Compensation of crosstalk in optical spatial division multiplexing systems employing multiple-input and multiple-output processing

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***Abstract*— The development of digital coherent optical communication technology has boosted the system capacity close to the nonlinear Shannon limit imposed by the combination of additive noise and nonlinear distortion. In this context, spatial division multiplexing (SDM) has emerged as a promising solution to overcome such a capacity limit. SDM systems, however, are affected by the channel-dependent loss as well as by interchannel crosstalk. In this paper, we obtain analytical expressions for the system capacity in presence of both impairments, showing that when multiple-input-and-multiple-output processing is implemented on the receiver side, the crosstalk can be compensated.**

***Keywords*— Spatial division multiplexing; channel capacity; optical digital coherent communications.**

I. INTRODUCTION

The irruption of digital coherent systems has revolutionized the way optical communication networks are designed [1]. Digital coherent systems not only allow the adoption of advanced modulation formats that exploit phase and polarization diversity but also open the possibility of mitigating the impact of the transmission impairments more efficiently [2]. However, even if the effects of chromatic dispersion, polarization mode dispersion, phase noise, and polarization fluctuation are satisfactorily compensated, the combination of nonlinear distortion and additive noise still poses a capacity bound to wavelength division multiplexing (WDM) systems [3], [4]. In this context, spatial division multiplexing (SDM) has emerged as a promising candidate to overcome this limitation [5]–[7].

In optical fiber communications, SDM can be implemented in different ways [8]: for instance, independent information can be transmitted in the different modes of a multi-mode fiber [9]. Alternatively, a fiber with multiple cores can be employed to transmit different signals in independent cores [10] or over multiple cores simultaneously in the so-called supermodes [11]. Furthermore, it is possible to combine both approaches, thus employing multi-core fibers where each core supports more than one mode [12]. In contrast to other multiplexing techniques, such as WDM, SDM systems are prone to stronger interchannel interference. This interference can be distributed, for instance in multi-mode fibers where fabrication-induced geometrical perturbations lead to intermodal coupling along the whole fiber or concentrated in the input/output couplers. This problem is not limited to fiber-based systems, but also to multi-mode integrated chips, which have attracted increasing attention in recent years.

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If not properly addressed, interchannel interference may severely affect the overall capacity, making SDM disappoint the expected capacity leverage [13]. A natural solution to this issue is implementing a multiple-input-multiple-output (MIMO) processing scheme, which has been extensively studied in wireless applications [14]. Nevertheless, it is worth noting that, due to the difference between the guided (fiber or integrated waveguides) and wireless channels, the former has some particularities that avoid the direct application of the knowledge acquired in wireless systems. In particular, the implementation of MIMO systems on the traditional intensity modulated with direct detection schemes is critically affected by the impossibility of retaining the phase of the received signal. This limitation, however, is not in present digital coherent systems, which are being adopted in applications with a progressively shorter range. On the other hand, guided channels are much more static than their wireless counterpart, showing much longer coherence times.

In this paper, we analyze and discuss the implications of implementing MIMO processing over an optical SDM system. In order to justify the use of MIMO processing, we compare the aggregated capacity of an SDM system in absence of this processing and the maximum capacity when a MIMO strategy is adopted. The rest of the paper is organized as follows: SDM and MIMO systems are introduced in Section II, paying special attention to the maximum capacity of each configuration. The numerical results are presented in Section III where we exemplify three scenarios: a crosstalk-free system with channel-dependent loss, a loss-less system with crosstalk and a system with channel-dependent loss and crosstalk. And finally, the main conclusions are drawn in Section IV.

II. SDM AND MIMO SYSTEMS

Let us consider an optical SDM system composed of n single-input-single-output (SISO) channels. Therefore, the number of input and output signals is the same. We denominate the i-th input as $s_i(t)$ and, assuming that it is a zero-mean wide-sense stationary signal, we can compute its power as $P_i = \langle s_i^2(t) \rangle$. We also assume that the inputs to the SDM system are mutually independent. These signals are coupled in, propagated, and coupled out, leading to interference among signals and channel-dependent loss (each signal may suffer different coupling and propagation losses). The interference and channel-dependent loss can be modeled using the following matrix [15]:

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1n} \\ h_{21} & h_{22} & \cdots & h_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ h_{n1} & h_{n2} & \dots & h_{nn} \end{bmatrix}, \tag{1}$$

h_{ij} being the complex field transmission coefficient from the j-th input to the i-th output. There are two particular cases of $\mathbf{H}$ that are of particular interest. On the one hand, a diagonal matrix ($h_{ij} = 0$ if $i \neq j$) represents an interference-free system where each channel has a transmittance coefficient of T_i. This matrix has the form of :

$$\mathbf{H}_{CDL} = \begin{bmatrix} T_1 & 0 & \cdots & 0 \\ 0 & T_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & T_n \end{bmatrix}. \tag{2}$$

Therefore, the loss associated with the i-th channel is given by $1/(T_i^2)$. On the other hand, another matrix with special interest is a lossless crosstalk matrix that describes a system where there is no loss, but the power of a given signal is coupled not to a single but to all outputs. The energy conservation principle implies that:

$$\sum_{i=1}^{n} |h_{ij}|^2 = 1, \forall j \in [1, n] \tag{3}$$

In this case, we can define the crosstalk XT_{ij} of the j-th input on the i-th output as $XT_{ij} = |h_{ij}|^2$.

Since in digital coherent systems impairments such as phase noise, frequency deviation, chromatic dispersion, and polarization-mode dispersion are compensated in the digital domain, and assuming that the system is operating in the linear transmission regime, where nonlinear distortion caused by the Kerr effect can be neglected, the dominant impairment is the additive noise [16]. Such noise can be caused either by the receiver or by any active optical device within the link (indeed, under certain conditions the nonlinear effects can also be modeled as an additive noise, which could expand the applicability of the model to the nonlinear regime). After adding the noise, the i-th output signal can be expressed as:

$$y_i = h_{ii}s_i + \sum_{j \neq i}^{n} (h_{ij} \cdot s_j) + \eta_i, \tag{4}$$

where, the first element on the left-hand side corresponds to the signal term, whereas the second and the third terms stand for the interference from the other channels and the additive noise. In a compact notation, we can write the set of the n output signals $\mathbf{y}$ in terms of the n input signals $\mathbf{s}$, the channel matrix $\mathbf{H}$ and the set of noise signals $\bar{\eta}$:

$$\mathbf{y} = \mathbf{H} \cdot \mathbf{s} + \bar{\eta}. \tag{5}$$

In absence of MIMO processing, each receiver has information of a single signal. In the case MIMO processing is included, receivers do not have information of just its received signal but also of the other receivers. Therefore, the received signals can be combined to improve the channel capacity. The output of the MIMO stage can then be written as:

$$\mathbf{y}' = \mathbf{H}_{\text{MIMO}} \cdot \mathbf{y}, \tag{6}$$

where $\mathbf{H}_{\text{MIMO}}$ is the MIMO processing matrix. Fig. 1 shows the block diagram of the SDM system including the MIMO processing stage.

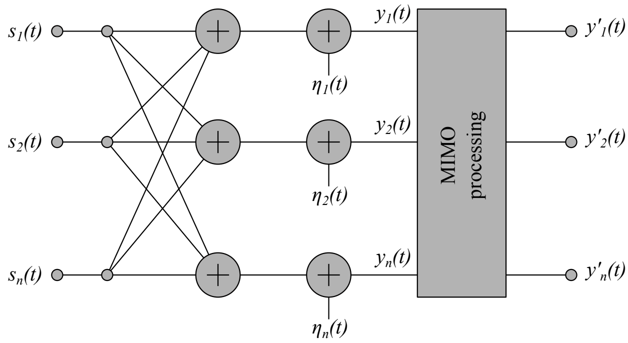

Fig. 1. Block diagram of an optical SDM system including the MIMO processing stage.

A. System capacity without MIMO processing

As previously mentioned, in absence of MIMO processing, the only useful contribution to a given output is that from its corresponding input, whereas the contributions from other inputs appear as an interference. Furthermore, the power of the i-th input coupled to the j-th output is lost. From Eq. 4, it is possible to obtain the signal to interference and noise ratio (SINR) of the i-th channel [16]:

$$SINR_i = \frac{|h_{ii}|^2 \cdot P_i}{\sum_{j \neq i}^{n} (|h_{ij}|^2 \cdot P_j) + P_{n,i}}, \tag{7}$$

where P_i and P_j are the powers of the i-th and j-th input signals and $P_{n,i}$ is the power of the noise contribution to the i-th output. Applying Shannon's formula for the channel capacity, we have that for the i-th channel, the maximum capacity is:

$$C_i = \log\left(1 + SINR_i\right), \tag{8}$$

and the total aggregated capacity of the SDM is:

$$\begin{aligned} C_{SDM} &= \sum_{i=1}^{n} C_i \\ &= \sum_{i=1}^{n} \log_2 \left(1 + \frac{|h_{ii}|^2 \cdot P_i}{\sum_{j \neq i}^{n} (|h_{ij}|^2 \cdot P_j) + P_{n,i}} \right). \end{aligned} \tag{9}$$

The previous expression can be simplified if we consider that all the input signals have uniform power P_s and that all noise signals have a power of P_n:

$$C_{SDM} = \sum_{i=1}^{n} \log_2 \left(1 + \frac{|h_{ii}|^2 \cdot P_s}{\sum_{j \neq i}^{n} (|h_{ij}|^2 \cdot P_s) + P_n} \right). \quad (10)$$

B. System capacity with MIMO processing

If MIMO processing is implemented, the total capacity of the system is given by:

$$C_{MIMO} = \log_2 \left[\det \left(I_n + \frac{P_s}{P_n} \mathbf{H} \cdot \mathbf{H}^{\dagger} \right) \right], \quad (11)$$

where I_n is the n-th order identity matrix, and $(\cdot)^{\dagger}$ denotes the complex transpose operation. It is also important to note that, to arrive into this expression, we also set uniform input and noise powers. After some algebra, it is possible to express the maximum capacity of the MIMO channel in terms of the eigenvalues of the matrix $\mathbf{H} \cdot \mathbf{H}^{\dagger}$, λ_i, [17]:

$$C_{MIMO} = \sum_{i=1}^{n} \log_2 \left(1 + \lambda_i \frac{P_s}{P_n} \right). \quad (12)$$

Therefore, when MIMO processing is implemented, the SDM system can be seen as a set of independent interference-free SISO channels whose input power is multiplied by λ_i.

III. Results

To understand the effect of the MIMO processing on the system capacity, we considered three different systems: (i) a system with channel-dependent loss but free of crosstalk, (ii) a system with crosstalk but with no loss, and (iii) a system affected by both channel-dependent loss and crosstalk. For the sake of simplicity, we limited our study to a 2×2 system.

A. Crosstalk-free system with channel-dependent loss

The first case under study is represented by a simple diagonal matrix of the form:

$$\mathbf{H}_{\text{CDL}} = \begin{bmatrix} T_1 & 0 \\ 0 & T_2 \end{bmatrix}. \quad (13)$$

Since there is no crosstalk, Eq. 10 acquires the form of:

$$C_{SDM} = \log_2 \left(1 + T_1^2 \frac{P_s}{P_n} \right) + \log_2 \left(1 + T_2^2 \frac{P_s}{P_n} \right), \quad (14)$$

where the case without MIMO processing is considered. If MIMO processing is adopted, the capacity can be calculated using the Eq. 12 by noting that the eigenvalues of $\mathbf{H}_{\text{CDL}} \cdot \mathbf{H}_{\text{CDL}}^{\dagger}$ are $|T_1|^2$ and $|T_2|^2$:

$$C_{MIMO} = \log_2 \left(1 + T_1^2 \frac{P_s}{P_n} \right) + \log_2 \left(1 + T_2^2 \frac{P_s}{P_n} \right). \quad (15)$$

As can be observed, in the case of a system affected uniquely by channel-dependent loss, the adoption of MIMO processing at the receiver does not represent any enhancement of the system capacity. It is important to note that we analyzed the processing only in the receiver. If we had contemplated MIMO processing also in the transmitter, some techniques could be implemented to allocate the power in a more efficient way, such as applying the water-filling algorithm.

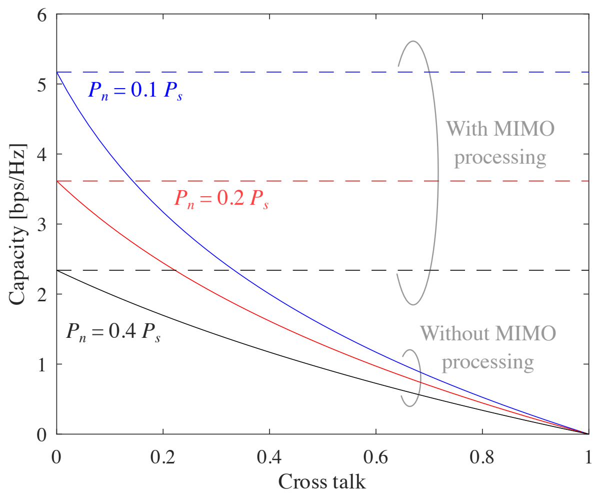

Fig. 2. System capacity considering MIMO processing and without considering it in terms of the power crosstalk for three different noise power levels, i.e. $P_n = 0.1P_s$, $P_n = 0.2P_s$, and $P_n = 0.4P_s$.

B. Loss-less system with crosstalk

The second considered scenario corresponds to an optical SDM system affected by crosstalk but without loss. In this particular case, the channel matrix $\mathbf{H}$ acquires the form of:

$$\mathbf{H}_{\text{XT}} = \begin{bmatrix} \sqrt{1 - XT} & \sqrt{XT} \\ -\sqrt{XT} & \sqrt{1 - XT} \end{bmatrix}, \quad (16)$$

where XT is the power crosstalk. The capacity of the SDM system if MIMO processing is not implemented is given by:

$$\begin{aligned} C_{SDM} &= \sum_{i=1}^{2} \log_2 \left(1 + \frac{(1 - XT) \cdot P_s}{XT \cdot P_s + P_n} \right) \\ &= 2 \log_2 \left(1 + \frac{(1 - XT) \cdot P_s}{XT \cdot P_s + P_n} \right). \end{aligned} \quad (17)$$

As expected, this expression shows that higher crosstalk leads to both signal power reduction and an increase in interference.

To calculate the capacity when MIMO processing is considered, we need to calculate the eigenvalues of $\mathbf{H}_{\text{XT}} \cdot \mathbf{H}_{\text{XT}}^{\dagger}$. The result of this expression returns an identity matrix, and thus both eigenvalues are equal to 1. Therefore, for these eigenvalues, the MIMO capacity given by Eq. 12 MIMO is:

$$C_{MIMO} = \sum_{i=1}^{2} \log_2 \left(1 + \frac{P_s}{P_n} \right) = 2 \log_2 \left(1 + \frac{P_s}{P_n} \right). \quad (18)$$

It is important to note that since the eigenvalues of the channel matrix do not depend on the crosstalk, neither does the capacity. In other words, the crosstalk does not affect the system capacity when adopting MIMO processing.

Fig. 2 shows the capacity of the SDM system with and without MIMO processing, for three different noise powers. As expected from Eq. 17, when MIMO processing is not applied, the system capacity decays as the crosstalk increases, whereas the system capacity remains constant if MIMO is implemented.

C. System with channel-dependent loss and crosstalk

The channel matrix of a system with a channel-dependent loss and interference can be constructed by concatenating a matrix representing a system with channel-dependent loss and an interference matrix:

$$\begin{aligned}\mathbf{H}_{\text{XT-CDL}} &= \begin{bmatrix}\sqrt{1-XT} & \sqrt{XT}\\ -\sqrt{XT} & \sqrt{1-XT}\end{bmatrix}\cdot\begin{bmatrix}T_1 & 0\\ 0 & T_2\end{bmatrix}\\ &= \begin{bmatrix}T_1\sqrt{1-XT} & T_2\sqrt{XT}\\ -T_1\sqrt{XT} & T_2\sqrt{1-XT}\end{bmatrix}. \end{aligned} \quad (19)$$

The capacity in absence of MIMO processing is:

$$\begin{aligned}C_{SDM} = \log_2\left(1+\frac{T_1^2(1-XT)\cdot P_s}{T_2^2 XT\cdot P_s + P_n}\right)\\ +\log_2\left(1+\frac{T_2^2(1-XT)\cdot P_s}{T_1^2 XT\cdot P_s + P_n}\right).\end{aligned} \quad (20)$$

To calculate the capacity of the system when MIMO processing is applied, we calculate once again the eigenvalues of the matrix $\mathbf{H}_{\text{XT-CDL}}\cdot\mathbf{H}^{\dagger}_{\text{XT-CDL}}$:

$$\begin{aligned}\mathbf{H}_{\text{XT-CDL}}\cdot\mathbf{H}^{\dagger}_{\text{XT-CDL}} &= \begin{bmatrix}T_1\sqrt{1-XT} & T_2\sqrt{XT}\\ -T_1\sqrt{XT} & T_2\sqrt{1-XT}\end{bmatrix}\\ &\cdot \begin{bmatrix}T_1\sqrt{1-XT} & -T_1\sqrt{XT}\\ T_2\sqrt{XT} & T_2\sqrt{1-XT}\end{bmatrix}\\ &= \begin{bmatrix}T_1^2 & 0\\ 0 & T_2^2\end{bmatrix},\end{aligned} \quad (21)$$

whose eigenvalues are T_1^2 and T_2^2. Consequently, the system capacity when employing MIMO is:

$$C_{MIMO} = \log_2\left(1+T_1^2\frac{P_s}{P_n}\right)+\log_2\left(1+T_2^2\frac{P_s}{P_n}\right). \quad (22)$$

As the reader can observe, the previous expression shows that the system capacity when MIMO processing is adopted is equal to that obtained for the crosstalk-free case (Eq. 15). In other words, the crosstalk does not affect the system capacity even when the channel-dependent loss is considered.

These results reveal that the MIMO processing compensates for the effect of the crosstalk in SDM systems, leading to an enhanced system capacity when this impairment is present. In order to quantify this enhancement, we consider a system with a $P_n = 0.1P_s$, XT = 0.1, and power transmission coefficients for the first and second channel covering the whole possible range, i.e. from 0 to 1. The obtained system capacities without considering MIMO and considering it are shown in Fig. 3(a) and Fig. 3(b), respectively. As expected, the lower the power transmittance of the channels is, the lower the system capacity is for both cases. However, the system in which MIMO processing is employed outperforms that without MIMO processing in terms of capacity. For instance, the maximum system capacity when no MIMO processing is used is 4 bps/Hz, whereas when MIMO is implemented, the system capacity is increased up to 5.17 bps/Hz. The aforementioned case corresponds to the configuration where a larger system capacity increase is achieved. As can be seen in Fig. 3(c), the capacity increase reduces for lower channel transference coefficients, which makes sense because

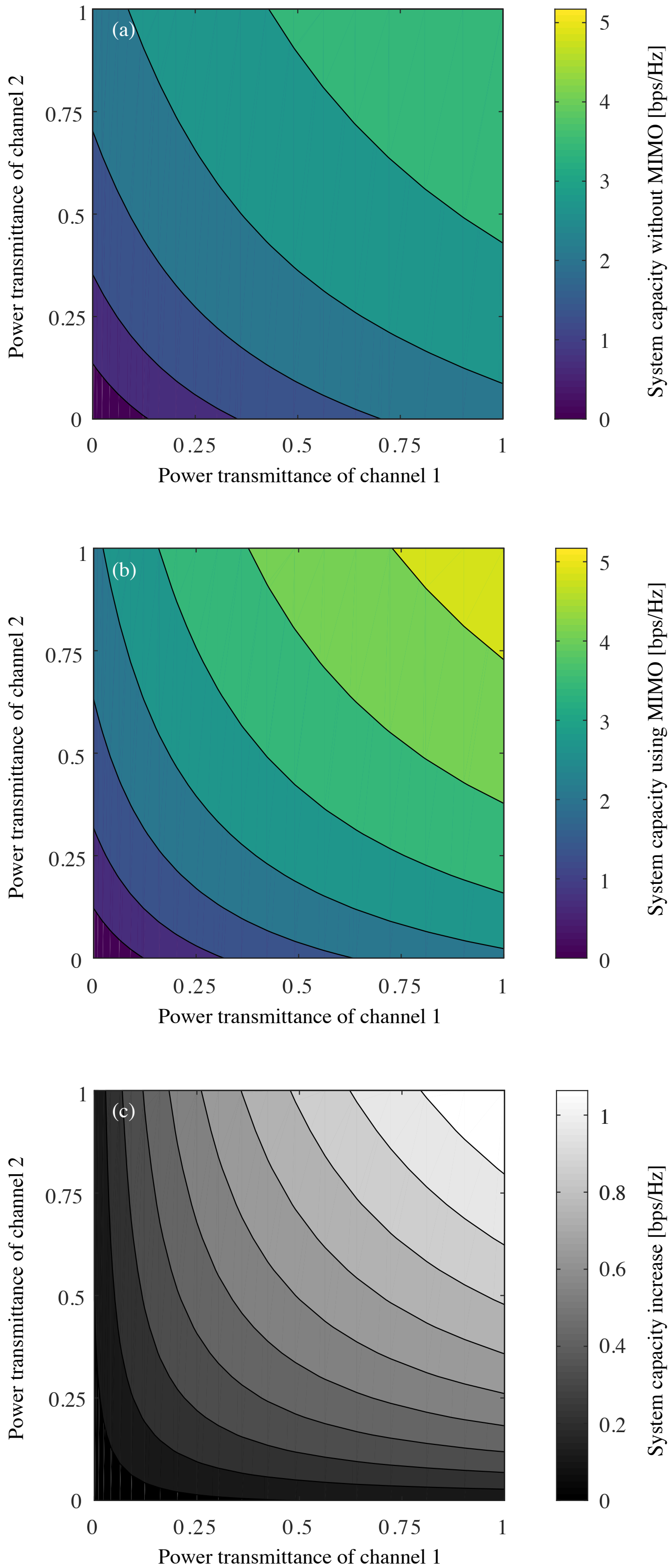

Fig. 3. System capacity in terms of the power transmittance of each channel when (a) no MIMO processing is adopted and (b) when MIMO processing is included. (c) Capacity increase as a function of the power transmittance when MIMO processing is used in the receiver. For the two systems, the noise power and crosstalk were set to $P_n = 0.1P_s$ and $XT = 1$, respectively.

the strength of the crosstalk depends on the power level. Thus, in configurations with large power levels, the effect of the crosstalk is more significant and, consequently, the adoption of MIMO processing leads to a larger capacity increase. For low signal power levels, the effect of the noise becomes dominant, and MIMO processing presents a marginal enhancement.

IV. Conclusions and future work

This paper analyzes the effect of MIMO processing in optical SDM systems based on digital coherent receivers. Theoretical and numerical results considering a 2×2 indicate that when incorporating MIMO processing in the receiver, the crosstalk among the channels can be compensated, leading to a system capacity increase. This increase is more notorious when the crosstalk is dominant, that is, for low noise power and small channel losses.

However, these results represent an initial study that must be deepened. In particular, extending the analysis to higher dimensional SDM systems considering more than two channels is essential. In addition, it is important to include the effect of MIMO processing not only in the receiver but also on the transmitter side, which will enable the implementation of intelligent power allocation schemes, such as the water-filling algorithm, to achieve further capacity increase.

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