The $\lambda - \mu$ General Fading Distribution

Gustavo Fraidenraich and Michel Daoud Yacoub

***Abstract*—This paper presents a general fading distribution, the λ–μ Distribution. The $\lambda - \mu$ Distribution includes the Hoyt, the Nakagami-m, the Rayleigh, and the One-Sided Gaussian distributions as special cases. Field measurement campaigns have been used to validate this new distributions. It has been observed that their fitting to experimental data outperforms that provided by the widely known fading distributions such as Rayleigh, Nakagami-m, and Weibull.**

***Index Terms*—fading distributions, Nakagami distribution, Rayleigh distribution, Rice distribution, One-sided Gaussian distribution, Hoyt distribution, Lognormal distribution.**

I. INTRODUCTION

The propagation of energy in a mobile radio environment is characterized by incident waves interacting with surface irregularities via diffraction, scattering, reflection, and absorption. The interaction of the wave with the physical structures generates a continuous distribution of partial waves [1], with these waves showing amplitudes and phases varying according to the physical properties of the surface. The propagated signal then reaches the receiver through multiple paths, and the result is a combined signal that fades rapidly, characterizing the short term fading. For surfaces assumed to be of the Gaussian random rough type, universal statistical laws can be derived in a parameterized form [1]. A great number of distributions exist that well describe the statistics of the mobile radio signal. The long term signal variation is well characterized by the Lognormal distribution whereas the short term signal variation is described by several other distributions such as Rayleigh, Rice, Nakagami-m, and Weibull. Among the short term distributions, the Nakagami-m distribution has been given a special attention for its ease of manipulation and wide range of applicability [2]. Although, in general, it has been found that the fading statistics of the mobile radio channel may well be characterized by Nakagami-m, situations are easily found for which other distributions such as Rice and Weibull yield better results [3] [4]. More importantly, situations are encountered for which no distributions seem to adequately fit experimental data, though one or another, may yield a moderate fitting. Some researches [4] even question the use of the Nakagami-m distribution because its tail does not seem to yield a good fitting to experimental data, better fitting being found around the mean or median. The well-known fading distributions have been derived assuming a homogeneous diffuse scattering field, resulting from randomly distributed point scatterers. With such an assumption, the central limit theorem leads to complex Gaussian processes with in-phase and quadrature Gaussian distributed variables x and y having zero means and equal standard deviations. The assumption of a homogeneous diffuse scattering field is certainly an approximation because the surfaces are spatially correlated characterizing a non-homogeneous environment [1]. More recently [5][6][7], three new fading distributions have been proposed that include or closely approximate the most common fading distributions. This paper presents a general fading distributions - the $\lambda - \mu$ Distribution. The λ-μ Distribution includes the Hoyt and the Nakagami-m distributions as special cases. Therefore, the One-Sided Gaussian and the Rayleigh distributions also constitute special cases and the Lognormal distribution may be well-approximated. Field measurement campaigns have been used to validate this new distribution. It has been observed that its fitting to experimental data outperforms that provided by the widely known fading distributions such as Rayleigh, Nakagami-m, and Weibull.

The authors are with The School of Electrical and Computer Engineering, Department of Communications University of Campinas CP 6101, 13083-970 Campinas, SP, Brazil

II. THE $\lambda - \mu$ DISTRIBUTION

The λ–μ distribution is a general fading distribution that can be used to better represent the small-scale variation of the fading signal. For a fading signal with envelope r and normalized envelope $\rho = r/\hat{r}$, $\hat{r} = \sqrt{E\left(r^2\right)}$ being the rms value of r, the $\lambda - \mu$ probability density function $p\left(\rho\right)$ is written as

$$p\left(\rho\right) = \frac{4\exp\left(\frac{2\mu\rho^2}{\lambda^2-1}\right)\mu^{\mu+\frac{1}{2}}\sqrt{\pi}\rho^{2\mu}I_{\mu-\frac{1}{2}}\left(\frac{2\mu\lambda\rho^2}{1-\lambda^2}\right)}{\lambda^{\mu-\frac{1}{2}}\sqrt{1-\lambda^2}\Gamma\left(\mu\right)} \tag{1}$$

where $\Gamma\left(.\right)$ is the Gamma function, $I_\nu\left(.\right)$ is the modified Bessel function of the first kind and arbitrary order ν (real), $\mu \geq 0$ and $0 \leq \lambda \leq 1$. For a fading signal with power $w = r^2/2$ and normalized power $\omega = w/\overline{w}$, where $\overline{w} = E\left[w\right]$, the $\lambda - \mu$ probability density function $p\left(\omega\right)$is given by

$$p\left(\omega\right) = \frac{2\mu^{\mu+\frac{1}{2}}\omega^{\mu-\frac{1}{2}}\sqrt{\pi}}{\Gamma\left[\mu\right]\lambda^{\mu-\frac{1}{2}}\sqrt{1-\lambda^2}}\exp\left[\frac{2\mu\omega}{\lambda^2-1}\right]I_{\mu-\frac{1}{2}}\left[\frac{2\mu\lambda\omega}{1-\lambda^2}\right] \tag{2}$$

In particular, we may also write $\mu = \left(1+\lambda^2\right)\frac{E^2[w]}{Var[w]}$ or equivalently $\mu = \left(1+\lambda^2\right)\frac{1}{Var[\omega]}$. The $n-th$ moment $E\left[\rho^n\right]$ of ρ is found in a closed-form formula as

$$E\left[\rho^n\right] = \frac{2^{-\frac{n}{2}}\left(1-\lambda^2\right)^{\frac{n}{2}+\mu}\mu^{-\frac{n}{2}}\Gamma\left(2\mu+\frac{n}{2}\right)}{\Gamma\left(2\mu\right)}\times {}_2F_1\left(\frac{n}{4}+\mu;\frac{2+n}{4}+\mu;\frac{1}{2}+\mu;\lambda^2\right) \tag{3}$$

where ${}_2F_1(.,.;.;.)$ is the hypergeometric function. Of course, $E\left[r^n\right] = \hat{r}^nE\left[\rho^n\right]$. The cumulative probability function is given by

$$P(\rho) = \sum_{j=0}^{\infty} \frac{2^{1-2j-2\mu}\sqrt{\pi}\lambda^{2j}\left(1-\lambda^2\right)^{\mu}}{\Gamma[\mu]\,\Gamma\left[\frac{1}{2}+j+\mu\right] j!} \times \left(\Gamma\left[2(j+\mu)\right] + \Gamma\left[2(j+\mu), \frac{2\rho^2\mu}{\lambda^2-1}\right]\right) \quad (4)$$

where $\Gamma[a,z] = \int_z^{\infty} t^{a-1}\exp(-t)\,dt$ is the incomplete Gamma function.

Figure 1, for a fixed μ ($\mu = 0.6$) and varying λ, and Figure 2, for a fixed λ ($\lambda = 0.333$) and varying μ, show the various shapes of the $\lambda-\mu$ probability density function $p(\rho)$. In Figure 1, the case in which $\lambda = 1$ ($\mu = 0.6$) coincides with that for Nakagami-m with $m = 0.6$, where m is the Nakagami parameter. Still in Figure 1, the case in which $\lambda = 0$ ($\mu = 0.6$) coincides with that for Nakagami-m with $m = 1.2$. In Figure 2, the case in which $\mu = 0.5$ ($\lambda = 0.333$) coincides with that for Hoyt.

The density $p(\theta)$ of the phase is given by

$$p(\theta) = \frac{\sqrt{1-\lambda^2}}{2\pi\left(1-\lambda\sin(2\theta)\right)} \quad (5)$$

A. Physical Model for the $\lambda-\mu$ Distribution

The fading model for the $\lambda-\mu$ Distribution considers a signal composed of clusters of multipath waves propagating in an homogeneous environment. Within any one cluster, the phases of the scattered waves are random and have similar delay times with delay-time spreads of different clusters being relatively large. The in-phase and quadrature components of the fading signal within each cluster are assumed to have zero mean and identical power but are correlated with correlation λ.

B. Derivation of the $\lambda-\mu$ Distribution

Given the physical model for the $\lambda-\mu$ Distribution, the envelope r can be written in terms of the in-phase and quadrature components of the fading signal as

$$r^2 = \sum_{i=1}^{n} r_i^2 = \sum_{i=1}^{n} x_i^2 + y_i^2 \quad (6)$$

where x_i and y_i are correlated Gaussian processes with $\mathrm{E}(x_i) = \mathrm{E}(y_i) = 0$, $\mathrm{E}\left(x_i^2\right) = \mathrm{E}\left(y_i^2\right) = \sigma^2$, and the correlation λ is given by

$$\lambda = \frac{\mu_{xy}}{\sigma_x\sigma_y} \quad (7)$$

with

$$\mu_{xy} = \mathrm{E}\left[(x-\mu_x)(y-\mu_y)\right] = \mathrm{E}[xy]\,. \quad (8)$$

That is, the in-phase and quadrature components of the multipath waves of cluster i, x_i and y_i are correlated through the parameter λ. By following a standard procedure, the density of r_i, $p(r_i)$, is found as

$$p(r_i) = \frac{r_i}{\sigma^2\sqrt{1-\lambda^2}} \exp\left(\frac{r_i^2}{2\sigma^2(\lambda^2-1)}\right) I_0\left(\frac{\lambda r_i^2}{2\sigma^2(1-\lambda^2)}\right) \quad (9)$$

It is possible to show that $\hat{r}_0^2 \triangleq E\left(r_i^2\right) = 2\sigma^2$, therefore

$$p(r_i) = \frac{2r_i}{\hat{r}_0^2\sqrt{1-\lambda^2}} \exp\left(\frac{r_i^2}{\hat{r}_0^2(\lambda^2-1)}\right) I_0\left(\frac{\lambda r_i^2}{\hat{r}_0^2(1-\lambda^2)}\right). \quad (10)$$

The density $p(w_i)$ of the power w_i is found by a simple transformation of variables ($w_i = \frac{r_i^2}{2}$) so that

$$p(w_i) = \frac{1}{\bar{w}_0\sqrt{1-\lambda^2}} \exp\left(\frac{w_i}{\bar{w}_0(\lambda^2-1)}\right) I_0\left(\frac{\lambda w_i}{\bar{w}_0(1-\lambda^2)}\right) \quad (11)$$

where $\bar{w}_0 = E[w_i] = \sigma^2$. The Laplace transform $\pounds[p(w_i)]$ of $p(w_i)$ if found in an exact manner as

$$\pounds[p(w_i)] = \frac{1}{\bar{w}_0\sqrt{(1-\lambda^2)}\sqrt{\left(s+\frac{1}{(1-\lambda)\bar{w}_0}\right)\left(s+\frac{1}{(1+\lambda)\bar{w}_0}\right)}} \quad (12)$$

Knowing that w_i , $i = 1, 2, ..., n$ are independent, the Laplace Transform $\pounds[p(w)]$ of $p(w)$ if found as

$$\pounds[p(w)] = \left(\frac{1}{\bar{w}_0\sqrt{(1-\lambda^2)}\sqrt{\left(s+\frac{1}{(1-\lambda)\bar{w}_0}\right)\left(s+\frac{1}{(1+\lambda)\bar{w}_0}\right)}}\right)^n \quad (13)$$

whose inverse is given by [8]

$$p(w) = \frac{\sqrt{\pi}}{(\bar{w}_0)^{\frac{n+1}{2}}\,\Gamma\left(\frac{n}{2}\right)\sqrt{(1-\lambda^2)}} \left(\frac{w}{2\lambda}\right)^{\frac{n-1}{2}} \times \exp\left(\frac{w}{(\lambda^2-1)\bar{w}_0}\right) I_{\frac{n-1}{2}}\left(\frac{\lambda w}{\bar{w}_0(1-\lambda^2)}\right) \quad (14)$$

We note that $\bar{w} = E(w) = n\bar{w}_0$. Therefore

$$p(w) = \frac{n^{\frac{n+1}{2}}\sqrt{\pi}}{(\bar{w})^{\frac{n+1}{2}}\,\Gamma\left(\frac{n}{2}\right)\sqrt{(1-\lambda^2)}} \left(\frac{w}{2\lambda}\right)^{\frac{n-1}{2}} \times \exp\left(\frac{nw}{(\lambda^2-1)\bar{w}}\right) I_{\frac{n-1}{2}}\left(\frac{n\lambda w}{\bar{w}(1-\lambda^2)}\right) \quad (15)$$

The corresponding density, $p(r)$, of the envelope r is found to be

$$p(r) = \frac{n^{\frac{n+1}{2}} r^n\sqrt{\pi}}{\Gamma\left(\frac{n}{2}\right)\hat{r}^{n+1}\lambda^{\frac{n-1}{2}} 2^{\frac{n-3}{2}}\sqrt{1-\lambda^2}} \times \exp\left(\frac{n}{(\lambda^2-1)}\left(\frac{r}{\hat{r}}\right)^2\right) I_{\frac{n-1}{2}}\left(\frac{n\lambda}{(1-\lambda^2)}\left(\frac{r}{\hat{r}}\right)^2\right) \quad (16)$$

It is possible to show that $E\left(r^2\right) = 2n\sigma^2$, and that $E\left(r^4\right) = 4n\sigma^4\left(1+n+\lambda^2\right)$. Therefore, $Var\left(r^2\right) = E\left(r^4\right) - E^2\left(r^2\right) = 4n\sigma^4\left(1+\lambda^2\right)$. Thus

$$\frac{E^2\left(r^2\right)}{var\left(r^2\right)} = \frac{n}{2} \times \frac{2}{(1+\lambda^2)} \quad (17)$$

Note from (17) that $n/2$ may be totally expressed in terms of physical parameters such as mean squared value of the envelope, variance of the power, and the correlation between the in-phase and quadrature components of the fading signal. Note also that whereas these physical parameters are of a continuous nature, $n/2$ is of a discrete nature (integer multiple of $1/2$). It is plausible to presume that if these parameters are to be obtained by field measurements, their ratios, as defined in (17), will certainly lead to figures that may depart from the exact $n/2$. Several reasons exist for this. One of them, probably the most meaningful one, is that, although the model proposed here is general, it is in fact an approximate solution to the so-called random phase problem, as are approximate solution to the random phase problem all the other well-known fading models. The limitation of the model can be made less stringent by defining μ as

$$\mu = \frac{E^2\left(r^2\right)}{var\left(r^2\right)} \frac{\left(1+\lambda^2\right)}{2} \tag{18}$$

μ being the real extension of $n/2$. Values of μ that differ from multiples of $1/2$ correspond to non-integer values of clusters and account for a) non-zero correlation among the clusters of multipath components; b) non-Gaussianity of the in-phase and quadrature components of the fading signal; among others. Non-integer values of clusters have been found in practice and are extensively reported in the literature. (See, for instance, [9] and the references therein) And of course, scattering occurs continuously throughout the surface and not at discrete points. (We note that in derivation of the Nakagami model [10], the parameter n, which describes the number of "component signals", therefore discrete, is also written in terms of the Nakagami continuous parameter m as $m = n/2$). It has been observed experimentally by Nakagami [10] that $\frac{E^2(r^2)}{var(r^2)} \geq \frac{1}{2}$. If this is to be kept, then we have $\frac{2\mu}{1+\lambda^2} \geq \frac{1}{2}$. On the other hand, the conditions $\mu \geq 0$ and $0 \leq \lambda \leq 1$ are less restrictive and comply with the physical meaning of the parameters. Using the definitions and the considerations as above and some algebraic manipulations, the $\lambda-\mu$ probability density function of the envelope can be written as

$$\begin{aligned} p\left(r\right) &= \frac{4(\mu)^{\mu+\frac{1}{2}}\sqrt{\pi}}{\Gamma\left(\mu\right)\hat{r}\,\lambda^{\mu-\frac{1}{2}}\sqrt{1-\lambda^2}} \left(\frac{r}{\hat{r}}\right)^{2\mu} \times \\ & \exp\left(\frac{2\mu}{(\lambda^2-1)}\left(\frac{r}{\hat{r}}\right)^2\right) I_{\mu-\frac{1}{2}}\left(\frac{2\mu\lambda}{(1-\lambda^2)}\left(\frac{r}{\hat{r}}\right)^2\right) \end{aligned} \tag{19}$$

In the same way, the probability density function of the power is obtained as

$$\begin{aligned} p\left(w\right) &= \frac{2\left(\mu\right)^{\mu+\frac{1}{2}}\sqrt{\pi}}{\bar{w}\,\Gamma\left(\mu\right)\lambda^{\mu-\frac{1}{2}}\sqrt{(1-\lambda^2)}} \left(\frac{w}{\bar{w}}\right)^{\mu-\frac{1}{2}} \times \\ & \exp\left(\frac{2\mu w}{(\lambda^2-1)\,\bar{w}}\right) I_{\mu-\frac{1}{2}}\left(\frac{2\mu\lambda w}{\bar{w}\,(1-\lambda^2)}\right) \end{aligned} \tag{20}$$

Equations (19) and (20) in their normalized forms are respectively given by (1) and (2).

The derivation of $p\left(\theta\right)$ is omitted here for brevity.

C. Estimating the parameters λ and μ

The parameters λ and μ can be estimated through the following relations

$$E[\rho^4] = \frac{2\mu+1+\lambda^2}{2\mu} \tag{21}$$

$$E\left[\rho^6\right] = \frac{\left(1+\mu\right)\left(1+3\lambda^2+2\mu\right)}{2\mu^2} \tag{22}$$

which can be solved to yield

$$\mu_1 = \frac{-3+3E[\rho^4]-\sqrt{1-6E[\rho^4]+9E^2[\rho^4]-4E[\rho^6]}}{4-6E[\rho^4]+2E[\rho^6]} \tag{23}$$

and

$$\mu_2 = \frac{-3+3E[\rho^4]+\sqrt{1-6E[\rho^4]+9E^2[\rho^4]-4E[\rho^6]}}{4-6E[\rho^4]+2E[\rho^6]} \tag{24}$$

with λ calculated through

$$\lambda_i = \sqrt{2\mu_i(E[\rho^4]-1)-1} \quad i=1,2 \tag{25}$$

Notice that there are two possible pairs, i.e, (λ_1, μ_1) and (λ_2, μ_2) which are solutions to (21, 22). Such an ambiguity can be resolved by estimating the mean phase of the distribution. In such case

$$E\left[\theta\right] = \frac{1}{4}\left[\pi + 2\,\arctan\left(\sqrt{\frac{1-\lambda^2}{\lambda^2}}\right)\right] \tag{26}$$

which yields

$$\lambda = \frac{1}{\sqrt{1+\left(\tan\left(\frac{4E[\theta]-\pi}{2}\right)\right)^2}}. \tag{27}$$

Therefore, from (21)

$$\mu = \frac{1+\lambda^2}{2\left(E\left[\rho^4\right]-1\right)} \tag{28}$$

D. The $\lambda-\mu$ Distribution and the Other Fading Distributions

The $\lambda-\mu$ Distribution is a general fading distribution that includes the Hoyt, the One-Sided Gaussian, the Rayleigh, and, more generally, the Nakagami-m distributions as special cases. Rice and Lognormal distributions may also be well-approximated by the $\lambda-\mu$ Distribution.

1) Hoyt, One-Side Gaussian, and Rayleigh: The Hoyt distribution can be obtained from the λ–μ Distribution in an exact manner by setting $\mu = 0.5$. From the Hoyt distribution the One-Sided Gaussian is obtained for $\lambda \to 1$. In the same way, from the Hoyt distribution the Rayleigh distribution is obtained in an exact manner for $\lambda \to 0$.

2) Nakagami-m, Rayleigh, and One-Sided Gaussian: The Nakagami-m distribution can be obtained in an exact manner from the $\lambda-\mu$ Distribution for $\mu = m/2$ and $\lambda \to 0$ or, in the same way, for $\mu = m$ and $\lambda \to 1$, as shown in the Appendix A. Through the Nakagami-m distribution the One-Sided Gaussian and the Rayleigh distributions can also be achieved. Therefore, the One-Sided Gaussian is obtained by setting $\lambda \to 0$ and $\mu = 0.25$, or equivalently by setting $\lambda \to 1$ and $\mu = 0.5$. In the same way, the Rayleigh distribution is obtained by setting $\lambda \to 0$ and $\mu = 0.5$, or equivalently by setting $\lambda \to 1$ and $\mu = 1$. Similarly, the Rice distribution can be approximated by setting $\lambda \to 0$ and $2\mu = (1+\kappa)^2/\left(1+2\kappa\right)$, or equivalently by setting $\lambda \to 1$ and $\mu = \left(1+\kappa\right)^2/\left(1+2\kappa\right)$. The Lognormal distribution, given as a function of m [10], can also be approximated by the $\lambda-\mu$ Distribution for $e^{-1} \leq \rho \leq e$, and for $\mu = m/2$ and $\lambda \to 0$, and for $\mu = m$ and $\lambda \to 1$.

E. Application of the $\lambda - \mu$ Distribution

The $\lambda - \mu$ Distribution, as implied in its name, is based on two parameters, λ and μ. It is possible to find estimators for both parameters. On the other hand, such a distribution has been successfully used to fit experimental data, and the procedure employed is rather similar to that utilized for other distributions, as explained next. From (18), it can be seen that the two parameters can be expressed in terms of the normalized variance of the power of the fading signal, which is usually defined as

$$m = \frac{2\mu}{1+\lambda^2} \tag{29}$$

For a given m, the parameters λ and μ are chosen that yield the best fitting. Note, on the other hand, that, for a given m, the parameter μ shall lie within the range $m/2$ and m, obtained for $\lambda = 0$ and $\lambda = 1$, respectively. Therefore, for a given m

$$\frac{m}{2} \leq \mu \leq m \tag{30}$$

The parameter μ is then chosen within the range of (30). Given that μ has been chosen, then λ is calculated from (29) as

$$\lambda = \sqrt{\frac{2\mu}{m} - 1} \tag{31}$$

F. The λ–μ Distribution for a Fixed m

Equation (29) shows that, for a given m, an infinite number of curves of the $\lambda - \mu$ Distribution can be found that present the same Nakagami parameter, conditioned on the fact that the constraints (30) and (31) are satisfied. The Nakagami curve is obtained for $\lambda \rightarrow 1$, in which case $\mu = m$, or, equivalently, for $\lambda \rightarrow 0$, in which case, $\mu = m/2$. The Hoyt distribution is obtained for $\mu = 0.5$. Figure 3 and Figure 4, respectively, depict a sample of the various shapes of the $\lambda - \mu$ probability density function $p(\rho)$ and probability distribution function $P(\rho)$ as a function of the normalized envelope ρ for the same Nakagami parameter $m = 0.75$. It can be seen that, although the normalized variance (parameter m) is kept constant for each figure, the curves are substantially different from each other. And this is particularly noticeable for the distribution function, in which case the lower tail of the distribution may yield differences in the probability of some orders of magnitude.

III. Validation Through Field Measurements

A series of field trials was conducted in Campinas downtown, Brazil, in order to investigate the short term statistics of the fading signal. Basically, the reception setup consisted of a vertically polarized omnidirectional antenna, a low noise amplifier, a spectrum analyzer, a data acquisition apparatus, a notebook computer, and a distance transducer. A forward control channel of an analog cellular system was used for the test runs. The spectrum analyzer was set to zero span and centered at a frequency of 870.9 MHz, and its video output used as the input of the data acquisition and processing equipment. The local mean was estimated through the moving average method, with the average being conveniently taken over samples symmetrically adjacent to every point, a procedure widely reported in the literature [11]. The practice used in order to adjust the distribution in accordance with the acquired data was that as described in sections II-E. We observed that, due to the versatility of this distributions, it was always possible to adequately fit experimental data through either the $\kappa - \mu$ Distribution [6] or the $\lambda - \mu$ Distribution. In order to illustrate the versatility of this distribution, in this paper we chose to show its fitting to experimental data other than those obtained by the author. We then turn our attention to Reference [11], in which a propagation measurement experiment at 10 GHz conducted in an indoor environment is reported. In Figure 4 of [11], where a logarithmic scale was used, it can be seen that the fitting provided by Rayleigh and Rice distributions to the experimental data is rather poor. Still in [11], the same set of data was then fitted to the Nakagam-m distribution and this is shown in Figure 1 of [11]. The fitting was better in this case, but a linear scale was used, the linear scale not being suitable to highlight the behavior of the tail of the distribution, though. In order to adjust the $\lambda - \mu$ Distribution to the data of [11], these data were carefully extracted from Figure 4 of [11]. Then, by using the respective Nakagami parameter m for the set of data, the same parameter as reported in [11], the parameters of the distributions under test were adjusted to yield the best fitting for each fixed. In all of the cases, the fitting given the $\lambda - \mu$ Distribution was better than that given by Rayleigh, Rice, Nakagami, or Weibull. Figure 5 shows this for the case in which $m = 2.1$ (Figure 4(b) and Figure 5(c) of [11]).

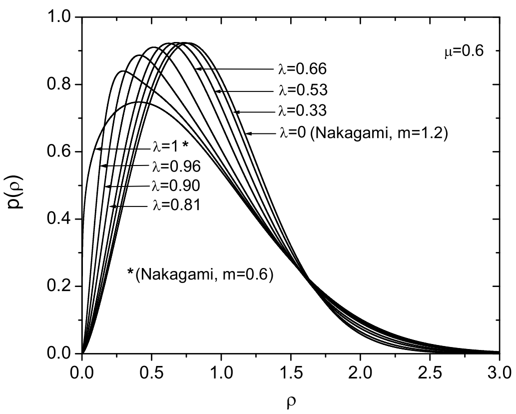

Fig. 1. The λ–μ probability density function for a fixed μ ($\mu = 0.6$).

IV. Conclusions

This paper presented a general fading distributions the $\lambda - \mu$ Distribution. The $\lambda - \mu$ Distribution includes the Hoyt (therefore, the Rayleigh and the One-Sided Gaussian) and the Nakagami-m (therefore, the Rayleigh and the One-Sided Gaussian) distributions as special cases. Because this distribution is more flexible than the other fading distributions it can yield better fitting to experimental data. And this has been observed in several field measurement campaigns.

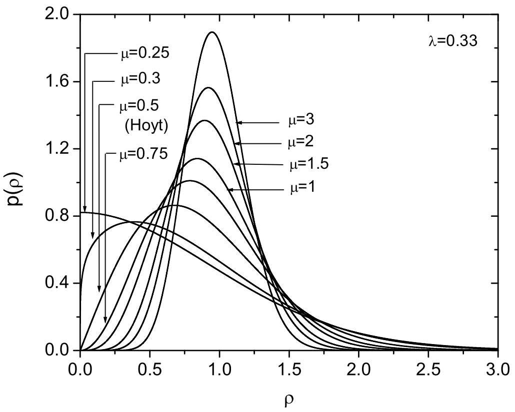

Fig. 2. The $\lambda - \mu$ probability density function for a fixed λ ($\lambda = 0.33$).

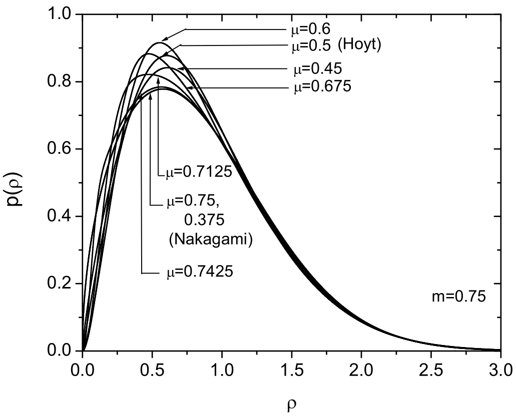

Fig. 3. The $\lambda - \mu$ probability density function for the same Nakagami parameter m (m=0.75).

APPENDIX

V. THE DISTRIBUTION FOR LIMITING VALUES OF THE PARAMETERS

This Appendix obtains the expressions of the distribution $\lambda - \mu$ for the cases in which the simple substitution of the parameters in the formulas leads to indeterminacy.

A. The $\lambda - \mu$ Distribution for$\lambda \rightarrow 0$

For small arguments of the Bessel function the relation $I_{v-1}(z) = (z/2)^{v-1}/\Gamma(v)$ holds [8, page 377, Eq. 9.6.7]. Using this in (1), and after some algebraic manipulation,

$$p(\rho) = \frac{2^{1+2\mu}\exp\left(\frac{2\mu\rho^2}{\lambda^2-1}\right)\mu^{2\mu}\rho^{-1+4\mu}}{(1-\lambda^2)^{\mu}\,\Gamma(2\mu)} \tag{32}$$

As $\lambda \rightarrow 0$ (32) reduces to

$$p(\rho) = \frac{2^{1+2\mu}\exp\left(-2\mu\rho^2\right)\mu^{2\mu}\rho^{-1+4\mu}}{\Gamma(2\mu)} \tag{33}$$

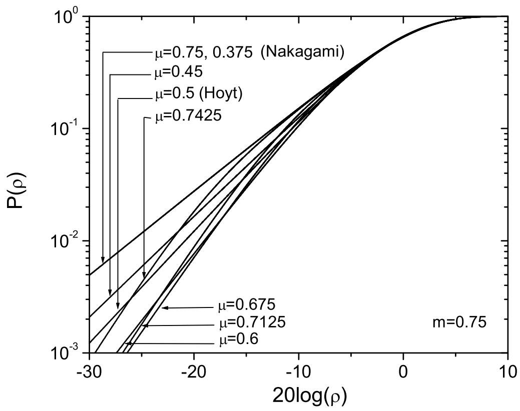

Fig. 4. The $\lambda - \mu$ probability distribution function for the same Nakagami parameter m (m=0.75).

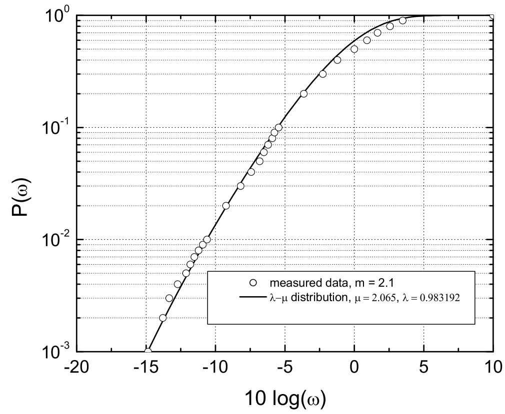

Fig. 5. The λ–μ distribution adjusted to data of a propagation measurement experiment at 10 GHz conducted in an indoor environment, as reported in [11].

which is the nakagami-m density function for the normalized envelope. In this case, the parameter μ and the Nakagami parameter are related to each other as $m = 2\mu$.

B. The $\lambda - \mu$ Distribution for$\lambda \rightarrow 1$

For large arguments of the the Bessel function, and utilizing the first term of the expansion, the relation $I_v(z) \simeq \frac{\exp(z)}{\sqrt{2\pi z}}$ holds [8, page 377, Eq. 9.7.1]. Using this in (1), and after tedius some algebraic manipulation,

$$p(\rho) = \frac{2\exp\left(-\frac{2\mu\rho^2}{1+\lambda^2}\right)\lambda^{-\mu}\mu^{\mu}\rho^{-1+2\mu}}{\Gamma(\mu)} \tag{34}$$

As $\lambda \rightarrow 1$ (34) reduces to

$$p(\rho) = \frac{2\exp\left(-\mu\rho^2\right)\mu^{\mu}\rho^{-1+2\mu}}{\Gamma(\mu)} \tag{35}$$

which is the nakagami-m density function for the normalized envelope. In this case, the parameter μ coincides with the well-know Nakagami parameter m.

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