Adaptive Regularization for Multiframes Resolution Restoration Considering Subpixel Contributions

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***Abstract*—In this work we propose an adaptive spatial resolution restoration algorithm for sequence of images. The Regularized Least-Squares (RLS) algorithm is modified to include an adaptive regularization that considers the subpixel contribution of each frame. This regularization is used to mitigate the distortions caused by the sub-sampling process. The contribution from additional frames is exploited according to its subpixel displacements. The pixels amplitudes from other frames, displaced by subpixel distances, provide the information to minimize sub-sampling distortions. The information about the displacements is used to control the adaptation. The regularization adapts in both ways: spatial and directional. In the motion estimation step only the reliable displacement vectors are chosen for the restoration process. The proposed model significantly improves the objective (SNR) and the subjective (visual) image quality.**

***Index Terms*—resolution restoration, interpolation, adaptive regularization, multiframe restoration, subpixel displacement.**

I. INTRODUCTION

The image resolution amplification has many important applications: scientific and medical imaging, satellite and aerial photography, astronomy and military use. However, image acquisition systems have an imaging resolution limit. Improving the spatial resolution through the use of a denser photo sensor, better lens, or more precise focus system, increase substantially the system cost [1,2]. A possible approach to solve this problem is by using multiframe resolution restoration, also known as super-resolution or resolution enhancement.

Most of the interpolation techniques used in spatial resolution amplification do not exploit the degradation model of the image acquisition device and the differential information among the frames in the sequence. The conventional interpolation methods [1,2], like the bilinear, the bicubic interpolation and high-order splines, are considered basic operations and only expand the low-resolution (LR) image without correcting the degradation. It is possible to achieve better results with the use of resolution restoration algorithms which consider the mixture of the pixels in the image sensor, motion and out-of-focus blur, aliasing in the sub-sampling process and noise from various sources. Also, the resolution restoration techniques based on the use of multiple frames can recover the components lost in the sub-sampling process, assuming that each frame in the sequence can contribute with new information about the high-resolution (HR) image [1,3,4,6].

In this work we estimate the relative motion among the frames and detects the outliers in the sequence [4,5]. The outliers are: i) regions that suffer complex movement and does not produce an apparent motion that can be well represented by the chosen motion model, ii) parts of objects that moves outside the image borders, iii) regions covered or exposed by the motion of other object. Due to the presence of outliers, not all the information provided by other frames can be used in the resolution restoration. The outlier regions as well as the unreliable motion vectors are not used [4].

Moreover, we achieve superior results using adaptive regularization. The adaptation depends on the subpixel contributions of the additional frames. The adaptive regularization can preserve the recovered details in the regions that received significantly subpixel contributions and properly mitigate the sub-sampling effects in regions that received little, or none, contributions. This additional adaptive regularization considering subpixel contributions has not been considered in previous works. This innovative approach can significantly improve the quality of the estimated images.

II. DEGRADATION MODEL

The degradation model can be represented as: i) One-Frame, which refers to the degradation of the same frame; ii) Additional Frames, which refers to the degradation related to other frames in the sequence.

A. One-frame model

The acquired image can be represented by the equation (1). The high-resolution image f_m suffers a degradation that includes: an optical degradation H_m, a sub-sampling process S_m, and an additive noise η_m.

$$g_m = S_m H_m f_m + \eta_m \tag{1}$$

Where f_m is a vector, with size $M_1M_2\times1$, that represents a digital HR image $f_m[m_1,m_2]$, with dimensions $M_1\times M_2$, lexicographically ordered. H_m is a matrix, with size $M_1M_2\times M_1M_2$, which represents the optical flux degradations like motion blur, out-of-focus blur and pixels mixture. S_m is a matrix of size $N_1N_2\times M_1M_2$ that represents the sub-sampling

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process in the photo sensor. g_m is a vector, with size $N_1N_2\times1$, which represents the LR image $g_m[n_1,n_2]$. The vector η_m represents the noise. The resolution ratio between the HR and the LR images is: $R^2=M_1M_2/N_1N_2$, where R is the amplification factor.

The equation (1) can be rewritten as:

$$g_m = D_m f_m + \eta_m \quad (2)$$

Where D_m is a matrix of size $N_1N_2\times M_1M_2$, and $S_mH_m=D_m$.

B. *Additional frames*

When additional frames are utilized, it is necessary to use a motion compensation operation. The operation is:

$$f_k = A_{k,m} f_m \quad (3)$$

Where f_k is another HR image of the sequence.

However, due to the existence of outliers, not all the image can be used. Then, the outliers, as well as the unreliable motion vectors, are removed. Only the useful regions are used, according to:

$$f_k^I = A_{k,m}^I f_m \quad (4)$$

Where f_k^I represent the useful pixels, they are named inliers. The f_m image can be related with the other LR frames through:

$$g_k^I = S_k H_k A_{k,m}^I f_m + \eta_k^I \quad (5)$$

In the vectors f_k^I, g_k^I and η_k^I, the pixels that correspond to outliers are set to zero. The degradation can also be represented by:

$$g_k^I = D_{k,m}^I f_m + \eta_k^I \quad (6)$$

Where $S_kH_kA^I_{k,m}=D^I_{k,m}$.

The motion among the frames and the outliers need to be estimated with an appropriated technique. [1,4,5].

III. INVERSE SOLUTION

A solution to f_m can be found either using only the correspondent LR frame g_m or using all the frames.

A. *One-frame inverse solution*

In this case, the linear system (1) has to be solved. However, the system is underdetermined, among other sources of ill-conditioning [1,7]. The sub-sampling process is the main source of the underdetermination. According to [7], it is required additional information about the solution in order to recover the image. The solution through the Regularized Least-Squares [8] method is adequate, because it is possible to use a constraint as additional information. The equation (7) presents the estimated solution:

$$\hat{f}_m = (D_m^T D_m + \alpha G^T G)^{-1} D_m^T g_m \quad (7)$$

Where G is a constraint operator, which is a filter chosen to mitigate the sub-sampling effects and to obtain a smooth solution of f_m. The term α is the regularization factor. The solution (7), in some cases, is computationally heavy to be solved directly and is usually carried out by an iterative method, as in [8]. The iterative method is shown in equation (8), where (n) is the current iteration:

$$f_m^{(n+1)} = f_m^{(n)} + \beta(D_m^T g_m - (D_m^T D_m + \alpha G^T G) f_m^{(n)}) \quad (8)$$

Where β is the relaxation factor, and it has to guarantee the convergence and the convergence rate [8].

B. *Multiframe inverse solution*

The use of multiple frames is advantageous. If the estimated motion, in any additional frame, is at subpixel displacement the additional frame contributes with new information about the details in the HR image.

In order to use the additional frames, a new linear system is built as:

$$\begin{bmatrix} g_1^I \\ \hline \varepsilon_1 \\ \vdots \\ g_L^I \\ \hline \varepsilon_L \end{bmatrix} = \begin{bmatrix} D_{1,m}^I \\ \hline \varepsilon_1 \\ \vdots \\ D_{L,m}^I \\ \hline \varepsilon_L \end{bmatrix} f_m + \begin{bmatrix} \eta_1^I \\ \hline \varepsilon_1 \\ \vdots \\ \eta_L^I \\ \hline \varepsilon_1 \end{bmatrix}, \text{or} \quad \mathbf{g} = \mathbf{D}_m f_m + \boldsymbol{\eta} \quad (9)$$

In this new system, the image f_m is related to L low-resolution frames, including the frame g_m and the additional frames g^I_k. The vector $\mathbf{g}$ has size $LN_1N_2\times1$ and the matrix $\mathbf{D}_m$ has size $LN_1N_2\times M_1M_2$. The factors ε_k represent a weight applied to each frame due to any motion estimation error [3].

The solution for the system (9) can be achieved by the use of Regularized Least-Squares:

$$\hat{f}_m = (\mathbf{D}_m^T \mathbf{D}_m + \lambda C^T C)^{-1} \mathbf{D}_m^T \mathbf{g} \quad (10)$$

In this case the system may not be underdetermined, or it can even be overdetermined. Then, the constraint C must avoid noise amplification. It also preferable that the solution of (10) is found by iteration, according to:

$$f_m^{(n+1)} = f_m^{(n)} + \beta(\mathbf{D}_m^T \mathbf{g} - (\mathbf{D}_m^T \mathbf{D}_m + \lambda C^T C) f_m^{(n)}) \quad (11)$$

In this situation, it is considered that the problems caused by the sub-sampling process are completely solved by subpixel contributions. In general, a discrete Laplacian operator is used as the constraint C, as in [6]. The same constraint is also used in restoration problems that do not involve the sub-sampling process, as in [8].

IV. ADAPTIVE REGULARIZATION

When the additional frames provide the complete solution to the problems caused by the sub-sampling process, the equation (10) is adequate. However, in many situations the additional frames do not provide enough contributions and the resulting images still has the distortions caused by the sub-sampling process. Usually, the additional frames are able to provide only partial contributions to the restoration of the details in the HR estimation. The regions that did not receive contributions from frames with subpixel displacement still have distortions caused by the sub-sampling process and have to be regularized. The Figure 1 illustrates an example of the sub-sampling distortions in a region without contribution. In this example the region that contains outliers where removed and do not contribute to the estimated image.

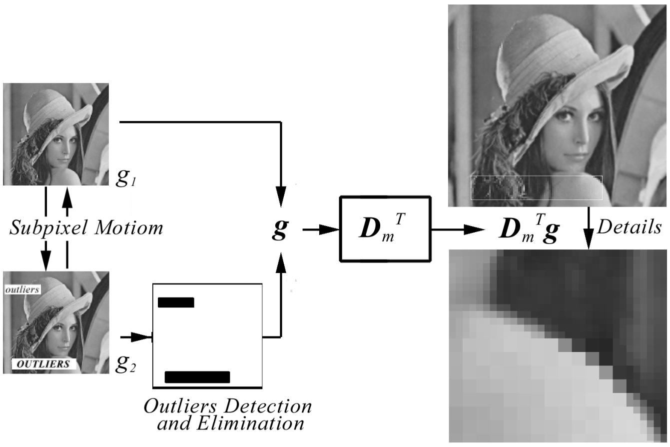
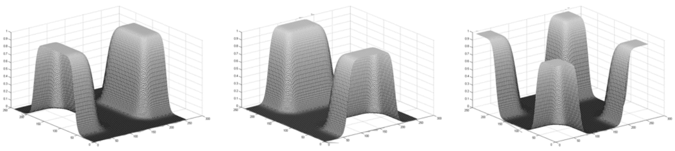


Figure 1: Example of distortions caused by the sub-sampling process.

This work proposes an adaptive regularization of the distortions caused by the sub-sampling process. This way, the regions that have more subpixel contribution from the additional frames receive less influence of the constraint. On the other hand, the regions that had less, or none, subpixel contributions, receive more influence of the constraint. The regularization also considers the direction of the subpixel contribution, since it depends on the direction of the subpixel displacement. The proposed solution is:

$$\hat{f}_m = \left(\mathbf{D}_m^T \mathbf{D}_m + \lambda C^T C + \alpha \sum_{i=1}^{P} W_i G_i^T G_i \right)^{-1} \mathbf{D}_m^T \mathbf{g} \qquad (12)$$

Where the G operator used to smooth the distortions caused by the sub-sampling process in (7) is divided in P filters. In an amplification factor of two (R=2) it is required P=3, where G_i can be: horizontal, vertical and diagonal filters. The filters for this case are illustrated in Figure 2.



(a) Horizontal (b) Vertical (c) Diagonal

Figure 2: Example of the filters used in the adaptive regularization.

The quantity of filters is related with the modulated base band components that appear in the Fourier spectrum during the sub-sampling process [1]. These components are not completely eliminated without subpixel contributions or proper regularization, and causes visible distortions in the restored image. The W_i matrix is a diagonal matrix where each value is the weight of the correspondent G_i filter in a specific pixel. Therefore, the regularization is spatially and directionally adaptive. The weights are chosen according to the subpixel contribution received in each region. If the additional frames provide enough subpixel contributions to cancel the effects of the sub-sampling process, all the weights are set to zero. If there were no contribution in the whole image (one-frame case) all the weights are set to one. The C operator is kept due to existence of noise. Figure 3 demonstrates a view of the necessary steps previews to the restoration. It illustrates that the regularization operators C and G_i, i=1…P, are chosen according to the image acquisition system and W_i, i=1…P, are chosen according to the sub-pixel motion analysis.

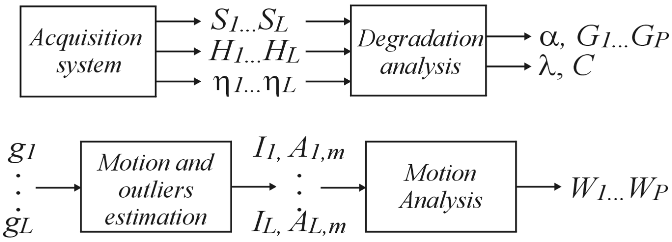


Figure 3: Steps previews to restoration.

A more detailed analysis about this adaptive regularization can be found in [9].

V. EXPERIMENTAL RESULTS

In order to evaluate the performance of the technique, the algorithm is used in an artificial and in a real degradation case.

A. *Artificial degradation*

The degradation is applied to a known HR sequence, followed by the restoration. A sequence composed by two frames is degraded by an optical degradation of a moving average 2×2 filter and a rectangular sub-sampling of factor R=2. To emphasize the need for regularization of the distortions caused by the sub-sampling process, only 3 dB of noise is added. This degradation is similar to the CCD degradation in amplifications of factor of R=2, [2]. The degraded sequence and the regions estimated as outliers are presented in Figure 4.

The goal is to improve the resolution of the Image 1 in the Figure 4. The proposed solution by equation (12) is compared to the solutions from equation (13) and the conventional interpolation methods. The Non-Adaptive Regularized Least-Squares, considering the same constraints is:

$$\hat{f}_m = \left(\mathbf{D}_m^T \mathbf{D}_m + \lambda C^T C + \alpha G^T G \right)^{-1} \mathbf{D}_m^T \mathbf{g} \qquad (13)$$

When the regularization factors, α and λ, are set to zero the equation becomes equivalent to the Least-Squares (LS) solution. The C operator is a discrete Laplacian operator and each G_i is a transformed version of the Bicubic low pass filter [1,8]. The Bicubic filter is preferable due to the better selectivity. Each G_i is high frequency transformed at horizontal, vertical, and both (diagonal) axis. The cutoff digital frequency is 1/R. The G_i's are the same shown in Figure 2.

The Table 1 shows the numerical results calculated according to equation (14).

$$PSNR = 10 * Log_{10}\left(\frac{255^2}{\frac{1}{M_1 M_2}\left\|f - f_e\right\|^2}\right) \quad (dB)$$
$$DSNR = 10 * Log_{10}\left(\frac{\left\|f - f_0\right\|^2}{\left\|f - f_e\right\|^2}\right) \quad (dB) \qquad (14)$$

Where f is the original image, f_e is the estimated image and f_0 is the zero order interpolated image.

The results in Table 1 show the importance of the regularization to produce better results. It can be seen comparing the values of the multiframe LS solution and the one-frame RLS. The Figure 5 shows some visual results presented in Table 1. The Figures 5a and 5b illustrate that the resolution restoration can recover optical degradations while conventional interpolation method cannot. Figure 5c, where no regularization was used, illustrates the distortions caused by the sub-sampling process in the region where there is no subpixel contribution from the additional frame due to the outliers (the car and the borders). It illustrates why the regularization process needs to adapt according to the contribution in the region. The solution using adaptive regularization is presented in Figure 5d. Only a small area of 180×120 of the experiment results are illustrated in Figure 5.

B. *Real degradation*

To demonstrate the application in real cases the technique is used in an image sequence captured by a commercial digital camera. Five frames are used in the restoration process. The degradation model assumed is the same of the last section. Figure 6 demonstrates a visual comparison of the results with another acquisition taken from a closer distance (better spatial resolution).

VI. CONCLUSION

In this work an adaptive regularization for the distortions caused by the sub-sampling process is proposed. This regularization provides superior results in the multiframe RLS resolution restoration method when the information provided by the sequence is not enough to cancel the effects of the sub-sampling process completely. The adaptive regularization provides the necessary smoothness in regions of low contribution and preserves details in regions of high contribution. Numerical and visual results illustrate the improvement achieved.

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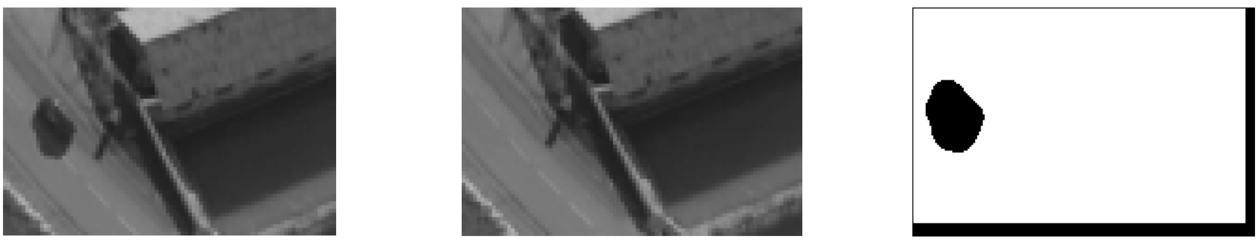
TABLE 1: NUMERICAL SNR VALUES.

AMPLIFICATION METHOD:		PSNR	DSNR	AMPLIFICATION METHOD:		PSNR	DSNR
ONE-FRAME	BILINEAR	36,63	1,88	MULTIFRAME	LS (λ=0.0, α=0.0)	40,02	5,46
	BICUBIC	38,18	3,44		NON-ADAPTIVE RLS (λ=0.01, α=0.2)	43,32	8,59
	NON-ADAPTIVE RLS (λ=0.01, α=0.2)	41,22	6,48		ADAPTIVE RLS (λ=0.01, α=0.2)	44,52	9,78



(a) Image 1 (b) Image 2 (c) Estimated outliers (black)

Figure 4: Low-resolution sequence and outliers.

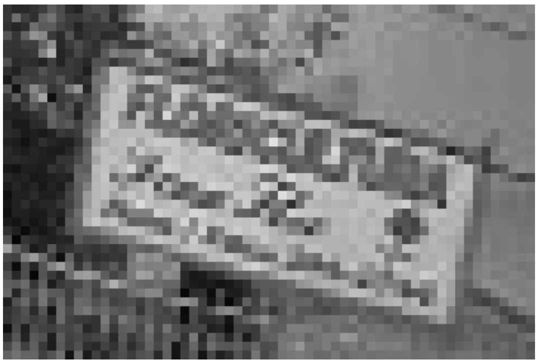
(a) Bilinear interpolation

(b) One-frame RLS

(c) Multiframe LS (no regularization)

(d) Multiframe RLS (proposed adaptive regularization)

Figure 5: Visual experimental results.



(a) One of the frames

(b) Bicubic interpolation

(c) Adaptive regularization

(d) Taken from a closer distance

Figure 6: Visual comparison in a real case.