PAPR mitigation in OFDM through Joint PTS and Side Information Optimization using Metaheuristics

Bruno F. Canale, Cristiano M. Panazio

***Abstract*— Partial transmit sequence (PTS) using metaheuristic optimization methods can achieve a very good compromise between peak-to-average power ratio (PAPR) mitigation of orthogonal frequency division multiplexing (OFDM) signals and complexity. However, one neglected problem is that the side information (SI) needed for demodulation leads to PAPR regrowth. To overcome this issue, we propose a joint optimization scheme called Side inforMation and pARtial Transmit sEquence optimizatioN (SMARTEN) in which the SI is taken into account. We show that the proposed SI allows to correctly demodulate the OFDM symbol with a small energy efficiency loss. Then, we evaluate SMARTEN with genetic algorithm and simulated annealing metaheuristics. It is shown that not only the PAPR regrowth is eliminated but PAPR can be smaller than the one achieved by the conventional PTS when SI is not added to the OFDM symbol. We also provide an efficient implementation scheme, where we can exchange complexity by precomputing and storing the SI signal.**

***Keywords*— PAPR, PTS, OFDM, joint optimization, combinatorial optimization, metaheuristics.**

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a modulation technique that allows high data rate transmission on frequency selective multipath channels with low computational complexity. However, it has one important drawback that is the high peak-to-average power ratio (PAPR). This incurs in low power efficiency since large power back-off values are needed due to linear amplification requirements, otherwise nonlinear amplification effects, such as saturation, will result in system performance degradation due to in-band and out-of-band interference [1].

In order to mitigate PAPR many methods have been devised [1]: clipping and filtering, tone reservation, tone injection, active constellation extension, selective mapping and partial transmit sequence (PTS). The latter stands out for achieving a good performance compromise considering PAPR reduction, spectrum and energy efficiency loss and computational complexity [2].

PTS works by rotating the phases of subcarrier groups for a given OFDM symbol. The rotations, which are limited to a few values to reduce complexity, are chosen in order to achieve the best PAPR reduction. Due to the usually huge number of rotation combinations and the computational cost involved to test each choice, an exhaustive search is usually infeasible. To overcome this issue and aiming maximal PAPR mitigation, many techniques were proposed such as iterative flipping [3], gradient descent [4], as well as the use of metaheuristic methods: genetic algorithm (GA) [5, 6], concentration- based immune network for combinatorial optimization [6], particle swarm optimization [7], harmony search [8] and simulated annealing (SA) [9]. Particularly, the metaheuristic approach presents an excellent compromise between performance and computational cost. Once the rotation ensemble is chosen, the set is encoded and transmitted as side information (SI) to let the receiver undo these rotations and recover the original OFDM symbol.

Although many contributions focused on finding the best rotation combination, one relevant issue is overlooked when the SI is embedded into the OFDM symbol: peak regrowth, increasing the PAPR that was once minimized by the chosen technique. Hitherto, to the authors knowledge, only [10] has addressed this issue. It proposes to send the SI of the current OFDM symbol in the next one, in a group of subcarriers that, without loss of the PAPR performance reduction, are not rotated. When optimized for the PAPR reduction, the OFDM symbol already contains the SI of the previous one and it can be taken into account in the minimization process. Since the SI is not rotated, it can be used to directly demodulate the previous symbol. The results show that it can eliminate the PAPR regrowth. However, this technique adds additional latency, since the demodulation of the current OFDM symbol can only be done when the next one arrives. Also, when working with bursts of OFDM symbols (i.e., packets), an additional OFDM symbol is needed, reducing spectral efficiency. It is worth noting that the authors of [10] discard the possibility of a technique that sends the SI in the same OFDM symbol as it would be too computationally costly. Another significant issue in [10] is that the SI is not protected against frequency selectivity and there is no performance analysis of the robustness and efficacy of the proposed arrangement. All these issues are addressed in the present paper.

We propose in this paper a computationally efficient joint technique, the so-called Side inforMation and pARtial Transmit sEquence optimizatioN (SMARTEN). Besides the use of metaheuristics to further reduce the complexity, we propose an implementation scheme that precomputes and stores the SI signals bringing the complexity of the proposed technique close to the conventional approach. Then, we show through bit-error rate (BER) simulations that the proposed SI format is robust enough to have no impact on the BER with low energy efficiency loss. Finally, PAPR simulation results are shown where the SMARTEN technique was implemented with two different metaheuristics, *i.e.* GA and SA, and compared to conventional PTS where SI is present. It is shown that SMARTEN can not only remove the PAPR regrowth after

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embedding the SI but can even further reduce the PAPR level when compared to the conventional PTS without SI.

This paper is organized as follows: In Section II, the system model is presented. The PTS and proposed SMARTEN technique are presented in Section III. In Section IV, simulation results are presented and analyzed and, finally, the paper is summarized and the conclusions are stated in Section V.

II. System Model

Consider a time-discrete OFDM symbol given by:

$$x_n = x\left(n\frac{T_s}{LN}\right) = \sum_{k=0}^{N-1} X_k e^{\frac{j2\pi kn}{LN}},\ 0 \leq n \leq LN-1 \quad (1)$$

where X_k is the k-th data symbol, taken from a digital modulation scheme (*e.g.*, QAM), transmitted through a duration of T_s in the k-th orthogonal subcarrier $\mathrm{e}^{j2\pi\frac{kn}{LN}}$, and L is an oversampling factor used to accurately represent the PAPR of the continuous time-domain version [1]. It is worth noting that eq. (1) can be efficiently obtained by applying an LN-point IFFT of the (zero-padded) data vector $\mathbf{X} = [X_0, X_1, \ldots, X_{N-1}]$.

Then, the PAPR is calculated by

$$\text{PAPR}(\mathbf{x}) = \frac{\max\limits_{0\leq n\leq LN-1}\left(|x_n|^2\right)}{E\left\{|x_n|^2\right\}}, \quad (2)$$

where $E\{\cdot\}$ is the expectation operator.

Given the random nature of this measure due to the randomness of the data symbols, a more meaningful measure is obtained by using complementary cumulative distribution function (CCDF) of the PAPR, *i.e.*, probability of the PAPR being larger than a given threshold.

III. The PTS and SMARTEN techniques

A. The PTS technique without SI

This technique reduces the PAPR of each OFDM symbol by partitioning the data symbols $\mathbf{X}$ into M disjoint subblocks $\mathbf{X}^{(m)}$, for $0 \leq m \leq M-1$, in which

$$\mathbf{X} = \sum_{m=0}^{M-1} \mathbf{X}^{(m)}. \quad (3)$$

Then, through an LN-point IFFT, we obtain the time domain version of each subblock:

$$\mathbf{x}^{(m)} = \text{IFFT}(\mathbf{X}^{(m)}). \quad (4)$$

where the vectors $\mathbf{x}^{(m)}$ are known as the partial transmit sequences. They are rotated by unitary norm complex values $\mathbf{b} = [b_0, b_1, \ldots, b_{M-1}]$ where $b_m = e^{j\theta^{(m)}}$ for $\theta^{(m)} \in [0, 2\pi)$, and recombined to form a new OFDM symbol $\mathbf{x}'$, given by

$$\mathbf{x}' = \sum_{m=0}^{M-1} b_m \mathbf{x}^{(m)}. \quad (5)$$

Clearly, the problem is to find the optimal set of M rotations $\mathbf{b}_{opt}$ that minimizes the PAPR of $\mathbf{x}'$. In general, in order to limit the complexity of the search procedure and reduce the amount of side information, $\theta^{(m)}$ can only assume a finite set of possible values. As shown in the literature, a good choice is usually $\{0, \pi/2, \pi, -\pi/2\}$. With such choice, without loss of generality and performance, we can make $b_0 = 1$ [11]. Thus, the total number of possible combinations of $\mathbf{b}$ is 4^{M-1} and the optimization problem can be written as

$$\mathbf{b}_{opt} = \arg\min_{[b_1,\ldots,b_{M-1}]}\left(\max\left(\left|\sum_{m=0}^{M-1} b_m \mathbf{x}^{(m)}\right|^2\right)\right). \quad (6)$$

Many techniques were proposed in order to reduce the computational complexity involved in the search of the optimal rotations. Methods that present an excellent compromise between performance and complexity are based in metaheuristics such as GA [5] and SA [9] when compared to other optimization tools used in this context. In this paper, we use both metaheuristics to provide performance results of the proposed technique.

B. A better PTS when taking into account SI: SMARTEN

In practice, the receiver has to make use of SI sent by the transmitter to identify which rotations $\mathbf{b}$ were used to correctly decode the data symbols. Considering 4 possible rotations and M subblocks, at least $\log_2(4^{M-1})$ bits are required to represent the SI. An error in one or more of these bits caused by the presence of additive noise and frequency selective fading in the channel would lead to unrecoverable data errors. Thus, these SI bits must be well protected and the BER associated to them must be considerably smaller than the BER of the data bits in order to not increase the latter. Also, differently from [10], we want to minimize latency by transmitting the SI in the same OFDM symbol that originated it. In this sense and based on the results of [12, 13], we propose a scheme where the SI bits are channel encoded and interleaved before being modulated into P QPSK symbols that are transmitted in evenly (N/P) spaced P unrotated subcarriers of the same OFDM symbol that originated the SI. These P subcarriers do not overlap with the set of $N-P$ rotated subcarriers allocated for data transmission and can be used to directly recover the SI. Additional power can be allocated to the SI symbols in order to further lower its BER. However, this additional power allocation must be kept as low as possible in order to not incur in substantial energy efficiency loss.

Therefore, with the SI embedded into the OFDM symbol, the latter has the following time domain representation

$$\mathbf{x}' = \sum_{m=0}^{M-1} b_m \mathbf{x}^{(m)} + \mathbf{z}, \quad (7)$$

where $\mathbf{z}$ is the LN-point IFFT of a vector with P SI symbols in their respective frequency bins and zeros in the other $LN-P$ elements, and $\mathbf{x}^{(m)}$ is similar to eq. (4), but $\mathbf{X}^{(m)}$ has zeros in the elements where the P SI symbols are inserted. In the rest of this paper we will simply refer to this approach as PTS with SI. When using a system without SI, where the rotations at the reception are perfectly guessed without any aid, we will call it PTS without SI as to make the distinction between the two configurations as clear as possible.

Despite having the SI embedded into the OFDM symbol, the optimization criterion used in PTS with SI is the same as the one used in PTS without SI, presented in eq. (6), which does not take into account the perturbations caused by the SI addition. As will be shown in IV, this will result in a PAPR regrowth by as much as 1.4 dB when compared to the PTS without SI. The SMARTEN method does not ignore this issue by including the SI, which intrinsically depends on the rotation set $\mathbf{b}$, in its PAPR minimization objective function:

$$\mathbf{b}_{opt} = \arg \min_{[b_1,\ldots,b_{M-1}]} \left(\max \left(\left| \sum_{m=0}^{M-1} b_m \mathbf{x}^{(m)} + \mathbf{z} \right|^2 \right) \right). \quad (8)$$

One interesting aspect is that the SI embedded in the same OFDM symbol that originated it can serve as additional degrees of freedom to further help mitigate the PAPR as shown in Section IV.

The inclusion of $\mathbf{z}$ in the optimization process adds for every tentative symbol an additional LN-point IFFT, considerably increasing the complexity even if we use a pruned IFFT. Hence, aiming to reduce this complexity we propose an architecture that precomputes all possible combinations and efficiently stores them, bringing the complexity close to the complexity of PTS without SI.

C. Complexity mitigation: precomputing and efficiently storing the time domain SI

This alternative implementation precomputes the time domain SI representation of all the possible combinations of $\mathbf{b}$ and stores them in memory. Then, the complexity involved in the generation and addition of the signal $\mathbf{z}$ into each tentative OFDM symbol $\mathbf{x}'$ becomes $2NL$ real summations, rather than $3LNlog_2(P) + 2LN$ real summations and $2LNlog_2(P)$ real multiplications already considering a pruned IFFT for lower complexity [14].

The memory in bytes needed to allocate the SI time domain representation for all possible combinations of $\mathbf{b}, b_0 = 1, b_k \in \{1, -1, +j, -j\}$ is given by $4^{M-2}LNq$, in which q is the number of bits chosen to represent each of the in-phase and quadrature elements of $\mathbf{z}$. To exemplify, considering $M = 8, L = 4, N = 256$ and an exaggerated $q = 32$, the allocated memory would be 134 MiB, which is well within any modern embedded system. However, as we consider that the P SI subcarriers are equally spaced by N/P subcarriers, $\mathbf{z}$ is periodic with a period of $\frac{LN}{P}$ samples and we can take advantage of this property. For the given example, the memory used to store the time domain representation can be reduced by 16 times, and it would take only 8 MiB to store all the SI, becoming even more feasible its implementation.

IV. Simulations and Results

In our simulations, we consider $M = 8$ and $b \in \{0, -1, +j, -j\}$ since these are typical parameters for PTS, and $b_0 = 1$ as it can be fixed without loss of generality and performance. This results in 16384 possible combinations, which translates into 14 SI bits. These bits plus two tail bits, for proper code termination, were encoded with a convolutional encoder $[5\ \ 7]_{octal}$, resulting in 32 coded bits, interleaved and mapped into $P = 16$ QPSK symbols. These symbols were allocated in equally spaced subcarriers, *i.e.*, with N/P subcarrier spacing. For the data bits, we used a convolutional code $[171\ 133]_{octal}$, an interleaver and mapped the coded bits into 16-QAM symbols. We define $\alpha = \mathrm{Power}_{\mathrm{SI-QPSK}}/\mathrm{Power}_{16-\mathrm{QAM}}$ as the power ratio between the average power of a SI QPSK symbol and the average power of a 16-QAM data symbol. A higher value of α means additional SI protection, but this incurs in additional energy efficiency loss. As we show in the simulations, there is a good value of α for which the efficiency penalty is small with a BER performance very close to an ideal system, *i.e.*, a PTS without SI where every rotation is perfectly guessed.

To illustrate the robustness and efficacy of the proposed SI configuration and the impact of α in terms of BER and energy efficiency, we provide some BER vs E_b/N_0 results for a 2-tap and 8-tap equal and independent average power profile Rayleigh block-fading channels and different values of α in Figs. 1 and 2 for $N = 256$ and $N = 1024$, respectively. Minimum length cyclic prefix and an ideal power amplifier were assumed.

From the $N = 256$ results in Fig. 1, it is clear that less protected SI with $\alpha = -6.99$ dB leads to higher performance degradation for both channels, particularly the 8-taps, since it tends to have more frequency nulls than the 2-taps channel. The $\alpha = -3.98$ dB and $\alpha = 2.55$ dB configurations have similar performance for the 2-tap channel, despite the latter being 0.38 dB more energy inefficient than the former as it has fewer rotation recovery errors. With respect to the ideal system, the $\alpha = 2.55$ dB case has a small additional performance loss of 0.3 dB besides the 0.5 dB loss due to power and subcarrier allocation. For the 8-tap channel, the additional SI protection given by the $\alpha = 2.55$ dB configuration is more relevant, as it becomes noticeable better than the $\alpha = -3.98$ dB scheme and keeps the 0.8 dB degradation to the ideal system.

The $N = 1024$ results, shown in Fig. 2, is similar to the $N = 256$ case, but with two noticeable differences. The first one is that the energy efficiency penalty with $\alpha = 2.55$ dB is of just 0.12 dB, making it even closer to the ideal system in both channel scenarios. This comes from the fact, when passing from $N = 256$ to $N = 1024$, the total power and the number of subcarriers allocated to the SI becomes much smaller than the values dedicated to the data subcarriers. The other difference is that the performance loss of the $\alpha = -3.98$ dB configuration in the 8-taps channel is exacerbated, since a subblock wrongly demodulated in the $N = 1024$ configuration generates a higher BER than in the $N = 256$ one. Thus, for the PAPR simulations, we are going to use the $\alpha = 2.55$ dB configuration, since it presents a good overall performance when compared to the ideal system where the SI is absent.

For the PAPR results, in order to demonstrate the efficiency of SMARTEN, we show in Figures 3 and 4 the complementary cumulative density function (CCDF) of the PAPR for N=256 and 1024 subcarriers, respectively, of a conventional OFDM, PTS SA and GA with SI (where it is present in the OFDM symbol, but not taken into account in $\mathbf{b}$ optimization for PAPR reduction), SMARTEN SA and with exhaustive search (ES) and, finally, PTS SA and ES without SI present in the OFDM symbols. It is worth noting that we only show a single result of GA since it was outperformed by SA in every simulation we

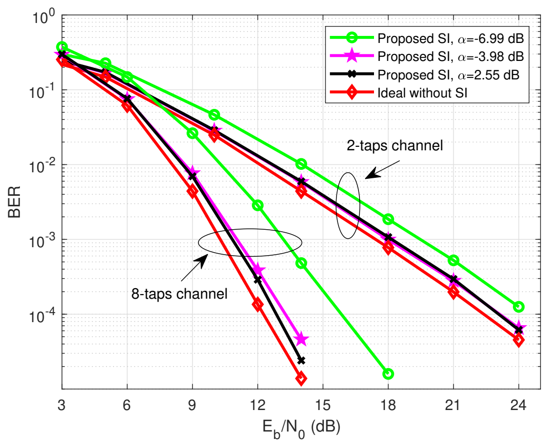

Fig. 1. BER values for $N = 256$ of different power allocation of the proposed SI configuration for SMARTEN and ideal system for 2-tap and 8-tap block-fading channel.

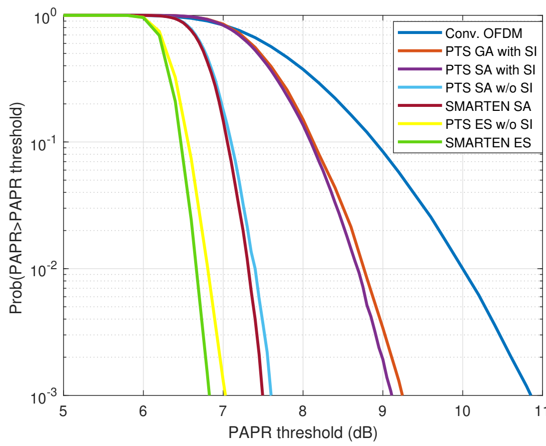

Fig. 3. PAPR CCDF values for $N = 256$ for conventional OFDM, PTS GA and SA with SI, PTS SA and ES without SI, SMARTEN GA, SA and ES optimization techniques. The results shown for the metaheuristics were obtained with 104 iterations per OFDM symbol.

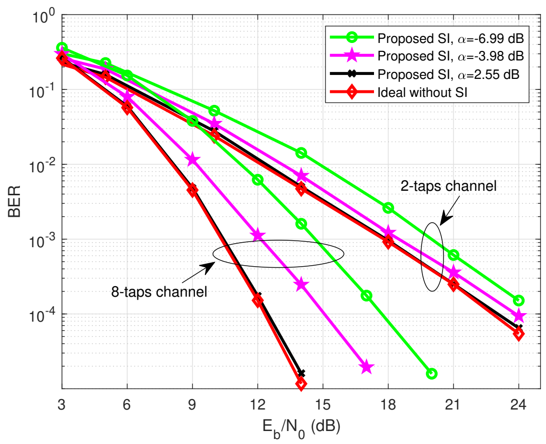

Fig. 2. BER values for $N = 1024$ of different power allocation of the proposed SI configuration for SMARTEN and ideal system for 2-tap and 8-tap block-fading channel.

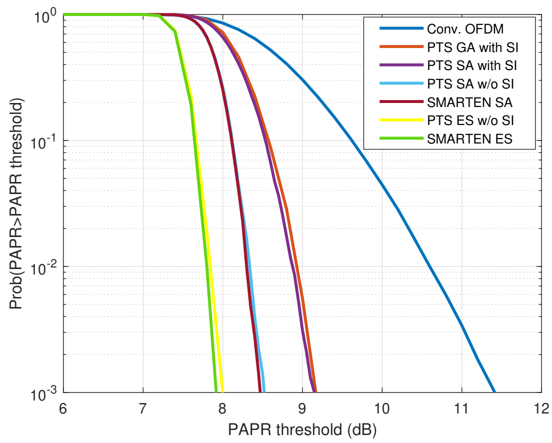

Fig. 4. PAPR CCDF values for $N = 1024$ for conventional OFDM, PTS and SMARTEN with GA, SA and exhaustive search (ES) optimization techniques. The results shown for the metaheuristics were obtained with 104 iterations.

did. The curves were obtained using 10^4 OFDM symbols and both metaheuristics used 104 iterations to obtain their results.

From Figs. 3 and 4, firstly, one of the most noticeable results is the expected problem of PAPR regrowth when SI is transmitted without being taking into account in the PAPR minimization through PTS (PTS with SI curves). The difference is less pronounced in Fig. 4 with $N = 1024$, since SI accounts for a smaller energy fraction of the OFDM symbol in this scenario than in the $N = 256$ case. Then, it is clear that the proposed SMARTEN technique can not only eliminate the PAPR regrowth but it can even achieve smaller PAPR values, particularly in the $N = 256$ scenario, where the SI accounts for a higher energy fraction of the OFDM symbol. This comes from the fact that the SI symbols provide additional degrees of freedom for mitigating PAPR, being similar to the tone reservation technique, where tones with proper phases in specific subcarriers are used to reduce the PAPR of an OFDM symbol [1]. Therefore, even if additional power (higher values of α) or subcarriers must be allocated to the SI, the SMARTEN technique will benefit from that to further reduce the PAPR. Finally, the performance of the SA is not too far from the ES solutions, which uses almost 160 times more iterations.

Finally, we analyze the convergence of the SA for PTS, PTS without SI and SMARTEN techniques for a fixed CCDF of 10^{-2} in Fig. 5. For all configurations, the PAPR reduction suffers from diminishing returns with the increase of iterations. However, recalling the results from Figs. 3 and 4, with about a 100 iterations, the performance of PTS SA without SI and SMARTEN SA is about 0.6 dB from their exhaustive search counterparts, which is about 160 times more complex. Also from these results, it seems that choosing 32 to 64 iterations, *i.e.*, the curve knee presents a good compromise between performance and complexity. It is also clear that the performance difference among PTS SA, PTS SA without SI and SMARTEN is constant for a given number of iterations. Finally, the SMARTEN SA can achieve the same performance of the PTS SA without SI with 30%-40% less iterations with $N = 256$ and 12%-20% less with $N = 1024$. The smaller difference in $N = 1024$ case comes again from the fact that SI

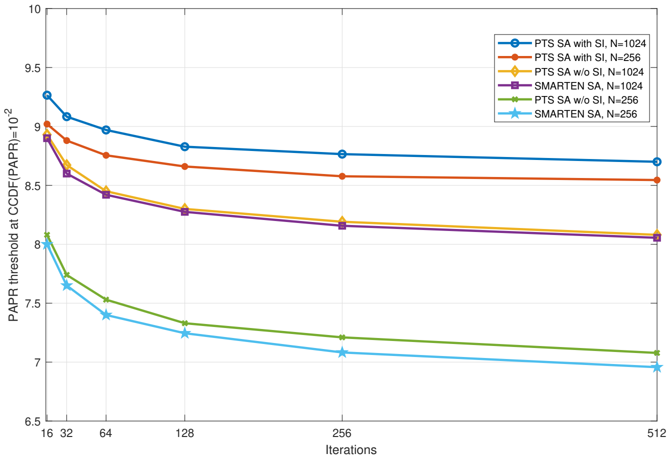

Fig. 5. PAPR threshold at CCDF(PAPR)=10^{-2} versus iterations for PTS SA, PTS SA without SI and SMARTEN SA.

accounts for less OFDM symbol energy than in the $N = 256$ case. This raises the question to be explored in future works that one may exchange complexity or better PAPR mitigation with more power allocated to the SI, when using SMARTEN, but at the expanse of some energy efficiency in terms of BER.

V. Conclusions

OFDM symbols suffer from high PAPR. Hence, more expansive and less energy efficient amplifiers are needed, otherwise we may end up with intolerable in and out-of-band distortions. PTS is an efficient PAPR reduction method, but it suffers of high complexity and PAPR regrowth when SI is embedded into the OFDM symbols.

In this paper, we tackle these PTS issues with the use of simulated annealing metaheuristic to reduce the computational complexity and a joint optimization of the PTS rotations and SI, called SMARTEN, to mitigate the PAPR regrowth. We also propose an efficient implementation where the SI is precomputed, leaving the complexity per iteration similar to the conventional PTS technique. Simulation results show the effectiveness of the proposed approach as it is able to even achieve lower PAPR values than the conventional PTS, since it can exploit SI as additional degrees of freedom to perform PAPR mitigation.

Future works may analyze if other SI configurations, power allocation and non adjacent sequences can lead to better results as well as test other metaheuristics besides the genetic algorithm and simulated annealing techniques tested in this paper.

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