Wavelet Shift-Keying: A New Digital Modulation

H.M. de Oliveira, H.A.N. Silva, E.A. Bouton

***Resumo*—Este trabalho introduz as idéias preliminares de um novo tipo de modulação digital, chamada de *Wavelet Shift Keying* (WSK), baseado em transformadas discretas de wavelet. A seqüência de bits de entrada é convertida em uma seqüência de funções escalonadas, indicando qual versão da wavelet-mãe deve ser transmitida em cada intervalo de símbolo. O sinal modulado consiste em uma seqüência de versões (superpostas ou não) escalonadas e deslocadas de uma wavelet. Tais esquemas podem ser vistos como uma generalização dos sistemas OFDM baseados em wavelets. São apresentados alguns exemplos desta modulação utilizando as wavelets deO e Sombrero.**

***Palavras-chave*—Modulação por chaveamento de wavelet, modulações digitais, wavelets, modulação multitom com wavelets discretas.**

***Abstract*—This note introduces the preliminary ideas of a new digital modulation scheme termed Wavelet Shift Keying (WSK), which is based on discrete wavelet transforms. The source bit stream is translated into a sequence of scales indicating which version of the mother wavelet should be transmitted in each symbol slot. The modulated signal therefore consists in a sequence of overlapping (or non-overlapping) scaled-translated versions of a wavelet. Such schemes can be viewed as a generalisation of "wavelet-based OFDM systems". Simple examples of deO-modulation and Mexican hat modulation are presented.**

***Index Terms*—wavelet shift keying, digital modulation, wavelets, discrete wavelet multitone modulation.**

I. INTRODUCTION

Several coherent and non-coherent modulation waveforms have been devised for transmitting binary information over bandpass channels [1]. *M*-ary signalling schemes are useful when one wishes to reduce bandwidth requirements of a baseband signal. Wavelet analysis has recently been playing a relevant role in quite a lot of areas, especially in Engineering. Wavelets set up now a well-established and powerful tool in signal analysis [2, 3]. They are even powerful for analysing signals over finite fields [4].

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Wavelets have already been proposed in the modulation framework, particularly in OFDM systems involving ×DSL applications [5-8]. Many interesting wavelets have been recently introduced such as new elliptic-cylinder orthogonal wavelets linked to Mathieu wave equation [9]. In Telecommunication, wavelets have be applied to various aspects of synchronisation [10-13]. They are also relevant in modulation [14-16]. A recent paper introduced new orthogonal wavelets related to the Nyquist raised cosine filters [17]. Although not being compact support wavelets, continuous deO (de Oliveira) wavelets are suitable to implement modulation techniques: their spectral requirements are well defined. In this case, overlapping orthogonal wavelet pulses are used. This is analogous to the idea behind OFDM systems. A further case concerns compact support wavelets such as db*N* (Daubechies), sym*N* (Symmlets) [2] or even cby*N* (Chebyshev) [18].

Common signalling schemes for transmitting digital information over bandpass channels are based on a perpetual carrier wave. Wavelets have been adopted with success in many practical situations to replace sinusoidal waveforms. It seems to be somewhat expected a replacement of a classical oscillator (carrier generator) by a waveform generator that produces a basic wavelet. This kind of modulation approach named Wavelet Shift Keying (WSK) is described in the next section. Details such as synchronisation, performance of WSK are currently being investigated.

II. THE WSK MODULATION

M-ary WSK signalling schemes can be used in data transmission application. Let $y(t)$ be a basic wavelet (preferentially compactly supported). The Wavelet Shift Keying (WSK) modulation is defined setting the number *M* of scale to be used as a power of two. Scaled versions of the mother wavelet are transmitted in each symbol slot. It is assumed that scale factor of every slot depends on the input binary data. The (normalised) non-overlapped modulated signal (*n*-WSK) based on a wavelet $y(t)$ is given by

$$j_{n-WSK}(t) = \sum_{m=-\infty}^{+\infty} 2^{n(m)/2} y\left(2^{n(m)}(t-m)\right), \qquad (1)$$

where *m* determine the index of the modulation symbol slot at a baud rate of 1 baud and $n(m) \in \{0,1,2,...,M-1\}$. The longest wavelet is $y(t)$ and its effective duration is supposed to be equals to one. Scaled versions $y(2^{n(m)}t)$, $n(m) \neq 0$ are shorter than $y(t)$ so the waveform $j_{WSK}(t)$ is a sequence of essentially non-overlapped wavelets.

Figure 1 displays a simple sketch of the WSK modulator. Data are segmented into blocks of length $\log_2 M$ bits. The selected block specifies the wavelet scaling factor n that should be implemented in a particular time slot.

Another much motivating case is (normalised) overlapped WSK modulated signal based (*o*-WSK) on a wavelet $y(t)$, which is given by

$$j_{o-WSK}(t) = \sum_{m=-\infty}^{+\infty} 2^{-n(m)/2} y\left(2^{-n(m)}(t-m)\right). \qquad (2)$$

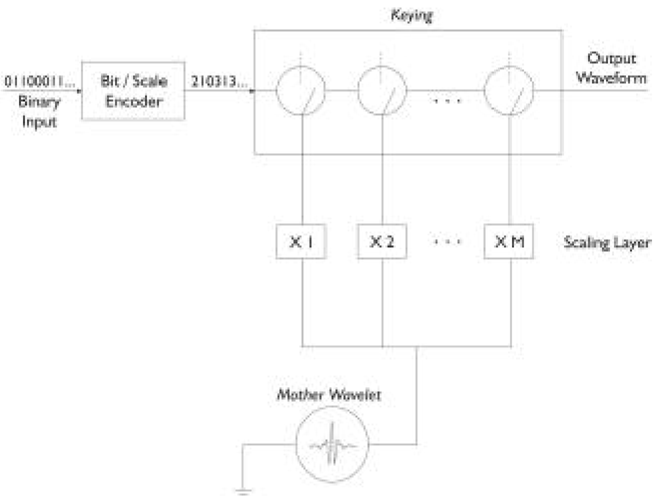

Figure 1. WSK digital modulator. The input binary string is converted into a stream of scale values that control the keying. The mother wavelet is locally generated and every scaled version is derived in a scaling layer. The notation ×1, ×2, ..., ×*M* specifies the scale factor according to eqn(1) or eqn(2).

The transmission rate for both overlapped and non-overlapped schemes is $\log_2 M$ bps (assuming a normalised baud). However *o*-WSK has higher spectral efficiency since that less bandwidth is required to achieve the same bit rate. Since the effective support of deO wavelet is [-10,10], the wavelet is formerly scaled to guarantee a normalised basic wavelet (effective duration equals to one.) Basic deO wavelets are shown in figure 2.

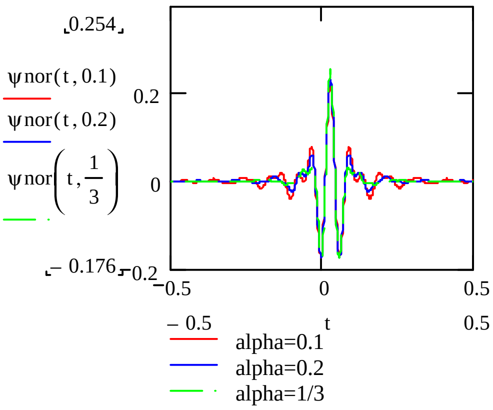

Figure 2. Normalised real deO wavelet with roll-off parameter *a*=0.1, 0.2 and 1/3.

Figure 3 and 4 illustrate *deO*-based modulated waveforms for non-overlapped and overlapped case.

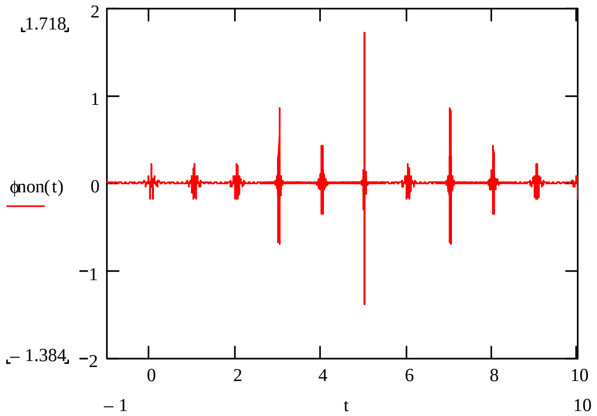

Figure 3. Typical (1 baud normalised) non-overlapped WSK waveforms: Examples based on *de Oliveira* wavelet with roll-off *a*=0.2. The *deO*-WSK scheme.

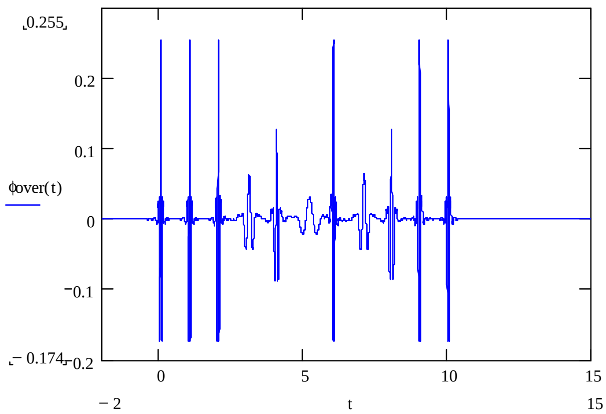

Figure 4. Typical overlapped WSK waveforms: Example based on *de Oliveira* wavelet (*deO*-WSK scheme with roll-off *a*=0.2.) M=4 scales were used.

A further example of digital modulation waveforms is shown in figures 5 and 6, which is based in Mexican wavelet. Random binary input data were generated in all cases and plots were drawn from Mathcad 2000®.

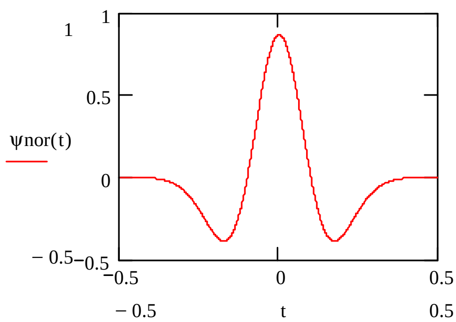

Figure 5. Normalised Mexican hat wavelet.

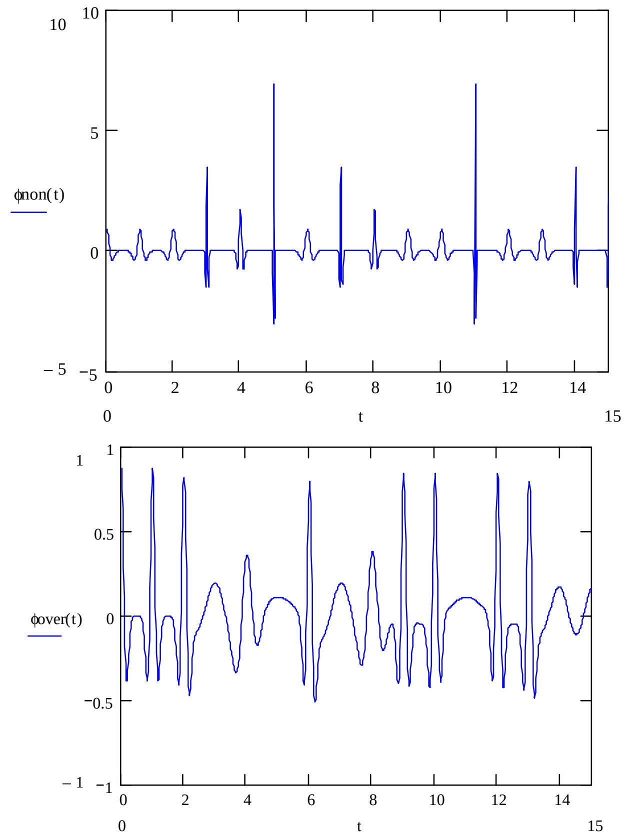

Figure 6. Typical (1 baud normalised) non-overlapped and overlapped WSK waveforms: Examples based on the *Mexh*-WSK scheme with M=4 scales.

III. Designing a Fitting Mother Wavelet for the Modulator

The mother wavelet controls the signal spectrum of the WSK modulation. As a preliminary discussion on WSK bandwidth requirements, let *esupp* $Y(w)$ denote the (effective) support of the mother wavelet spectrum. Overlapped WSK yields then

$$\mathrm{esupp}\Psi\left(2^{n(m)}w\right)\subseteq \mathrm{esupp}\Psi(w), \qquad (3)$$

so that the bandwidth requirements for $j_{WSK}(t)$ are essentially controlled by the mother wavelet spectrum $Y(w)$. It is associated with the shortest emitted wave.
For non-overlapped WSK systems

$$\mathrm{esupp}\Psi\left(2^{n(m)}w\right)\subseteq \mathrm{esupp}\Psi\left(2^{M-1}w\right), \qquad (4)$$

so that the bandwidth requirements for $j_{WSK}(t)$ are essentially controlled by the wavelet at the (M-1)th scale, that is, the $y(2^{-(M-1)}t)$ version. Although $y(t)$ is a pass-band signal, it is as a rule, "normalised" as conceived in the wavelet definition. Assume that the passband spectrum Y is essentially confined in a bandwidth of length B_0, centred at w_0. For instance, the spectrum of deO wavelet with roll-off $\boldsymbol{a}$ is given by [17]

$$\Psi^{(deO)}(w)=e^{-jw/2}S^{(deO)}(w). , \qquad (5)$$

where

$$S^{(deO)}(w)=\begin{cases} 0 & \text{if } w<p(1-a) \\ \frac{1}{\sqrt{2p}}\cos\frac{1}{4a}\left(w-p(1+a)\right) & \text{if } p(1-a)\le w<p(1+a) \\ \frac{1}{\sqrt{2p}} & \text{if } p(1+a)\le w<2p(1-a) \\ \frac{1}{\sqrt{2p}}\cos\frac{1}{8a}\left(w-2p(1-a)\right) & \text{if } 2p(1-a)\le w<2p(1+a) \\ 0 & \text{if } w\ge 2p(1+a). \end{cases} \qquad (6)$$

Therefore, the spectrum of deO wavelet is confined within the closed interval $[(1-a)p<w<(1+a)2p]$ so that

$$w_0=\sqrt{2(1-a^2)}p \text{ and } B_0=p.[1+3a]\le 2p. \qquad (7)$$

Suppose now that the communication channel (pass band) is centred at the frequency w_c and has a bandwidth B. For the sake of simplicity, it is adopted an ideal pass-band model. The mother wavelet must be modified to guarantee that $j_{WSK}(t)$ copes with the channel characteristic.

Starting with a given mother wavelet $y(t)$, an appropriated transmitted wavelet $y_{TX}(t)$ is fabricated simply by translating the original spectrum into the channel frequencies and then scaling it in order to fulfil the total bandwidth of the channel. The first operation can be easily accomplished by means of a mixer. The second step requires the scaling $y(at)$, where $a=B/B_0$.

Figure 7 displays as an example, the spectra of deO wavelets: the normalised version $|Y(w)|$, and the transmitted wavelet $|Y_{TX}(w)|$ adapted for a passband channel of parameters (w_c,B).

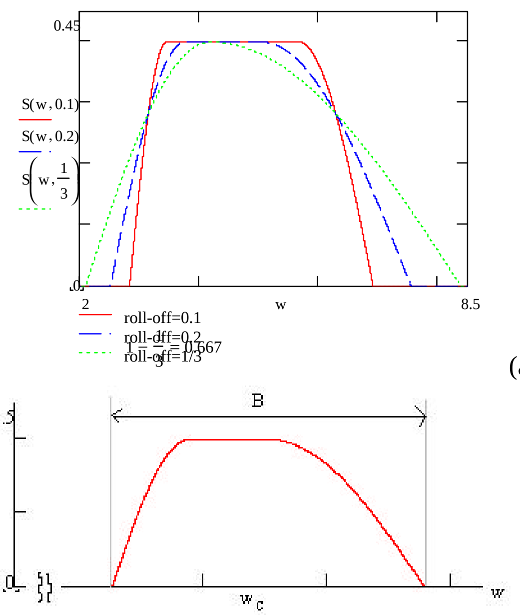

Figure 7. Design of wavelets for WSK: adapting the spectrum of to the channel: (a) normalised mother wavelet spectrum and (b) transmitter mother wavelet spectrum derived from (a).

Figure 8 below illustrates the strategy of generating transmitter wavelets. The block diagram shows the two steps carried out to adapt the wavelet to the channel.

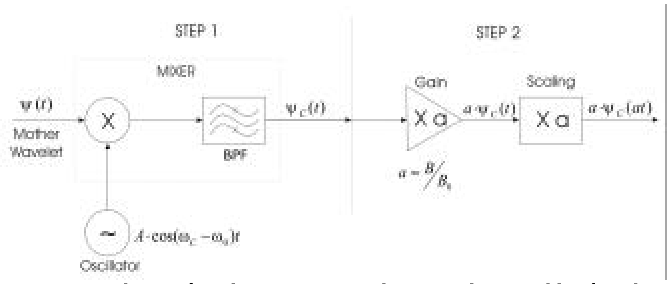

Figure 8. Scheme for designing a mother wavelet suitable for the communication channel. The task in implemented in two steps: *i*) a frequency translation by a mixer and *ii*) a scaling the wavelet to fulfil the channel bandwidth.

IV. CONCLUDING REMARKS

Although much has been left to be done, some chief ideas of new digital modulation schemes were introduced. Non-overlapping schemes can be seen as some sort of generalisation on the *M*-FSK modulation, but now taking into accounts the time-frequency resolution. Despite orthogonal WSK can be interpreted as Discrete wavelet multitone modulation (DWMT) [5], the approach here described closed follows standard digital modulations such as ASK, FSK, PSK, APK. Non-orthogonal wavelets can be used so these schemes can be viewed as a generalisation of "wavelet-based OFDM systems". They are very flexible since they can be based on an infinity of different wavelet families. The spectrum of overlapped WSK should be evaluated by investigating the autocorrelation function of the modulated signal. Bandwidth requirements can be found through the power spectral density. The peak-to-average power ratio of such schemes should also be examined. A demodulation scheme based on the pyramidal algorithm [19] can probably be devised, which will be especially attractive. A comparison between the bit error rate performance of *M*-WSK schemes and other equivalent (same spectral efficiency) conventional modulation such as *M*-PSK or *M*-FSK should be carried on different communication channels. WSK relative immunity to channel impairments also remains to be analysed. Synchronisation issues are not addressed in this preliminary investigation. However, efficient timing information can probably be derived from the modulated waveform itself because all waveforms are resulting from the same basic wavelet.

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