Robust probabilistic constrained power control for 3G networks

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***Abstract*— This paper deals with the power control problem for wireless cellular networks. The problem is modelled by a stochastic optimization problem with probabilistic constraints considering the link gains as random vectors whose probability distribution satisfies uncertain mean and covariance constraints. Specifically, we consider that the mean vector and covariance matrix are not perfectly known, however they belong to convex hull of a fixed set. The solution is established in terms of an optimization problem over linear matrix inequalities (LMIs) and it is robust in the sense that from the set of probability distributions matching the given bounds on the mean vector and covariance matrix, we take the one that produces the lowest tail probability.**

***Keywords*— Quality of service, radio resource management, power control, generalized Chebychev inequalities, linear matrix inequalities.**

I. INTRODUCTION

For cellular wireless networks, the control of transmitted power such that the link quality, expressed by signal-to-interference ratio (SIR), is maintained above a specified threshold, can be established mathematically as follows (cf. [1]–[4])

$$\begin{cases} \min & \displaystyle\sum_i p_i \\ \text{s.t. :} & \gamma_i(p, g_i) = \dfrac{g_{ii}p_i}{\sum_{j\neq i} g_{ij}p_j + \nu_i} \geq \alpha_i, \\ & 0 < p_i \leq p_{i,max}, \quad i = 1, \ldots, n, \end{cases} \qquad (1)$$

where $p = [p_1, \ldots, p_n]' \in \mathbb{R}^n$ is the transmitted power vector and $p_{max} = [p_{1,max}, \ldots, p_{n,max}]' \in \mathbb{R}^n$ is the maximum allowable transmitted power vector. $g_i = [g_{i1}, \ldots, g_{in}]' \in \mathbb{R}^n$ for $i = 1, \ldots, n$ are the channel gain vectors where g_{ij} is the power gain from the transmitter of j-th link to the receiver of the i-th one and ν_i is the background noise power at the receiver of i-th link. Finally, γ_i and α_i are the received SIR[1] and the required SIR threshold for the i-th link, respectively.

In this framework, the solution of optimization problem (1) is determinate under the assumption that the channel gains g_i for $i = 1, \ldots, n$ are exactly known. In applications however, they are difficult to estimate precisely. This is due, for example, to noise corruptions. As consequence, the aforementioned power control model falls short of handling the uncertain case.

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[1]It is important to point out that the γ_i model, defined in (1), is general enough to describe DS-CDMA systems with matched-filter receivers as well as one-channel TDMA/FDMA systems giving specific interpretations to their parameters (see [5]). As consequence, for theoretical studies, there is no need to distinguish between multiple access schemes.

In this paper, the power control problem is reformulated by considering the channel gains g_i for $i = 1, \ldots, n$ as random vectors. We assume that only partial information regarding the probability distributions of g_i for $i = 1, \ldots, n$ are available. In particular, this paper studies the case where they satisfies given bounds on the mean and covariance constraints. In this setting, a robust stochastic optimization problem with probabilistic constraints is used to model the power control problem. This formulation allows us achieve the specified level of link quality satisfaction (SIR reliability requirements).

We approach the problem via generalized Chebychev inequalities, i.e., optimal bounds on the probability that a certain vector random variable belongs to a given set, under moment constraints (see [6], [7]). The solution is presented in terms of a convex optimization problem with linear objective and positive definite constraints involving symmetric matrices that are affine in the decision variables, namely a LMI problem [8]. Using interior-point methods for convex optimization, the LMI problem can be solved very efficiently.

The paper is organized as follows. In Section II some notations and basic definitions are presented. Additionally, the stochastic power control problem is precisely stated. In Section III, we present the solution of proposed problem. A numerical example and some conclusions are finally presented in Sections IV and V, respectively.

II. NOTATION AND PROBLEM FORMULATION

Throughout this paper, the following notation is adopted. $\mathbb{R}^n$ denotes the n-dimensional real space and $\mathcal{M}^{r\times m}$ ($\mathcal{M}^r$) the normed linear space of all $r \times m$ ($r \times r$) real matrices. The superscript $'$ stands for the matrix transpose and the notation $U \geq 0$ ($U > 0$) for $U \in \mathcal{M}^r$, means that U is semi-definite (definite) positive. Then, the closed (opened) convex cone of all positive semi-definite (definite) matrices in $\mathcal{M}^r$ is denoted by $\mathcal{M}^{r0} = \{U \in \mathcal{M}^r : U = U' \geq 0\}$ ($\mathcal{M}^{r+}$). The convex hull of the fixed set $\{U^1, \ldots, U^l\}$ is represented by

$$\mathbf{Co}\{U^1, \ldots, U^l\} = \Big\{ U = \sum_{k=1}^{l} \delta_k U^k : \\ \delta_k \geq 0,\ k = 1, \ldots, l, \quad \sum_{k=1}^{l} \delta_k = 1 \Big\}$$

where $U^k \in \mathbb{R}^{r\times m}$ for $k = 1, \ldots, l$. Finally, for a matrix $U \in \mathcal{M}^r$, the trace operator is denoted by $tr\{U\}$ and for a vector $u \in \mathbb{R}^r$, the diagonal matrix where u is the vector of its diagonal entries is represented by $diag\{u\}$.

In [6], it was established that for a given vector $\overline{g}_i \in \mathbb{R}^n$ and matrix $\Sigma_{g_i} \in \mathcal{M}^{n0}$, there exists a random vector

$g_i = [g_{i1}, \ldots, g_{in}]' \in \mathbb{R}^n$, defined in the probability space $(\Omega, \mathfrak{F}, Pr)$, whose mean vector and covariance matrix are given by $\overline{g}_i$ and Σ_{g_i} respectively, i.e.,

$$E[g_i] = \overline{g}_i \quad \text{and} \quad E[(g_i - \overline{g}_i)(g_i - \overline{g}_i)'] = \Sigma_{g_i},$$

where $E[\cdot]$ stands for the mathematical expectation with respect to the basic probability space. Let $g_i \sim (\overline{g}_i, \Sigma_{g_i})$ denotes a random vector g_i which has a feasible probability distribution matching the specified mean $\overline{g}_i$ and covariance Σ_{g_i}.

In this scenario, we formulate the control power problem as the following robust stochastic program with probabilistic constraints

$$\begin{cases} \min & \sum_i p_i \\ \text{s.t. :} & \inf_{g_i \sim (\overline{g}_i, \Sigma_{g_i})} Pr(\gamma_i(p, g_i) \geq \alpha_i) \geq \beta_i, \\ & 0 < p_i \leq p_{i,max}, \\ & \overline{g}_i \in \mathbf{Co}\{\overline{g}_i^1, \ldots, \overline{g}_i^l\},\ \overline{g}_i^k \in \mathbb{R}^n, \\ & \Sigma_{g_i} \in \mathbf{Co}\{\Sigma_{g_i}^1, \ldots, \Sigma_{g_i}^l\},\ \Sigma_{g_i}^k \in \mathcal{M}^{n+}, \\ & i = 1, \ldots, n, \quad k = 1, \ldots, l, \end{cases} \tag{2}$$

where $\beta = [\beta_1, \ldots, \beta_n]' \in \mathbb{R}^n$, with $\beta_i \in (0, 1]$ for $i = 1, \ldots, n$, is the target reliability vector.

Remark 1: Note that the stochastic optimization problem (2) can be considered as a strategic game, where the power controller chooses a decision p while nature picks the worst possible probability distribution of g_i for $i = 1, \ldots, n$ matching the given bounds on the mean $\overline{g}_i$ and covariance Σ_{g_i}.

III. MAIN RESULT

Before showing our main result, the following lemmas will be used as support.

Lemma 1: [7] Consider the vector $\overline{x} \in \mathbb{R}^n$ and the matrix $\Sigma_x \in \mathcal{M}^{n+}$. Then,

$$\sup_{x \sim (\overline{x}, \Sigma_x)} Pr(a'x \geq b) = \frac{1}{1 + c^2},$$

where

$$c^2 = \inf_{a'x \geq b} (x - \overline{x})' \Sigma_x^{-1} (x - \overline{x})$$

is the squared distance from $\overline{x}$ to the set $\{x \in \mathbb{R}^n : a'x \geq b\}$ under the norm induced by the matrix Σ_x^{-1}.

Lemma 2: [8] (*Schur complement*) Consider the matrices A and C symmetric. Then,

$$\begin{bmatrix} A & B \\ \star & C \end{bmatrix} > 0 \quad \Longleftrightarrow \quad C > 0, \qquad A > BC^{-1}B',$$

where, for symmetric matrices, the symbol $\star$ indicates the corresponding off diagonal term.

Using matrix-vector notation, the shorthand formulation of optimization problem (2) is

$$\begin{cases} \min & tr\{P\} \\ \text{s.t. :} & 0 < P \leq P_{max}, \\ & \inf_{g_i \sim (\overline{g}_i, \Sigma_{g_i})} Pr(d_i' P g_i \geq \alpha_i \nu_i) \geq \beta_i, \\ & \overline{g}_i \in \mathbf{Co}\{\overline{g}_i^1, \ldots, \overline{g}_i^l\},\ \overline{g}_i^k \in \mathbb{R}^n, \\ & \Sigma_{g_i} \in \mathbf{Co}\{\Sigma_{g_i}^1, \ldots, \Sigma_{g_i}^l\},\ \Sigma_{g_i}^k \in \mathcal{M}^{n+}, \\ & i = 1, \ldots, n, \quad k = 1, \ldots, l, \end{cases} \tag{3}$$

where the matrix $P = diag\{p = [p_1, \ldots, p_n]'\} \in \mathcal{M}^{n+}$, the matrix $P_{max} = diag\{p_{max} = [p_{1,max}, \ldots, p_{n,max}]'\} \in \mathcal{M}^{n+}$, and the vector $d_i = [\mathbb{1}_{\{i=1\}} - \alpha_i \mathbb{1}_{\{i \neq 1\}}, \ldots, \mathbb{1}_{\{i=n\}} - \alpha_i \mathbb{1}_{\{i \neq n\}}]' \in \mathbb{R}^n$ being $\mathbb{1}_{\{\cdot\}}$ the Dirac measure.

Considering that $\Delta_{g_i}^k \in \mathcal{M}^{n+}$ is the unique square root of matrix $\Sigma_{g_i}^k$ (i.e., $\Delta_{g_i}^{k\prime} \Delta_{g_i}^k = \Sigma_{g_i}^k$) and representing the identity matrix by I, the solution of proposed power control problem is determinate in the following theorem.

Theorem 1: The LMI problem

$$\begin{cases} \min & tr\{P\} \\ \text{s.t. :} & 0 < P < P_{max}, \\ & \begin{bmatrix} d_i' P \overline{g}_i^k - \alpha_i \nu_i & \left(\frac{\beta_i}{1-\beta_i}\right) d_i' P \Delta_{g_i}^k \\ \star & \left(\frac{\beta_i}{1-\beta_i}\right)(d_i' P \overline{g}_i^k - \alpha_i \nu_i) I \end{bmatrix} > 0, \\ & i = 1, \ldots, n, \quad k = 1, \ldots, l, \end{cases} \tag{4}$$

provides the solution for the stochastic optimization problem (3).

Proof: The inner optimization problem in (3) can be rewritten as

$$\sup_{g_i \sim (\overline{g}_i, \Sigma_{g_i})} Pr(-d_i' P g_i \geq -\alpha_i \nu_i) < 1 - \beta_i, \tag{5}$$

for $i = 1, \ldots, n$.

Applying the Lemma 1 on the optimization problem above, we have

$$\sup_{g_i \sim (\overline{g}_i, \Sigma_{g_i})} Pr(-d_i' P g_i \geq -\alpha_i \nu_i) = \frac{1}{1 + c_i^2}, \tag{6}$$

where

$$c_i^2 = \inf_{-d_i' P g_i \geq -\alpha_i \nu_i} (g_i - \overline{g}_i)' \Sigma_{g_i}^{-1} (g_i - \overline{g}_i). \tag{7}$$

Note that the values of β_i are usually different from zero thus we have to select the matrix P such that $\overline{g}_i \notin \{g_i : -d_i' P g_i \geq -\alpha_i \nu_i\}$.

The optimal solution for the optimization problem (7) can be obtained applying the Karush-Kuhn-Tucker conditions. To this end, define the Lagrange function

$$L(g_i, \lambda_i) = (g_i - \overline{g}_i)' \Sigma_{g_i}^{-1} (g_i - \overline{g}_i) + \lambda_i(-d_i' P g_i + \alpha_i \nu_i),$$

for $\lambda_i \geq 0$. The conditions for g_i to be a global minima are

$$\frac{\partial L(g_i, \lambda_i)}{\partial g_i} = 2\Sigma_{g_i}^{-1} (g_i - \overline{g}_i) - \lambda_i P d_i = 0, \tag{8}$$

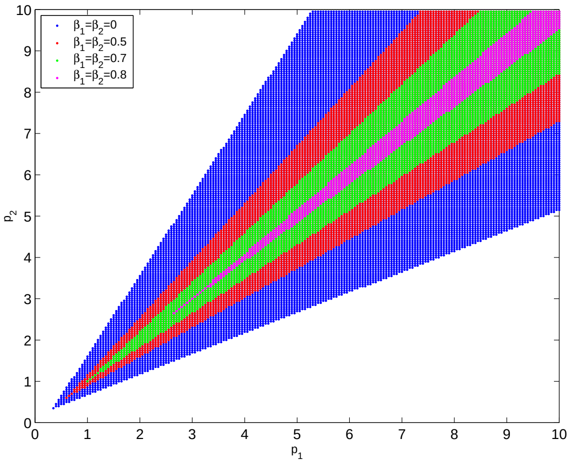

Fig. 1. Robust feasibility set.

and

$$\frac{\partial L(g_i, \lambda_i)}{\partial \lambda_i} = -d_i' P g_i + \alpha_i \nu_i = 0. \tag{9}$$

From equations (8) and (9) we have that

$$\lambda_i = \frac{2(d_i' P \overline{g}_i - \alpha_i \nu_i)}{d_i' P \Sigma_{g_i} P d_i}.$$

Thus,

$$c_i^2 = \frac{\lambda_i^2}{4} d_i' P \Sigma_{g_i} P d_i = \frac{(d_i' P \overline{g}_i - \alpha_i \nu_i)^2}{d_i' P \Sigma_{g_i} P d_i}. \tag{10}$$

By replacing equations (6) and (10) into inequality (5), we have

$$(d_i' P \overline{g}_i - \alpha_i \nu_i)^2 > \Big(\frac{\beta_i}{1-\beta_i}\Big) d_i' P \Sigma_{g_i} P d_i, \tag{11}$$

Finally applying the Schur complement (see Lemma 2) in (11), the LMI problem (4) is justified. ■

Remark 2: Note that when $\beta_i = 0$ for $i = 1, \ldots, n$ and $l = 1$, the stochastic optimization problem (4) is reduced to the usual deterministic optimization problem (1) if the channel gains g_i are replaced by their mean value $\overline{g}_i$.

Remark 3: Notice that if there exist an upper bound for the transmitted power then high reliability requirements may become the optimization problem (4) infeasible. This is due to fast increase of $\sqrt{\beta_i/(1-\beta_i)}$ when β_i tends to 1.

IV. Numerical example

To illustrate the previous discussion, consider a wireless cellular network with 2 active users. The target SIR and the receiver noise are 5dB and 0.05W, respectively. Two design cases are considered:

(i) For this case, the mean vector and covariance matrix are assumed to be exactly know ($l = 1$), such that

$$\overline{g}_1 = \begin{bmatrix} 0.986 & 0.160 \end{bmatrix}, \quad \overline{g}_2 = \begin{bmatrix} 0.150 & 0.956 \end{bmatrix},$$
$$\Sigma_{g_1} = \Sigma_{g_2} = \begin{bmatrix} 0.004 & 0 \\ 0 & 0.004 \end{bmatrix}.$$

Figure 1 shows the feasibility set of optimization problem (4) for several reliability thresholds values. Clearly, the system reliability can be increased by allocating power in such a way that each transmitter has extra margin of SIR. As result, the system capacity is reduced. Applying the Theorem 1, the allocation of transmitted power is $p = \begin{bmatrix} 2.60 & 2.60 \end{bmatrix}$ for $\beta = \begin{bmatrix} 0.8 & 0.8 \end{bmatrix}$.

(ii) For this case we assume that the mean vector and covariance matrix are not exactly known, but they are such that

$$\overline{g}_1 \in \mathbf{Co}\{\begin{bmatrix} 1.00 & 0.18 \end{bmatrix}, \begin{bmatrix} 0.96 & 0.14 \end{bmatrix}\},$$
$$\overline{g}_2 \in \mathbf{Co}\{\begin{bmatrix} 0.17 & 0.97 \end{bmatrix}, \begin{bmatrix} 0.13 & 0.93 \end{bmatrix}\},$$
$$\Sigma_{g_1} = \Sigma_{g_2} \in \mathbf{Co}\{\begin{bmatrix} 0.003 & 0 \\ 0 & 0.003 \end{bmatrix}, \begin{bmatrix} 0.005 & 0 \\ 0 & 0.005 \end{bmatrix}\}.$$

The transmitted power in this case is $p = \begin{bmatrix} 3.28 & 3.29 \end{bmatrix}$ for $\beta = \begin{bmatrix} 0.8 & 0.8 \end{bmatrix}$.

V. Conclusion

We address the problem of allocating power in a wireless network considering the power gains as random vectors whose probability distribution is partially known. We assume that, it satisfies imprecise mean and covariance constraints. In this framework, the power control problem is modelled by a stochastic optimization problem with probabilistic constraints and the robust solution is established in terms of LMI problem.

A further research topic is constrain the underlying distribution to get less conservative controllers.

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