An Analytical Method for BER Determination in Multi-Rate DS-WCDMA Cellular Systems and Frequency-Selective Rayleigh Fading Channels

André Fagundes da Rocha and Paul Jean Etienne Jeszensky

***Abstract* — An analytical method for BER determination in DS-WCDMA cellular systems which support services with different requirements on bit rate and performance is proposed, assuming Gaussian approximations for multiple access interference and multipath interference, and modelling interference from neighbour cells as a function of path loss model, mobile user densities on cells and cellular architecture.**

***Index Terms* — CDMA, multi-rate, multiple access interference, multipath, performance**

***Resumo* — Um método analítico para determinação da taxa de erro de bit em sistemas celulares DS-WCDMA que suportam serviços com diferentes requisitos de taxa de bits e desempenho é proposto, assumindo aproximações gaussianas para interferência de múltiplo acesso e interferência multi-percurso, e modelando a interferência proveniente de células vizinhas como função da perda de percurso, densidade de usuários nas células e arquitetura celular.**

***Palavras-Chave* — CDMA, multitaxa, interferência de múltiplo acesso, multi-percurso, desempenho**

I. INTRODUCTION

The convergence between data services and mobility is a key step in the evolution of communication systems, as proposed by third generation cellular systems. One of the main objectives of these emerging systems is to support simultaneously several types of applications with different requirements on bit rate and performance, such as speech, data and video services. On investigating multi-rate systems [1] [2], it is important to be able to evaluate their performance so that their capability of supporting some application requirements can be verified. This paper addresses bit error rate (BER) or bit error probability of multi-rate DS-WCDMA (Direct Sequence Wideband Code Division Multiple Access) cellular systems and characterizes multiple access interference and multipath interference in frequency-selective Rayleigh fading channels, based on [3] and [4]´s approaches.

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This paper is organized as follows. Section II provides a description of the reverse link of a multi-rate DS-WCDMA cellular system where each cell supports several applications with different requirements on bit rate and bit error probability. In sections III and IV we assume Gaussian approximations for characterization of multiple access interference (MAI) and multipath interference (MPI) respectively, obtaining their mean and variance. In addition to that, a mathematical model which represents MAI from neighbour cells as a function of path loss model, mobile user densities on cells, and cellular architecture and geometry is presented in section III. In section V, a bit error probability expression for frequency-selective Rayleigh fading channel assuming QPSK modulation is derived. Finally, numerical results are presented in section VI and conclusions are drawn in section VII.

II. MULTI-RATE CELLULAR SYSTEM MODEL

The reverse link of a multi-rate DS-WCDMA cellular system is represented in Fig.1. For convenience, the system is modelled as a group of subsystems, each one representing a specific application. In this model, when an user develops an application, it is characterized as an user of a specific subsystem, what means that some transmitter and receiver parameters are adjusted in a way that it can support required bit rate and performance. According to Fig. 1, subsystem i from cell g has U_{gi} users and supports bit rate R_{gi} and bit error rate or probability $P_{B,gi}$ specificated for its correspondent application.

The multi-rate transmitter gik translates each group of $\log_2 M_{gi}$ bits of the binary sequence from user k in subsystem i of a cell g into a complex valued symbol according to a mapping rule. It generates a symbol sequence $A_{gik}(m) = A^I_{gik}(m) + jA^Q_{gik}(m)$, which modulates the amplitude of rectangular pulses with unitary amplitude and duration $T_{gi} = \log_2 M_{gi} / R_{gi}$. The resulting information signals $b^I_{gik}(t)$ and $b^Q_{gik}(t)$ are multiplied by spreading codes $c^I_{gik}(t)$ and $c^Q_{gik}(t)$, which are periodic signals consisting of rectangular pulses (chips) with a constant duration T_c assumed the same in all subsystems. The resulting signals modulate the amplitude and phase of a carrier with central frequency ω, amplitude $\sqrt{2P_{gik}}$ and phase θ_{gik}, where P_{gik} is adjusted according to power control requirements and ω depends on the frequency band available for the mobile cellular service. Thus, the transmitted signal for user *gik* is

$$s_{gik}(t) = \sqrt{2P_{gik}}\, b^I_{gik}(t) c^I_{gik}(t) \cos\left(\omega t + \theta_{gik}\right) - \sqrt{2P_{gik}}\, b^Q_{gik}(t) c^Q_{gik}(t) sin\left(\omega t + \theta_{gik}\right). \quad (1)$$

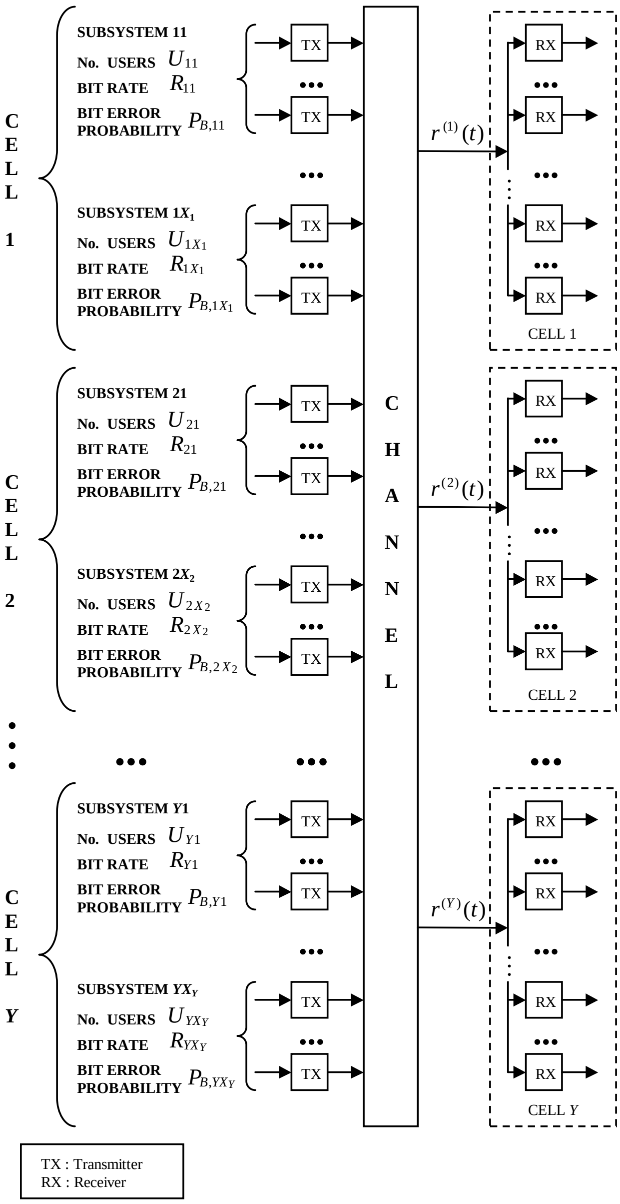

Fig. 1. Multi-rate cellular system

The signal provided by the frequency-selective Rayleigh channel with slow fading to receivers located at the base station in cell *h*, assuming asynchronous transmission, is

$$r^{(h)}(t) = n(t) + \sum_{g=1}^{Y}\sum_{i=1}^{X_g}\sum_{k=1}^{U_{gi}}\sum_{x=1}^{L} \alpha^{(h)}_{gikx}\; s_{gik}(t - \tau^{(h)}_{gikx}), \quad (2)$$

where *n(t)* is a zero-mean additive white Gaussian random process with two-sided spectral density $N_0/2$; Y is the number of cells in the multi-rate system; X_g is the number of subsystems in a cell *g*; U_{gi} is the number of mobile users in subsystem *gi*; *L* is the number of resolvable paths assumed the same for all users in the multi-rate system because all signals are spread to the same bandwidth; $\alpha^{(h)}_{gikx}$ is the attenuation factor and $\tau^{(h)}_{gikx}$ is the propagation delay that the channel introduces into signal $s_{gik}(t)$ at the *x*-th path between the transmitter *gik* and base station in cell *h*, modelled respectively as a time-invariant Rayleigh-distributed variable and a time-invariant random variable uniformly distributed over zero and T_{gi}.

The multi-rate receiver *hjl* detects the binary sequence sent by user *hjl* based on a decision variable obtained from despreading and demodulation of signal $r^{(h)}(t)$. If the receiver does not use diversity techniques then only one multipath component is used as reference for demodulation while the others are treated as an interfering component named multipath interference.

The components of the decision variable $\hat{A}_{hjlz} = \hat{A}_{hjlz}(m_0)$ at the multi-rate receiver *hjl* matched to the *z*-th multipath component of signal $s_{hjl}(t)$, assuming that $\tau^{(h)}_{hjlz} = 0$ and $\phi^{(h)}_{hjlz} = 0$ are references for the other propagation delays $\tau^{(h)}_{gikx}$ and phases $\phi^{(h)}_{gikx} = \theta_{gik} - \omega\tau^{(h)}_{gikx}$, are got from (1) and (2) as

$$\hat{A}^I_{hjlz} = \int_{m_0 T_{hj}}^{(m_0+1)T_{hj}} r^{(h)}(t) c^I_{hjl}(t) \cos\omega t\, dt = \eta^I_{hjlz} + \beta^I_{hjlz} + \gamma^I_{hjlz} + \chi^I_{hjlz} \quad (3)$$

and

$$\hat{A}^Q_{hjlz} = -\int_{m_0 T_{hj}}^{(m_0+1)T_{hj}} r^{(h)}(t) c^Q_{hjl}(t) sin\omega t\, dt = \eta^Q_{hjlz} + \beta^Q_{hjlz} + \gamma^Q_{hjlz} + \chi^Q_{hjlz} \quad (4)$$

where η^I_{hjlz} and η^Q_{hjlz} are the zero-mean Gaussian noise terms with variance $N_0 T_{hj}/4$ [4]; β^I_{hjlz} and β^Q_{hjlz} are the terms which contain the information sent by user *hjl*; γ^I_{hjlz} and γ^Q_{hjlz} represent the multiple access interference; and χ^I_{hjlz} and χ^Q_{hjlz} represent the multipath interference. These variables are given as follows:

$$\eta^I_{hjlz} = \int_{m_0 T_{hj}}^{(m_0+1)T_{hj}} n(t)\; c^I_{hjl}(t) \cos\omega t\; dt\,; \quad (5)$$

$$\eta^Q_{hjlz} = -\int_{m_0 T_{hj}}^{(m_0+1)T_{hj}} n(t)\; c^Q_{hjl}(t)\; sin\,\omega t\; dt\,; \quad (6)$$

$$\beta^{I}_{hjlz} = \sqrt{P_{hjl}/2}\,\alpha^{(h)}_{hjlz}\, A^{I}_{hjl}(m_0)\; T_{hj} \;; \tag{7}$$

$$\beta^{Q}_{hjlz} = \sqrt{P_{hjl}/2}\;\alpha^{(h)}_{hjlz}\; A^{Q}_{hjl}(m_0)\, T_{hj} \;; \tag{8}$$

$$\begin{aligned}\gamma^{I}_{hjlz} = & \sum_{\substack{g=1\\ g\neq h,}}^{Y}\;\sum_{\substack{i=1\\ i\neq j,}}^{X_g}\sum_{\substack{k=1\\ k\neq l}}^{U_{gi}}\sum_{x=1}^{L}\sqrt{\frac{P_{gik}}{2}}\,\alpha^{(h)}_{gikx}\psi^{II}_{gikx,hjlz}\cos\phi^{(h)}_{gikx}\\ & - \sum_{\substack{g=1\\ g\neq h,}}^{Y}\;\sum_{\substack{i=1\\ i\neq j,}}^{X_g}\sum_{\substack{k=1\\ k\neq l}}^{U_{gi}}\sum_{x=1}^{L}\sqrt{\frac{P_{gik}}{2}}\,\alpha^{(h)}_{gikx}\psi^{QI}_{gikx,hjlz}\sin\phi^{(h)}_{gikx}\;;\end{aligned} \tag{9}$$

$$\begin{aligned}\gamma^{Q}_{hjlz} = & \sum_{\substack{g=1\\ g\neq h,}}^{Y}\;\sum_{\substack{i=1\\ i\neq j,}}^{X_g}\sum_{\substack{k=1\\ k\neq l}}^{U_{gi}}\sum_{x=1}^{L}\sqrt{\frac{P_{gik}}{2}}\,\alpha^{(h)}_{gikx}\psi^{IQ}_{gikx,hjlz}\sin\phi^{(h)}_{gikx}\\ & + \sum_{\substack{g=1\\ g\neq h,}}^{Y}\;\sum_{\substack{i=1\\ i\neq j,}}^{X_g}\sum_{\substack{k=1\\ k\neq l}}^{U_{gi}}\sum_{x=1}^{L}\sqrt{\frac{P_{gik}}{2}}\,\alpha^{(h)}_{gikx}\psi^{QQ}_{gikx,hjlz}\cos\phi^{(h)}_{gikx}\;;\end{aligned} \tag{10}$$

$$\chi^{I}_{hjlz} = \sum_{\substack{x=1\\ x\neq z}}^{L}\sqrt{\frac{P_{hjl}}{2}}\;\alpha^{(h)}_{hjlx}\left[\psi^{II}_{hjlx,hjlz}\cos\phi^{(h)}_{hjlx} - \psi^{QI}_{hjlx,hjlz}\sin\phi^{(h)}_{hjlx}\right]; \tag{11}$$

$$\chi^{Q}_{hjlz} = \sum_{\substack{x=1\\ x\neq z}}^{L}\sqrt{\frac{P_{hjl}}{2}}\;\alpha^{(h)}_{hjlx}\;\left[\psi^{IQ}_{hjlx,hjlz}\sin\phi^{(h)}_{hjlx} + \psi^{QQ}_{hjlx,hjlz}\cos\phi^{(h)}_{hjlx}\right]; \tag{12}$$

with

$$\psi^{H_1H_2}_{gikx,hjlz} = \int_{m_0T_{hj}}^{(m_0+1)T_{hj}} b^{H_1}_{gik}(t-\tau^{(h)}_{gikx})\; c^{H_1}_{gik}(t-\tau^{(h)}_{gikx})\; c^{H_2}_{hjl}(t)\; dt \tag{13}$$

where H_1 and H_2 can be I or Q, denoting In-phase or Quadrature-phase terms, respectively.

III. MULTIPLE ACCESS INTERFERENCE

Assuming that the multiple access interference components γ^{I}_{hjlz} and γ^{Q}_{hjlz} are Gaussian random variables, their behaviour are completely described by their mean and variance. Although this assumption is not necessarily correct because the number of interfering users is finite, Gaussian approximation has been shown to be relatively good if the number of interfering users is large so Central Limit Theorem can be invoked [5]. Therefore, we determine mean and variance of γ^{I}_{hjlz} in this section. From analogous assumptions and derivations it can be shown that the results for γ^{Q}_{hjlz} are the same.

A. *Mean*

It is easy to see from (9) that if random variables $\alpha^{(h)}_{gikx}$, $\phi^{(h)}_{gikx}$, $\psi^{II}_{gikx,hjlz}$ and $\psi^{QI}_{gikx,hjlz}$ are independent and phases $\phi^{(h)}_{gikx}$ are uniformly distributed over 0 and 2π then $\mathrm{E}\{\gamma^{I}_{hjlz}\} = 0$ because $\mathrm{E}\{\cos\phi^{(h)}_{gikx}\}$ and $\mathrm{E}\{\sin\phi^{(h)}_{gikx}\}$ are null.

B. *Variance*

The variance of γ^{I}_{hjlz} is determined from the probability distributions of $b^{I}_{gik}(t=t_0)$, $b^{Q}_{gik}(t=t_0)$, $c^{I}_{gik}(t=t_0)$, $c^{Q}_{gik}(t=t_0)$, $\tau^{(h)}_{gikx}$, $\phi^{(h)}_{gikx}$ and $\alpha^{(h)}_{gikx}$, based on the assumption that these random variables in one or various multipath components of signals from one or various users are independent.

Since phases $\phi^{(h)}_{gikx}$ are uniformly distributed over 0 and 2π, we have from (9) that

$$\begin{aligned}Var\{\gamma^{I}_{hjlz}\} = & \sum_{\substack{g=1\\ g\neq h,}}^{Y}\;\sum_{\substack{i=1\\ i\neq j,}}^{X_g}\sum_{\substack{k=1\\ k\neq l}}^{U_{gi}}\sum_{x=1}^{L}\left(P_{gik}/4\right)\mathrm{E}\left\{\left(\alpha^{(h)}_{gikx}\right)^2\right\}\mathrm{E}\left\{\left(\psi^{II}_{gikx,hjlz}\right)^2\right\}\\ & + \sum_{\substack{g=1\\ g\neq h,}}^{Y}\;\sum_{\substack{i=1\\ i\neq j,}}^{X_g}\sum_{\substack{k=1\\ k\neq l}}^{U_{gi}}\sum_{x=1}^{L}\left(P_{gik}/4\right)\mathrm{E}\left\{\left(\alpha^{(h)}_{gikx}\right)^2\right\}\mathrm{E}\left\{\left(\psi^{QI}_{gikx,hjlz}\right)^2\right\}.\end{aligned} \tag{14}$$

Then, let us evaluate $\mathrm{E}\left\{\left(\psi^{II}_{gikx,hjlz}\right)^2\right\}$. For convenience $\psi^{II}_{gikx,hjlz}$ can be written as the sum of integrals over the intervals where $b^{I}_{gik}(t-\tau^{(h)}_{gikx})$ is constant. According to Fig. 2 and (13) we get

$$\begin{aligned}\psi^{II}_{gikx,hjlz} = & \; b^{I}_{gik}(t_2 - T_{gi})\,\Re^{II}_{gikx,hjlz}\left(t_1,t_2\right)\\ & + \left(\sum_{p=0}^{H} b^{I}_{gik}(t_2 + pT_{gi})\,\Re^{II}_{gikx,hjlz}\left(t_2 + pT_{gi}, t_2 + (p+1)T_{gi}\right)\right)\\ & + b^{I}_{gik}(t_{W-1})\,\Re^{II}_{gikx,hjlz}\left(t_{W-1},t_W\right)\end{aligned} \tag{15}$$

where $H = \left(\left(t_{W-1} - t_2\right)/T_{gi}\right) - 1$ and

$$\Re^{II}_{gikx,hjlz}(t_a,t_b) = \int_{t_a}^{t_b} c^{I}_{gik}(t-\tau^{(h)}_{gikx})\,c^{I}_{hjl}(t)\,dt\;. \tag{16}$$

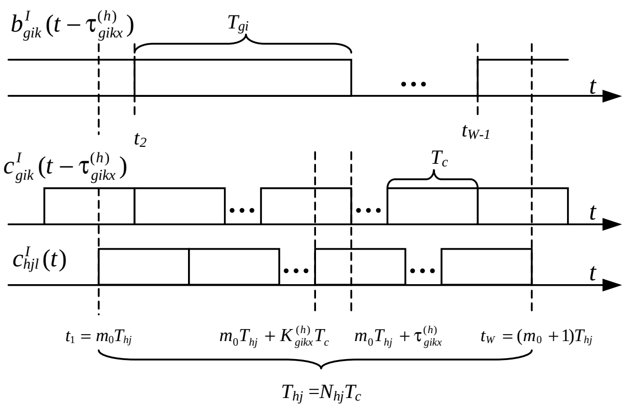

Fig. 2. Chip and symbol durations

Observe that (15) describes the three possible cases of relation between bit rates of users *gik* and *hjl*: if $T_{gi} < T_{hj}$ then $t_2 = m_0T_{hj} + \tau^{(h)}_{gikx}$ and $t_{W-1} = t_2 + \left\lfloor\left(T_{hj} - \tau^{(h)}_{gikx}\right)/T_{gi}\right\rfloor T_{gi}$; if

$T_{gi} = T_{hj}$ then $t_2 = t_{W-1} = m_0 T_{hj} + \tau^{(h)}_{gikx}$ and the second term in (15) is null; and if $T_{gi} > T_{hj}$ then $t_2 = t_{W-1} = m_0 T_{hj} + \tau^{(h)}_{gikx}$ and the second term in (15) is null, or $t_W = t_{W-1} = t_2 = (m_0 + 1) T_{hj}$ and second and third terms in (15) are null.

Assuming that $b^I_{gik}(t)$ is a stationary random process, and also that its symbols are equiprobable and zero-mean, and consecutive symbols are independent, we have from (15) that

$$\mathrm{E}\left\{\left(\psi^{II}_{gikx,hjlz}\right)^2\right\} = \mathrm{E}\left\{\left(b^I_{gik}(t)\right)^2\right\}\mathrm{E}\left\{\sum_{q=1}^{W-1}\left(\Re^{II}_{gikx,hjlz}\left(t_q, t_{q+1}\right)\right)^2\right\}. \quad (17)$$

Then, assuming $c^I_{hjl}(t)$ and $c^I_{gik}(t)$ are stationary random processes with equiprobable zero-mean chips, consecutive chips are independent, and also observing that $\Re^{II}_{gikx,hjlz}(t_q, t_{q+1})$ can be written as the sum of integrals over the intervals where both $c^I_{hjl}(t)$ and $c^I_{gik}(t - \tau^{(h)}_{gikx})$ are constant, we have

$$\mathrm{E}\left\{\sum_{q=0}^{W-1}\left(\Re^{II}_{gikx,hjlz}(t_q, t_{q+1})\right)^2\right\} = \sum_{u=0}^{N_{hj}-1}\mathrm{E}\left\{\left(\tau^{(h)}_{gikx} - K^{(h)}_{gikx} T_c\right)^2\right\} + \sum_{u=0}^{N_{hj}-1}\mathrm{E}\left\{\left(T_c - \tau^{(h)}_{gikx} + K^{(h)}_{gikx} T_c\right)^2\right\} \quad (18)$$

where $K^{(h)}_{gikx}$ is a positive constant integer that satisfies $0 \le \tau^{(h)}_{gikx} - K^{(h)}_{gikx} T_c < T_c$ and $N_{hj} = T_{hj} / T_c$ is the processing gain of subsystem *hj*, as illustrated in Fig. 2.

Finally, assuming propagation delays $\tau^{(h)}_{gikx}$ are random variables uniformly distributed over 0 and T_{gi} and observing that $\mathrm{E}\left\{\left(\tau^{(h)}_{gikx} - K^{(h)}_{gikx} T_c\right)^2 + \left(T_c - \tau^{(h)}_{gikx} + K^{(h)}_{gikx} T_c\right)^2\right\}$ corresponds to the sum of expectations on intervals from uT_c to $(u+1)T_c$ where $K^{(h)}_{gikx} = uT_c$ we obtain

$$\mathrm{E}\left\{\left(\tau^{(h)}_{gikx} - K^{(h)}_{gikx} T_c\right)^2 + \left(T_c - \tau^{(h)}_{gikx} + K^{(h)}_{gikx} T_c\right)^2\right\} = 2T_c^2 / 3. \quad (19)$$

From (17), (18) and (19) we determine $\mathrm{E}\left\{\left(\psi^{II}_{gikx,hjlz}\right)^2\right\} = 2 N_{hj} T_c^2 \mathrm{E}\left\{\left(b^I_{gik}(t)\right)^2\right\}/3$ and from analogous assumptions and derivations it can be shown that $\mathrm{E}\left\{\left(\psi^{QI}_{gikx,hjlz}\right)^2\right\} = 2 N_{hj} T_c^2 \mathrm{E}\left\{\left(b^Q_{gik}(t)\right)^2\right\}/3$. Thus, from (14) we get

$$Var\left\{\gamma^I_{hjlz}\right\} = S^{(hjl)}_{MAI} T_{hj} T_c / 6 \quad (20)$$

where $S^{(hjl)}_{MAI}$ is the total average interference power from all interfering users on receiver *hjl*, which is discussed as follows.

C. *Interference Power*

Consider a path loss model where the relation between average transmitted power and average received power is inversely proportional to the γ-th power of the distance between transmitter and receiver, where γ depends on the cellular environment. The power on the base station in cell *h* received from all U_{gi} users of subsystem *i* in a cell *g* is given by

$$S^{(h)}_{gi} = \sum_{k=1}^{U_{gi}} S^{(g)}_{gik}\left(\frac{y^{(g)}_{gik}}{\sqrt{\left(y^{(g)}_{gik}\right)^2 + \left(d_{g,h}\right)^2 + 2\, d_{g,h}\, y^{(g)}_{gik} \cos\varphi^{(g)}_{gik}}}\right)^{\gamma} \quad (21)$$

where $y^{(g)}_{gik}$, $\varphi^{(g)}_{gik}$ and $d_{g,h}$ are defined in Fig. 3, and $S^{(g)}_{gik}$ is the power on cell *g* received from an user *gik*, which must be the same for all U_{gi} users in subsystem *gi* since they have to support the same application requirements.

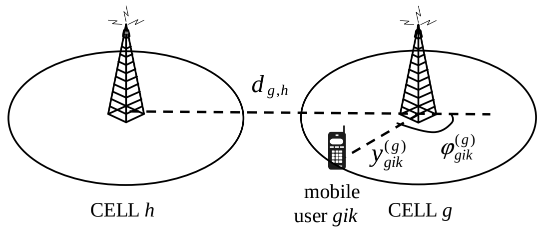

Fig. 3. Interference to neighbour cell

As positions of users in cell *g* is seldom known due to their mobility, it is convenient to define $S^{(h)}_{gi}$ as the sum of powers from dU_{gi} users of subsystem *gi*, which are in a region with area *dA* in cell *g*, that is to say, $S^{(h)}_{gi} = S^{(g)}_{gi}\ w^{(h)}_{gi}$, where $S^{(g)}_{gi}$ is the power on cell *g* received from all users of subsystem *gi*;

$$w^{(h)}_{gi} = \frac{1}{U_{gi}} \int_{\substack{cell\ g \\ area}} \left(\frac{y^{(g)}_{gik}}{\sqrt{\left(y^{(g)}_{gik}\right)^2 + \left(d_{g,h}\right)^2 + 2\, d_{g,h}\, y^{(g)}_{gik} \cos\varphi^{(g)}_{gik}}}\right)^{\gamma} \rho_{gi}\, dA \quad (22)$$

is a function that characterizes how the total power from users of subsystem *gi* interfers to a neighbour cell *h*; and $\rho_{gi} = dU_{gi} / dA$ is the user density in subsystem *gi*. If a cell *g* is circular with radius λ_g, its base station is located at its centre, and the U_{gi} users in subsytem *gi* are uniformly distributed over its area, then $\rho_{gi} = U_{gi} / \pi\lambda_g^2$. Assuming γ = 4, we get from (22) that [6]

$$w^{(h)}_{gi} = \frac{4 d^2_{g,h}}{\lambda^2_g} \ln\left(\frac{d^2_{g,h}}{d^2_{g,h} - \lambda^2_g}\right) - \frac{4 d^4_{g,h} - 6 d^2_{g,h}\lambda^2_g + \lambda^4_g}{\left(d^2_{g,h} - \lambda^2_g\right)^2}. \quad (23)$$

Defining

$$w^{(h)}_g = \left(\sum_{i=1}^{X_g} S^{(g)}_{gi}\ w^{(h)}_{gi}\right) \Big/ \sum_{i=1}^{X_g} S^{(g)}_{gi} \quad (24)$$

as a function that characterizes how the total power from all users of cell *g* interfers to a neighbour cell *h*, we see from (23) that $w^{(h)}_g = w^{(h)}_{gi}$ if all subsystems in cell *g* have uniform user distribution over the cell because in this case $w_{gi}^{(h)}$ depends only on λ_g and $d_{g,h}$. Evaluating $w_g^{(h)}$ for all neighbour interferent cells from (23) based on the cellular system architecture and geometry, we can determine the total multiple access interference power on receiver *hjl* from

$$S_{MAI}^{(hjl)} = \left(\sum_{g=1}^{Y} S_g^{(g)} w_g^{(h)} \right) - S_{hjl}^{(h)} \qquad (25)$$

where $S_g^{(g)}$ is the power on cell *g* received from all multipath components of signals from all users of cell *g*.

A cellular architecture based on hexagonal geometry is described in Fig. 4. All cells have the same radius λ and are grouped in rings so that the number of cells in ring-*n* is 6*n* and the total number of rings is *G*. Base stations are assumed to be at the centre of each cell so that the distance between base stations of cell *h* and the cell at ring-*n* and coordinate *m* is given by $2\lambda\sqrt{n^2+m^2-nm}$. In this case, the total multiple access interference power $S_{MAI}^{(hjl)}$ on multi-rate receiver *hjl* is given by

$$S_{MAI}^{(hjl)} = S_g^{(g)} \left(1 + \sum_{n=1}^{G} \sum_{m=1}^{n} 6 w_g^{(h)} \right) - S_{hjl}^{(h)} \,. \qquad (26)$$

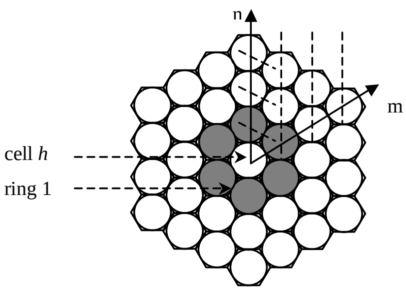

Fig. 4. Hexagonal cellular geometry

IV. MULTIPATH INTERFERENCE

Assuming from Central Limit Theorem that multipath interference components χ_{hjlz}^I e χ_{hjlz}^Q defined in (11) and (12) are Gaussian random variables, their behaviour are completely characterized by their mean and variance. Moreover, if random variables $b_{hjl}^I(t=t_0)$, $b_{hjl}^Q(t=t_0)$, $c_{hjl}^I(t=t_0)$, $c_{hjl}^Q(t=t_0)$, $\tau_{hjlx}^{(h)}$, $\phi_{hjlx}^{(h)}$ and $\alpha_{hjlx}^{(h)}$ in one or various multipath components of signal from user *hjl* are independent; information signals $b_{hjl}^I(t)$ and $b_{hjl}^Q(t)$ are stationary random processes with equiprobable zero-mean symbols; consecutive symbols are independent; spreading codes $c_{hjl}^I(t)$ and $c_{hjl}^Q(t)$ are stationary random processes with equiprobable zero-mean chips; consecutive chips are independent; attenuation factors $\alpha_{hjlx}^{(h)}$ are Rayleigh-distributed variables; propagation delays $\tau_{hjlx}^{(h)}$ are uniformly distributed over 0 and T_{hj}; and phases $\phi_{hjlx}^{(h)}$ are uniformly distributed over 0 and 2π; then we can determine analogously to what was done in the previous section for multiple access interference that $E\{\chi_{hjlz}^I\}$ and $E\{\chi_{hjlz}^Q\}$ are null and

$$Var\{\chi_{hjlz}^I\} = Var\{\chi_{hjlz}^Q\} = S_{MPI}^{(hjlz)} T_{hj} T_c / 6 \qquad (27)$$

where $S_{MPI}^{(hjlz)}$ is the average interference power on the receiver matched to the *z*-th multipath component of signal $s_{hjl}(t)$ which results from the other *L*–1 multipath components of this signal. As the average power of each multipath component, denoted by $S_{hjlx}^{(h)}$, are the same, we have $S_{MPI}^{(hjlz)} = (L-1)S_{hjlx}^{(h)}$.

V. BIT ERROR PROBABILITY

The bit detection errors at a multi-rate receiver *hjl* matched to the *z*-th multipath component of signal $s_{hjl}(t)$ occur when a decision variable $\hat{A}_{hjlz}$ is not mapped into the correct decision region D_y that corresponds to the transmitted symbol $Z_y = Z_y^I + jZ_y^Q$. Thus, the bit error probability at multi-rate receiver *hjl* when no diversity technique is used is given by

$$P_{B,hjl} = \sum_{y=1}^{M_{hj}} P\left(A_{hjl} = Z_y\right) \sum_{\substack{x=1 \\ x \neq y}}^{M_{hj}} \frac{n(x,y)}{\log_2 M_{hj}} P\left(\hat{A}_{hjlz} \in D_x \middle| A_{hjl} = Z_y\right) \quad (28)$$

where $n(x,y)$ is the number of different bits between the $\log_2 M_{hj}$-bit sequences corresponding to symbols Z_x and Z_y.

Consider QPSK modulation (M_{hj} = 4) described in Fig. 5 and assume the difference between adjacent symbols is one bit and the difference between opposite symbols is two bits. Then, for equiprobable symbols, the optimum detection solution is achieved when the decision boundaries μ^I and μ^Q are zero.

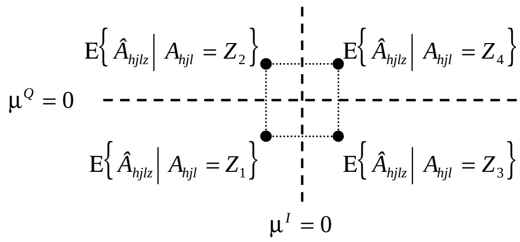

Fig. 5. Decision regions in QPSK modulation

As η_{hjlz}^I, η_{hjlz}^Q, γ_{hjlz}^I, γ_{hjlz}^Q, χ_{hjlz}^I and χ_{hjlz}^Q are independent Gaussian random variables, decision variables $\hat{A}_{hjlz}^I$ and $\hat{A}_{hjlz}^Q$

defined in (3) and (4) are also Gaussian random variables and due to symmetry we get from (28) that

$$P_{B,hjl} = Q\left(\frac{\mathrm{E}\left\{\hat{A}^{I}_{hjlz}\middle| A^{I}_{hjl} = Z^{I}_{4}\right\}}{\sqrt{Var\left\{\hat{A}^{I}_{hjlz}\right\}}}\right) \qquad (29)$$

where from (3), (7), (20) and (27) we have

$$\mathrm{E}\left\{\hat{A}^{I}_{hjlz}\middle| A^{I}_{hjl} = Z^{I}_{4}\right\} = \sqrt{P_{hjl}/2}\ \alpha^{(h)}_{hjlz}\ Z^{I}_{4}\ T_{hj} \qquad (30)$$

and

$$Var\left\{\hat{A}^{I}_{hjlz}\right\} = \left(N_0 T_{hj}/4\right) + \left(\left(S^{(hjl)}_{MAI} + S^{(hjlz)}_{MPI}\right) T_{hj} T_c/6\right). \qquad (31)$$

Since attenuation factors $\alpha^{(h)}_{gikx}$ are Rayleigh-distributed random variables, bit error probability is obtained averaging (29) over Rayleigh probability density function. Hence, the bit error probability at receiver ***hjl*** is given by

$$P_{B,hjl} = \frac{1}{2}\left(1 - \sqrt{\frac{\mathrm{E}\left\{\lambda^{(h)}_{hjlz}\right\}}{2 + \mathrm{E}\left\{\lambda^{(h)}_{hjlz}\right\}}}\right) \qquad (32)$$

with

$$\mathrm{E}\left\{\lambda^{(h)}_{hjlz}\right\} = \left(\frac{1}{2\,(SNR)_{hjlz}} + \frac{T_c\left(S^{(hjl)}_{MAI} + S^{(hjlz)}_{MPI}\right)}{3\,N_0\,(SNR)_{hjlz}}\right)^{-1} \qquad (33)$$

where $(SNR)_{hjlz} = S^{(h)}_{hjlz}/R_{hj}N_0$ is the received signal-to-noise ratio per bit per path of the signal of user *hjl*.

VI. Numerical Results

In this section we present some results that illustrate bit error probability in multi-rate systems derived in (32).

Consider the multi-rate cellular system proposed on previous sections and assume that all cells of the system have the same power $S^{(g)}_{g}$ resulting from the several multipath components from its several users who develop various applications. If a new user ***hjl*** transmitting with bit rate R_{hj} is introduced in cell ***h***, his bit error rate $P_{B,hjl}$ can be evaluated from (32) as a function of his received signal-to-noise ratio per bit per path $(SNR)_{hjlz}$, as shown in Fig. 6. Multiple access interference power $S^{(hjl)}_{MAI}$ is assumed fixed and given by (26), with $w^{(h)}_{g}$ given by (23), *G*=5, and the values of $S^{(g)}_{g}/N_0 = S/N_0$ defined in Fig. 6. Multipath interference power increases as signal-to-noise ratio per bit per path increases, according to $S^{(hjlz)}_{MPI} = (L-1)(SNR)_{hjlz} N_0 R_{hj}$.

Observe that an user needs more energy to support a performance requirement when multiple access interference is higher. It should also be noted that bad performance presented in Fig. 6 shows the importance of the use of diversity techniques at multi-rate receivers.

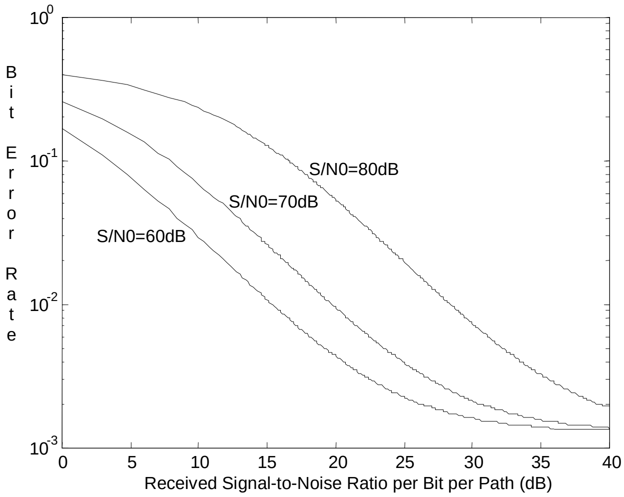

Fig. 6. User performance with fixed multiple access interference, *L*=3, R_{hj}=14400 bps and T_c = 2 x (512 x 14400)$^{-1}$s

VII. Conclusions

We derived a bit error probability expression for multi-rate DS-WCDMA cellular system in frequency-selective Rayleigh channels with slow fading, assuming random spreading codes and Gaussian approximation for multiple access interference and multipath interference. A mathematical model that describes interference from neighbour cells as a function of path loss model, user distributions over the cells and cellular architecture was also presented. At last, some numerical results illustrated analytical results obtained and helped to show the importance of the use of diversity techniques at multi-rate receivers. Models and expressions proposed have proved to be very convenient to the investigation of multi-rate systems and can be useful in the study of more complex systems.

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