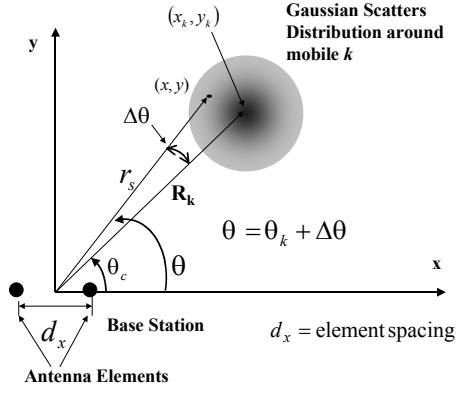


Fig.1- Modeling of scattering elements using gaussian model

Consider the general case of scattering area situated at a point  $(x_k, y_k)$  in space with the base station located at origin as shown in fig.1. In this case, the density of the scattering elements at a distance  $r_s$  and angle  $\theta$  from the mobile can be described by the bivariate Gaussian distribution [3].

$$f_{x,y}(x,y) = \frac{1}{2\pi\sigma_k^2} e^{-\frac{((x-x_k)^2 + (y-y_k)^2)}{2\sigma_k^2}} \Rightarrow f_{r_s,\theta}(r_s,\theta) = \frac{r_s}{2\pi\sigma_k^2} e^{-\frac{((r_s \cos\theta - x_k)^2 + (r_s \sin\theta - y_k)^2)}{2\sigma_k^2}} \quad (1)$$

Where  $\theta$  and  $r_s$  are showed at fig.1 and  $\sigma_k$  represents the standard deviation of the local scattering elements. To calculate the DOA probability density function is necessary to calculate the following integral.

$$f_{\theta_k}(\theta) = \int_0^{\infty} f_{r_s,\theta}(r_s,\theta) dr_s = \frac{1}{2\sqrt{2\pi}\sigma_k} \operatorname{erfc}\left(\frac{-R_k \cos(\theta - \theta_k)}{\sqrt{2\sigma_k}}\right) e^{-\frac{R_k^2 \sin^2(\theta - \theta_k)}{2\sigma_k^2}} R_k \cos(\theta - \theta_k) + \frac{e^{-\frac{R_k^2}{2\sigma_k^2}}}{2\pi} \quad (2)$$

Where  $\operatorname{erfc}(z) = \frac{2}{\sqrt{\pi}} \int_z^{\infty} e^{-t^2} dt$   $x_k, y_k$  is the user location, with  $\theta_k = \arctan\left(\frac{y_k}{x_k}\right)$   $R_k = \sqrt{x_k^2 + y_k^2}$ .

Consider now a crowd of users (hereon referred to as a *cluster*) concentrated around a point  $(X,Y)$  where a facility of public relevance is located. Let's assume the DOA of the  $k$ -th user's paths are determined by a Gaussian probability density function given by (1). Based in this information we have that all users scattering distribution will also be gaussian, with variance  $\sigma_s^2$ . Therefore the DOA probability density function of all users will also be given by

$$f_{\theta}(\theta) = \frac{1}{2\sqrt{2\pi}\sigma_s} \operatorname{erfc}\left(\frac{-R \cos(\theta - \theta_c)}{\sqrt{2\sigma_s}}\right) e^{-\frac{R^2 \sin^2(\theta - \theta_c)}{2\sigma_s^2}} R \cos(\theta - \theta_c) + \frac{e^{-\frac{R^2}{2\sigma_s^2}}}{2\pi} \quad (3)$$

Where:  $\theta_c = \arctan(Y/X)$ ;  $R = \sqrt{X^2 + Y^2}$

If we consider  $\sigma_s \ll R$  the distribution of  $\theta$  will be concentrated around  $\theta_c$ , making the approximations:

$$\sin(\theta - \theta_c) \cong (\theta - \theta_c) = \Delta\theta$$

$$\cos(\theta - \theta_c) \cong 1, \operatorname{erfc}\left(\frac{-R}{\sqrt{2\sigma_s}}\right) \cong 2, e^{-\frac{R^2}{2\sigma_s^2}} \cong 0 \quad (4)$$

Then

$$f_{\theta}(\theta) \cong \frac{1}{\sqrt{2\pi}\Delta\theta_c} e^{-\frac{-(\theta - \theta_c)^2}{2\Delta\theta_c^2}} \quad (5)$$

Where  $\Delta\theta_c = \frac{\sigma_s}{R}$ ,  $\theta_c - \pi \leq \theta \leq \theta_c + \pi$

At every sampling instant, a random number  $K$  of users is simultaneously transmitting. The signal of the  $k$ -th user is spread into  $P_k$  paths whose DOA probability density is ruled by (1) only with  $(\theta_k, \Delta\theta_k)$  in place of  $(\theta_c, \Delta\theta_c)$ . In the simulations, in hypothesis, assume that each amplitude path,  $A_{pk}$ , is a random variable whose mean follows a gaussian distribution that depends on DOA:

$$f_{\bar{A}_{pk}/\theta}(A/\theta) = \delta\left(A - A_0 e^{-\frac{-(\theta - \theta_c)^2}{2\Delta\theta_c^2}}\right) \quad (6)$$

Where  $\theta_c$  is mean angle,  $\Delta\theta_c$  is the angle spread and

$\bar{A}_{pk}$  is the  $p$ -th path,  $k$ -th user's mean amplitude and  $A_0$  is a normalization factor. Each multipath received at base station is independent random process, with temporal Rayleigh distribution.

Let the phase of each path  $(\psi_{pk}(t))$  be a random variable uniformly distributed in  $[-\pi, \pi]$ . The signal of the  $p_k$ -th path is defined as

$$s_{p_k}(t) = A_{p_k}(t) = |A_{p_k}| e^{j\psi_{p_k}(t)} \quad (7)$$

Finally, the steering vector of a Uniform Linear Array (ULA) of  $N$  elements towards the  $p_k$ -th path is given by:

$$\mathbf{U}_{p_k} = A_{p_k}(t) \begin{bmatrix} \dots & e^{-j2\pi(N-1)\Delta e_n \cos(\theta_{p_k})} \end{bmatrix}^T \quad (8)$$

Where  $\Delta e_n$  is the element spacing in wavelengths.

The spatial signature of the  $k$ -th user is therefore:

$$\mathbf{U}_k = \sum_{p=1}^{P_k} \mathbf{U}_{p_k} \quad (9)$$

Suppose that a Uniform linear array antenna receives the signals from  $K$  users and each user with  $P$  paths and that angles, amplitudes and phases be random variables and suppose one user's concentration. The received covariance matrix will be given by

$$\mathbf{R} = \sum_{k=1}^{KP} \sum_{j=1}^{KP} E \left\{ A_{p_k} e^{j\psi_{p_k}} A_{p_j}^* e^{-j\psi_{p_j}} \begin{bmatrix} 1 \\ \vdots \\ e^{-j\alpha(N-1)\cos(\theta_{p_k})} \end{bmatrix} \begin{bmatrix} 1 \\ \vdots \\ e^{j\alpha(N-1)\cos(\theta_{p_j})} \end{bmatrix} \right\} + E\{\mathbf{nn}^H\} \quad (10)$$

Where  $\alpha = 2\pi\Delta e_n$

According to equations (5) and (6), the DOA-pdf  $f(\theta)$  of a concentration is gaussian and Amplitudes-pdf  $f(A/\theta)$  is DOA gaussian dependent, respectively. Phases distribution  $f(\psi)$  are constant from  $-\pi$  to  $\pi$ , i.e.:

$$f(\theta) = \frac{1}{\sqrt{2\pi\Delta\theta_c}} e^{-\frac{(\theta-\theta_c)^2}{2\Delta\theta_c^2}}, f(A/\theta) = \delta \left( A_k - A_0 e^{-\frac{(\theta-\theta_c)^2}{2\Delta\theta_c^2}} \right),$$

$$f(\psi) = \frac{1}{2\pi}, \psi \in [-\pi, \pi] \quad (11)$$

Applying this information on the covariance matrix calculation, the element of line  $k$ , column  $l$  from  $\mathbf{R}$  can be written as

$$R(k,l) = KPA^2 \int_{-\pi}^{\pi} \frac{1}{\sqrt{2\pi\Delta\theta_c^2}} e^{-\frac{(\theta-\theta_c)^2}{2\Delta\theta_c^2}} e^{-j\alpha(k-l)\cos(\theta)} d\theta + \sigma_n^2 \delta(k-l) \quad (12)$$

Making  $\theta - \theta_c = \xi \rightarrow \theta = \xi + \theta_c$

$$\begin{aligned} \text{Since } \xi \text{ is small, we have for Taylor approximation} \\ \cos(\xi + \theta_c) = \cos(\xi)\cos(\theta_c) - \sin(\theta_c)\sin(\xi) \approx \\ \approx -\xi\sin(\theta_c) + \cos(\theta_c) \end{aligned} \quad (13)$$

Therefore, we have

$$R(k,l) \cong KPA^2 e^{-j\alpha(k-l)\cos(\theta_c)} e^{-\frac{\alpha^2(k-l)^2 \sin^2(\theta_c) \Delta\theta_c^2}{2}} + \sigma_n^2 \delta(k-l) \quad (14)$$

We have, therefore, in matricial form:

$$\mathbf{R}(\theta_c, \Delta\theta_c) \approx \mathbf{S} \mathbf{D}_a(\theta_c) \mathbf{B}(\Delta\theta_c \sin(\theta_c)) \mathbf{D}_a^*(\theta_c) + E[\mathbf{nn}^H] \quad (15)$$

Where

$$\mathbf{D}_a(\theta_c) = \text{diag}[\mathbf{a}(\theta_c)] \text{ and } \mathbf{B} \text{ is a Toeplitz matrix with } \mathbf{B}(\Delta\theta_c \sin(\theta_c))_{k,l} = b((k-l)\Delta\theta_c \sin(\theta_c)),$$

$$\text{With } b(x) = e^{-\frac{\alpha^2 x^2}{2}} \text{ and } \mathbf{a}(\theta_c) = [1 \dots e^{j\alpha(N-1)\cos(\theta_c)}]$$

And,  $S = KPA^2$ , a mean power of users concentration. Making

$$\mathbf{R}_v(\theta_c, \Delta\theta_c) = \mathbf{D}_a(\theta_c) \mathbf{B}(\Delta\theta_c \sin(\theta_c)) \mathbf{D}_a^*(\theta_c) \quad (16)$$

We have, finally

$$\mathbf{R}(\theta_c, \Delta\theta_c) \approx \mathbf{S} \mathbf{R}_v(\theta_c, \Delta\theta_c) + E[\mathbf{nn}^H], \quad (17)$$

### III. CLUSTER FINDING ALGORITHM

Although the covariance matrix for concentrated scattered sources has full rank, it is possible to apply a subspace algorithm such as root-MUSIC. The case of a single concentration is analyzed first with gaussian DOA distribution where it is desired to estimate the mean angle and spread angle. A generalization to the case of several concentrated sources is given at the end of the section.

From (17) the covariance matrix for a single concentrated sources with power  $S$  is given by

$$\mathbf{R} = \mathbf{S} \mathbf{R}_v(\theta_c, \Delta\theta_c) + \sigma_n^2 \mathbf{I} \quad (18)$$

Making a singular value decomposition on  $\mathbf{B}$ ,

$$\begin{aligned} \mathbf{B} = \mathbf{E}_B \mathbf{\Lambda}_B \mathbf{E}_B^*, \text{ then} \\ \mathbf{R} = (\mathbf{D}_a(\theta_c) \mathbf{E}_B) (\mathbf{S} \mathbf{\Lambda}_B + \sigma_n^2 \mathbf{I}) (\mathbf{D}_a(\theta_c) \mathbf{E}_B)^* = \mathbf{E}_R \mathbf{\Lambda}_R \mathbf{E}_R^* \end{aligned} \quad (19)$$

Is a singular value decomposition of  $\mathbf{R}$ .

Collect, the noise eigenvectors, i.e. the eigenvectors corresponding to the  $N-CN$  (*Clusters' Number*) smallest eigenvalues of  $\mathbf{R}$ , in the matrix  $\mathbf{E}_n$  and define

$\mathbf{a}(z) = [1, z, \dots, z^{N-1}]^T$ . Then the root-MUSIC [7] estimates are the arguments of the  $CN$  roots inside the unit circle with largest modulus of the polynomial  $f(z) = \mathbf{a}^T(z^{-1}) \mathbf{E}_n \mathbf{E}_n^* \mathbf{a}(z)$ .

Select an arbitrary signal subspace dimension  $CN$ . From (19), the noise subspace of  $\mathbf{R}$  is  $\mathbf{E}_{n,R} = \mathbf{D}_a(\theta_c) \mathbf{E}_{n,B}$ , where  $\mathbf{E}_{n,B}$  is the corresponding "noise subspace" of  $\mathbf{B}$ . Since

$$\mathbf{D}_a^*(\theta_c) \mathbf{a}(z) = \mathbf{a} \left( z e^{-j2\pi \frac{\Delta\theta_c}{\lambda} \cos(\theta_c)} \right),$$

the root-MUSIC cost function applied to  $\mathbf{R}$  and  $\mathbf{B}$  respectively,  $f_R(z)$  e  $f_B(z)$  are related through

$$\begin{aligned} f_R(z) &= \mathbf{a}^T(z^{-1}) \mathbf{E}_{n,R} \mathbf{E}_{n,R}^* \mathbf{a}(z) \\ &= \mathbf{a}^T(z^{-1}) \mathbf{D}_a(\theta_c) \mathbf{E}_{n,B} \mathbf{E}_{n,B}^* \mathbf{D}_a^*(\theta_c) \mathbf{a}(z) \\ &= \mathbf{a}^T \left( z^{-1} e^{j2\pi \frac{\Delta\theta_c}{\lambda} \cos(\theta_c)} \right) \mathbf{E}_{n,B} \mathbf{E}_{n,B}^* \mathbf{a} \left( z e^{-j2\pi \frac{\Delta\theta_c}{\lambda} \cos(\theta_c)} \right) \\ &= f_B \left( z e^{-j2\pi \frac{\Delta\theta_c}{\lambda} \cos(\theta_c)} \right) \end{aligned} \quad (20)$$

And the only difference between the root loci of

$f_R(z)$  e  $f_B(u)$  is a rotation  $z = ue^{j2\pi \frac{\Delta\theta_c}{\lambda} \cos(\theta_c)}$ , independently of  $\Delta\theta$ .

In [8] is shown that for  $CN=1$ , root-MUSIC applied to  $\mathbf{B}$  will give a positive real root, i.e., the argument is zero and for  $CN=2$ , the roots from  $\mathbf{B}$  will be complex conjugated,  $z = \alpha e^{\pm j\gamma}$ . This, together with the rotational property, proves that ordinary root-MUSIC ( $CN=1$ ) for a single concentrated scattered sources, will yield consistent DOA estimates [9]. It also proves that if root-MUSIC is applied to locate  $CN=2$  concentrations from the covariance matrix  $\mathbf{R}$ , the found roots will be  $z = \beta e^{j(2\pi \frac{\Delta\theta_c}{\lambda} \cos(\theta_c) \pm \gamma)}$ , where  $\gamma$  is function of  $\Delta\theta$ .

Exploiting this structure of the root loci, the following algorithm, 'B-root-MUSIC' is suggested to estimate  $\theta_c$  e  $\Delta\theta$  for a single concentration:

- Estimate  $\theta_c$  consistently. Ordinary root-MUSIC could be used, but the following estimate is expected to give better accuracy, since two roots are used instead of a single one.

$$\hat{\theta} = \cos^{-1} \left( \frac{1}{2\pi \frac{\Delta\theta_c}{\lambda}} \left( \frac{\arg(z_1) + \arg(z_2)}{2} \right) \right) \quad (21)$$

- Estimate  $\Delta\theta$  by

$$\Delta\hat{\theta} = \frac{\gamma^{-1} \left( \left| \frac{\arg(z_1) - \arg(z_2)}{2} \right| \right)}{\sin(\hat{\theta})} \quad (22)$$

Typically,  $\gamma(x)$  is a monotonously increasing function and can be pre-calculated for a given array. The inverse function is then easily interpolated from the tabulated values.

For well-separated clusters, the algorithm can be generalized to the case of several clusters. Use root-MUSIC to estimate twice the number of true sources clusters, pair the estimates together and apply (21) and (22) for each pair of estimates.

If the number of paths of all users together is large enough, the summation in (9) can be modeled as an integral and (23) can be used to determine the Gaussian envelope of the cluster.

$$\mathbf{U}_c = \int_{-\pi}^{\pi} \frac{\begin{bmatrix} 1 \\ \vdots \\ e^{-j2\pi(N-1)\Delta e_r \cos(\theta_c)} \end{bmatrix}}{\sqrt{2\pi} \hat{\Delta} \hat{e}_c} e^{-\frac{(\theta-\theta_c)^2}{2\Delta\theta_c^2}} d\theta, \quad (23)$$

#### IV. BEAM FORMING

The optimum weights computation by the Wiener solution can be readily extended to use the array vector model with the continuous Gaussian envelope. Suppose that parameters for the desired signal and interference are respectively:  $(\theta_d, \Delta\theta_d)$  and  $(\theta_i, \Delta\theta_i)$ , so that  $\mathbf{U}_{dc}$  (desired continuous vector) and  $\mathbf{U}_{ic}$  (interference continuous vector) can be calculated from (23). If power control is assumed, the total interfering power is proportional to the number  $K$  of interfering users, known to the base station. Thus, the optimum estimated weight vector would then be:

$$\mathbf{W}_{opt} = \left\{ \mathbf{U}_{dc}^* \mathbf{U}_{dc}^T + K \mathbf{U}_{ic}^* \mathbf{U}_{ic}^T + SNR^{-1} \mathbf{I}_N \right\}^{-1} \mathbf{U}_{dc}^* \quad (24)$$

Where  $\mathbf{I}_N$  is the  $N \times N$  identity matrix.

On the other hand, if the spatial signatures of the desired and interfering users are known, so that  $\mathbf{U}_d$  (desired vector) and  $\mathbf{U}_i$  (interference vector) are calculated from (9) the optimum theoretical weight vector is given by:

$$\mathbf{W}_{opt} = \left\{ \mathbf{V}_d + \frac{K}{PI} \sum_{p=1}^{PI} \mathbf{V}_{ipk} + SNR^{-1} \mathbf{I}_N \right\}^{-1} \mathbf{Ref}^* \quad (25)$$

Where  $\mathbf{V} = \mathbf{U}^* \mathbf{U}^T$  and  $\mathbf{Ref}$  is the reference (desired) vector.

#### V. OPTIMUM SPREADING FACTOR ESTIMATION

If perfect power control is assumed, and the parameters of the desired user's  $(\theta_d, \Delta\theta_d)$  and of interferer's  $(\theta_i, \Delta\theta_i)$  path distributions are available, the power of desired and interfering signals can be estimated by:

$$P_S(\theta, \Delta\theta) = \left| \mathbf{W}_{opt}^T \mathbf{U}_S(\theta, \Delta\theta) \right|^2 \quad (26)$$

Where  $\mathbf{U}_d = \mathbf{U}_S(\theta_d, \Delta\theta_d)$ ;  $\mathbf{U}_i = \mathbf{U}_S(\theta_i, \Delta\theta_i)$  and  $\mathbf{W}_{opt}$  is as in (24).

Then, for a desired Bit Error Rate (BER), we have:

$$SNR = Q^{-1}(BER)^2 \quad (27)$$

Where  $Q^{-1}$  is the inverse of the well-known  $Q$ -function. If the noise power is known, the optimum spreading factor can be estimated by [6]:

$$SF = \frac{P_i}{\frac{P_d}{SNR} + \sigma^2} \quad (28)$$

Where  $P_d = P_S(\theta_d, \Delta\theta_d)$  and  $P_i = P_S(\theta_i, \Delta\theta_i)$ .

On the other hand, if the power of every path and the spatial signature of every user in the cell are known, (26) and (28) can be computed exactly using (9) in place of  $\mathbf{U}_S$  and (25) in place of (24).

In the next section, however, computer simulations will show that the spreading factors given by (28) in both cases are nearly the same, provided that the signals are distributed approximately according to the Gaussian model. The reason for this is that an array antenna with few elements does not have strong spatial filtering capabilities. Therefore, if the number of users in a cell is larger than the number of elements, optimum performance is achieved in terms of average (envelope) spatial filtering rather than path-by-path nulling.

#### VI. SIMULATION RESULTS

As example, consider signals coming from 2 users' concentrations, each with 100 signals, with amplitude Rayleigh distribution and uniform phases distribution between  $-\pi$  to  $\pi$ . The signals are detected for a linear array antenna with 8 elements, with  $\lambda/2$  between elements, and 3000 samples are used to estimate the covariance matrix. The results for three different situations are showed at Table 1.

| Clusters | $\theta / \Delta\theta$ | Measured $\hat{\theta} / \Delta\hat{\theta}$ |
|----------|-------------------------|--|
| 1        | $80^\circ / 5^\circ$    | $80.01^\circ / 4.5^\circ$                    |
|          | $30^\circ / 20^\circ$   | $29.03^\circ / 23.78^\circ$                  |
| 2        | $85^\circ / 5^\circ$    | $85.005^\circ / 5.3^\circ$                   |
|          | $25^\circ / 20^\circ$   | $25.18^\circ / 23.1012^\circ$                |
| 3        | $75^\circ / 5^\circ$    | $75.025^\circ / 5.0625^\circ$                |
|          | $20^\circ / 10^\circ$   | $19.3^\circ / 10.0257^\circ$                 |

Table 1- Mean angle and spread angle for different clusters distributions.

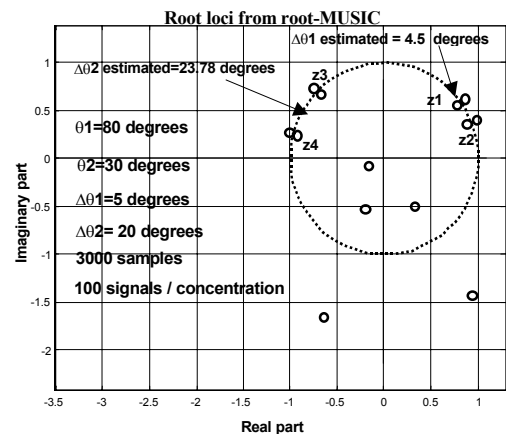


Fig.2 - Root loci from root-MUSIC considering 2 concentrations with 100 signals at  $\theta_{c1} = 80^\circ$  e  $\theta_{c2} = 30^\circ$  with spreads  $\Delta\theta_1 = 5^\circ$  and  $\Delta\theta_2 = 20^\circ$  respectively.

Further, Figure 3 displays the optimum receiving radiation beam-pattern obtained with the information estimated by the cluster finder algorithm contrasted to the one that would theoretically be obtainable with the full knowledge of the spatial signatures of all users, as in Section IV.

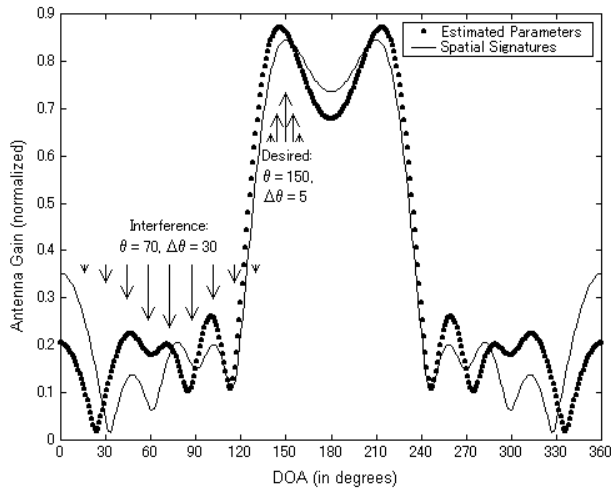


Fig. 3 – Radiation Patterns

Figure 4 shows how the SINR of a desired signal varies according to its location due to dependence of spatial filtering performance on the steering angle and the interference DOA spread. It can be seen that the spatial filtering performances of beam-forming based on the estimated parameters and that based on the users' spatial signatures are virtually the same, confirming the relevance of the information acquired by the cluster finding algorithm.

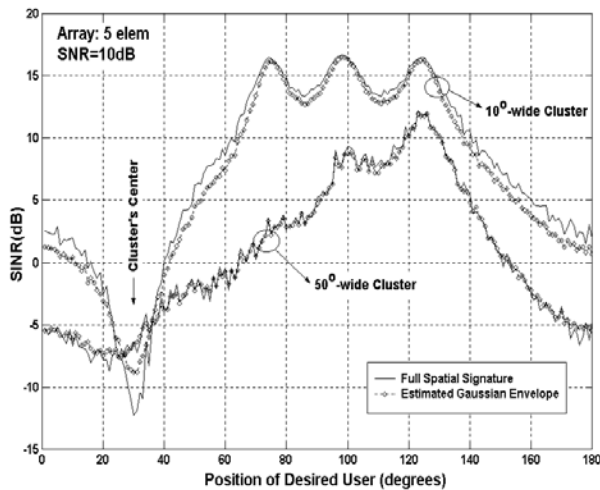


Fig. 4 – Spatial Filtering Performance.

Finally, Figure 5 shows how the necessary spreading gain to achieve a desired BER can be estimated considering scenario of Gaussian distributed paths and a small array antenna, having its value nearly the same as the obtained with the full knowledge of all users' spatial signatures.

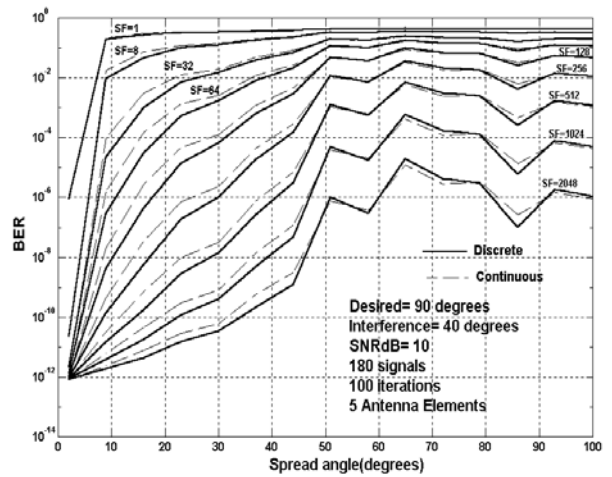


Fig. 5 – BER x Spread Angle

## VII. CONCLUSIONS

In this paper, a method to estimate the average and the spread of the direction of arrival for signals Gaussianly distributed in angle and in power is outlined. This method is based on the root-MUSIC estimation algorithm. The algorithm extracts useful information on the channel of users in a largely populated cell with a relatively small array antenna. It was also shown that successful beam forming results from the estimated parameters acquired.

Further, based upon the spatial filtering specifications estimation results a scheme to support the adaptive selection of the spreading factor to be used by each user in a future cellular system was proposed, considering the little resolution of a small array, with the well-accepted assumption that the path distributions of cellular systems' users is Gaussian.

The algorithm was designed over the assumption that the signals of all users feature an identical and synchronized training sequence with a large spreading factor, what is fully compliant with the IMT2000 and UMTS standards for future CDMA systems.

In the future, we intend to extend the present algorithm so to estimate several clusters simultaneously.

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