Effects of mutual coupling and amplitude/phase mismatch on a WCDMA downlink receiver with multiple antennas

Fabbryccio A. C. M. Cardoso, Marcelo A. C. Fernandes, José G. Chiquito and Dalton S. Arantes

***Abstract*— In this paper we study the effects of mutual coupling and amplitude/phase mismatch on a WCDMA downlink receiver using multiple antennas in the user terminal. Analytical expressions are obtained for the Zero-Forcing and for the Minimum Mean-Square Error receiver, in which the implicit effects of the mutual couplig and amplitude/phase mismatch matrices are taken into account. Simulation results for a time-variant COST-259 channel model, using an adaptive LMS receiver, show that these non-ideal behaviors in a co-linear array have negligible effects on performance when the antenna spacing is $\lambda/2$. For $\lambda/4$ spacing there is a loss in receiver performance for low values of signal-to-noise ratios.**

***Keywords*— Antenna coupling, antenna array, wcdma downlink.**

I. INTRODUCTION

The effects of mutual coupling and amplitude/phase mismatch in an array of receiving antennas can be represented with good accuracy by linear transformations operating on the antenna output signals [1][2][3], as shown in Figure 1. In this model the receiver noise is added after these linear operations.

Consider a Multi-Target Space-Time Receiver (MT-STR) for the user terminal in WCDMA systems as shown in Figure 2, which is described in detail in [4]. The usual approach in the design of such receiver considers an ideal co-linear antenna array, in which the antenna coupling and other effects, such as amplitude and phase mismatches, are not taken into account. In the present work we study the effects of antenna coupling and amplitude/phase mismatch on the performance of this space-time receiver.

In this linear model the coupling matrix $\mathbf{C}$ is obtained according to the "induced electromagnetic field method" [5], whereas the mismatch matrix is simply a diagonal matrix $\mathbf{M}$, given by

$$\mathbf{M} = \begin{bmatrix} b_0 e^{j\phi_0} & 0 & \cdots & 0 \\ 0 & b_1 e^{j\phi_1} & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & & b_{M-1} e^{j\phi_{M-1}} \end{bmatrix}$$

where ϕ_i, the phase mismatches, can assume any value between 0 and 2π, and b_i, the amplitude mismatches, are real numbers around 1. We will assume that $\sum_{i=0}^{i=M-1} (b_i)^2 = M$, to ensure that the overall input signal power remains the same at the output.

The combined effect of mutual coupling and amplitude/phase mismatch can then be represented by a single $M \times M$ matrix $\mathbf{D} = \mathbf{M}\,\mathbf{C}$.

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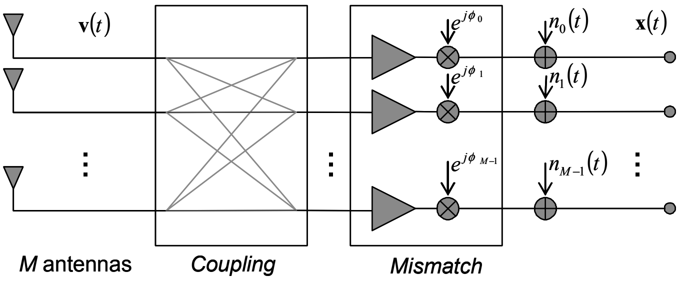

Fig. 1. Model for Mutual Coupling and Amplitude/Phase Mismatch in an antenna array.

In the present work we assume a co-linear array with six antenna elements, i.e., $M = 6$. According to results presented in [4], the complex-valued elements of the coupling matrix can be calculated for any value of antenna spacing. In particular, for a load impedance of 50Ω, the magnitudes of the entries in matrix $\mathbf{C}$, for a $\lambda/2$ equally-spaced co-linear array, are given by the matrix $\mathbf{C}_m$ as follows

$$\mathbf{C}_m = [|c_{ij}|] = \begin{bmatrix} 0.4822 & 0.0922 & 0.0374 & 0.0226 & 0.0163 & 0.0137 \\ 0.0922 & 0.4816 & 0.0871 & 0.0348 & 0.0212 & 0.0163 \\ 0.0374 & 0.0871 & 0.4822 & 0.0862 & 0.0348 & 0.0226 \\ 0.0226 & 0.0348 & 0.0862 & 0.4822 & 0.0871 & 0.0374 \\ 0.0163 & 0.0212 & 0.0348 & 0.0871 & 0.4816 & 0.0922 \\ 0.0137 & 0.0163 & 0.0226 & 0.0374 & 0.0922 & 0.4822 \end{bmatrix}$$

For a $\lambda/4$ equally-spaced co-linear array the matrix $\mathbf{C}_m$ is given by

$$\mathbf{C}_m = [|c_{ij}|] = \begin{bmatrix} 0.4776 & 0.1683 & 0.0679 & 0.0371 & 0.0259 & 0.0152 \\ 0.1683 & 0.4752 & 0.1904 & 0.0785 & 0.0472 & 0.0259 \\ 0.0679 & 0.1904 & 0.4784 & 0.1833 & 0.0785 & 0.0371 \\ 0.0371 & 0.0785 & 0.1833 & 0.4784 & 0.1904 & 0.0679 \\ 0.0259 & 0.0472 & 0.0785 & 0.1904 & 0.4752 & 0.1683 \\ 0.0152 & 0.0259 & 0.0371 & 0.0679 & 0.1683 & 0.4776 \end{bmatrix}$$

Notice that the off-diagonal elements in matrix $\mathbf{C}_m$ for $\lambda/4$-spacing are considerably larger than the corresponding values in matrix $\mathbf{C}_m$ for $\lambda/2$-spacing.

For the simulations performed in this work, the amplitudes (satisfying $\sum_{i=0}^{i=5} (b_i)^2 = 6$) were arbitrarily chosen as $(1.124,$ $0.749,$ $0.749,$ $1.030,$ $1.405,$ $0.656)$ and the phase mismatches (in degrees) were chosen as $(60.00,$ $72.00,$ $-160.00,$ $154.28,$ $-98.18,$ $-90.00)$.

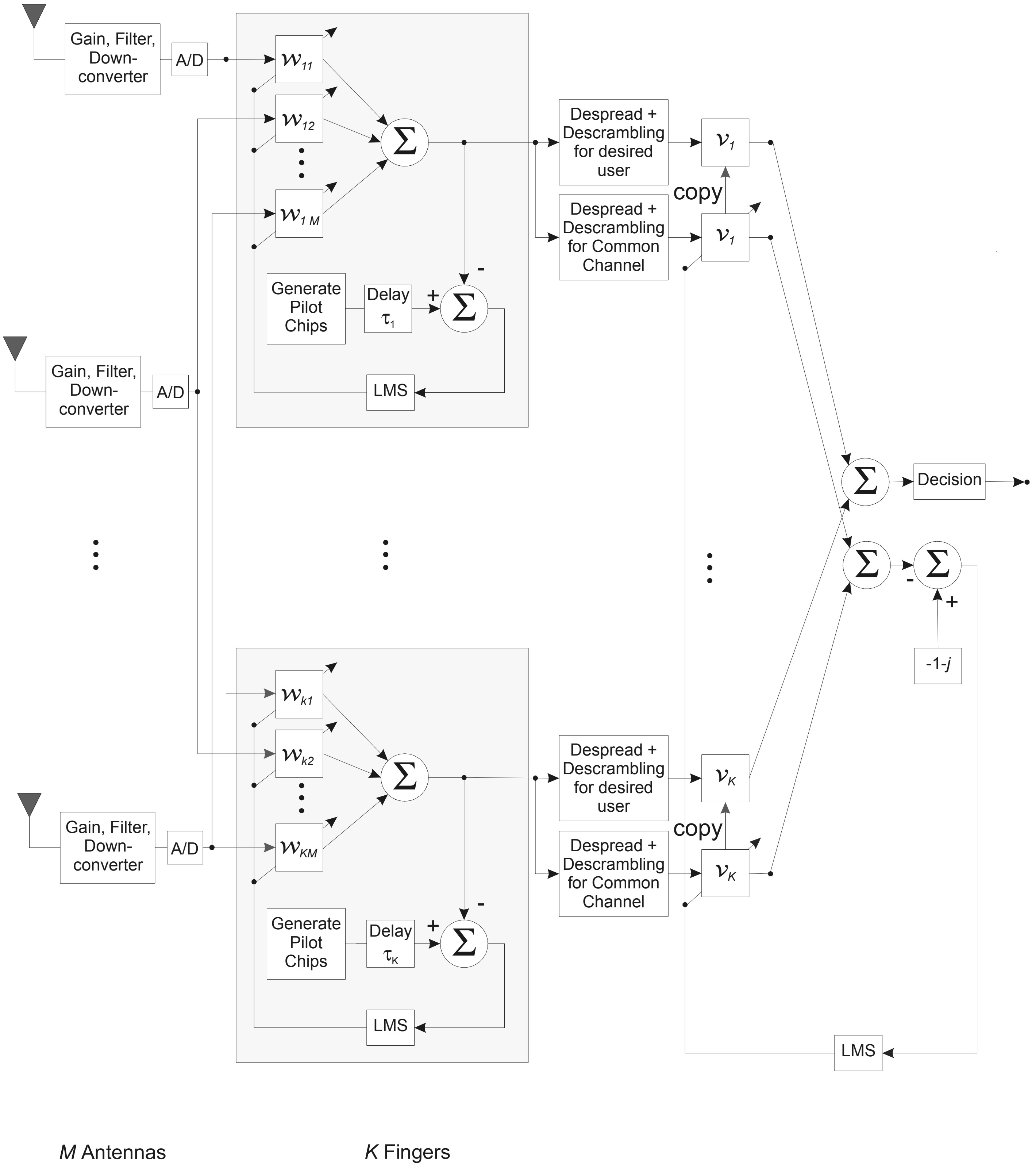

Fig. 2. Details of the adaptation processes in the MT-STR receiver, where possible equalizers in each finger have been suppressed for convenience.

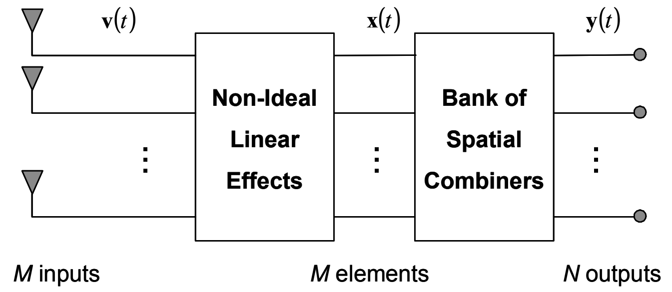

Fig. 3. Front-end of the MT-STR receiver with incorporated non-ideal effects.

With the above results, the magnitudes of the elements in matrix $\mathbf{D} = \mathbf{M}\,\mathbf{C}$, for $\lambda/2$-spacing and load impedance of 50Ω, are given by

$$\mathbf{D}_m{=}[|d_{ij}|]{=}\begin{bmatrix} 0.5419 & 0.1036 & 0.0421 & 0.0254 & 0.0183 & 0.0154 \\ 0.0691 & 0.3609 & 0.0652 & 0.0261 & 0.0159 & 0.0122 \\ 0.0316 & 0.0734 & 0.4064 & 0.0727 & 0.0293 & 0.0191 \\ 0.0233 & 0.0359 & 0.0888 & 0.4968 & 0.0897 & 0.0386 \\ 0.0229 & 0.0298 & 0.0489 & 0.1223 & 0.6766 & 0.1295 \\ 0.0090 & 0.0107 & 0.0148 & 0.0245 & 0.0604 & 0.3161 \end{bmatrix}$$

The ideal MT-STR receiver in Figure 2 can then be extended in order to incorporate the non-ideal effects of mutual coupling and phase/amplitude mismatch. If we consider only the bank of spatial combiners in the front-end of the ideal MT-STR receiver, we obtain the non-ideal front-end shown in Figure 3.

The discrete values of $\mathbf{x}\,(t)$ in Figure 1 are given by

$$\mathbf{x}\,(t_k) = \mathbf{D}\mathbf{v}\,(t_k) + \mathbf{n}\,(t_k)\,, \tag{1}$$

where $\mathbf{n}\,(t_k)$ are the discrete values of the additive Gaussian noise. The output signal vector $\mathbf{y}\,(t_k)$ is then given by

$$\mathbf{y}\,(t_k) = \mathbf{W}^H\,(t_k)\;\mathbf{x}\,(t_k)\,, \tag{2}$$

where $\mathbf{W}\,(t_k)$ collects the N spatial combiners, each one with M elements, that is,

$$\mathbf{W} = \begin{bmatrix} \mathbf{w}_0 & \mathbf{w}_1 & \cdots & \mathbf{w}_{N-1} \end{bmatrix}_{M\times N}\,.$$

The generic channel model in Figure 4 incorporates the wireless multipath channel and the ideal antennas, and its output is given by

$$\mathbf{v}\,(t_k) = \mathbf{A}^H\,(t_k)\;\mathbf{u}\,(t_k)\,, \tag{3}$$

where $v_m(t_k)$ is the signal received from antenna m at time t_k for the transmitted signal $\mathbf{u}\,(t_k)$, and $\mathbf{A}\,(t_k)$ is a known time-variant matrix representing the vector channel (with spatial information), as given by the COST-259 model (with projected signals on the antenna elements). Each column of matrix $\mathbf{A}\,(t_k)$ represents the multipath channel model seen from the antenna perspective, and vector $\mathbf{u}\,(t_k)$ collects the delayed samples of the transmitted signal due to the multipath channel, that is,

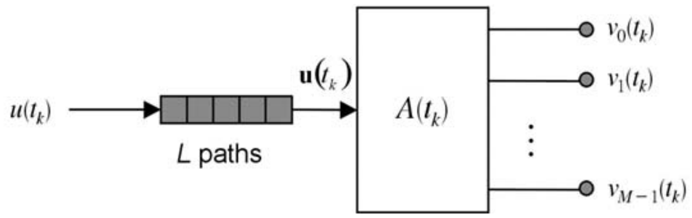

Fig. 4. Generic multipath channel model.

$$\mathbf{u}^T\,(t_k) = \begin{bmatrix} u\,(t_k) & u\,(t_k - T) & \cdots & u\,(t_k - (L-1)\,T) \end{bmatrix}.$$

II. Zero-Forcing Solution

To simplify notation, from now on we will drop the index k in the t_k. By direct manipulation of the above equations we obtain the output vector $\mathbf{y}\,(t)$ as a function of $\mathbf{u}\,(t)$ and $\mathbf{n}\,(t)$, as follows

$$\mathbf{y}\,(t) = \mathbf{W}^H\,(t)\;\mathbf{D}\,\mathbf{A}^H\,(t)\;\mathbf{u}\,(t) + \mathbf{W}^H\,(t)\;\mathbf{n}\,(t)\,.$$

This means that the resulting channels corresponding to each output terminal in the spatial combiners, are given by the rows of the matrix

$$\mathbf{W}^H\,(t)\;\mathbf{D}\,\mathbf{A}^H\,(t)\,.$$

The Zero-Forcing solution is then given by

$$\mathbf{W}^H\,(t)\;\mathbf{D}\,\mathbf{A}^H\,(t) = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & \cdots & 0 \end{bmatrix},$$

that is,

$$\mathbf{W}^H\,(t)\;\mathbf{D}\,\mathbf{A}^H\,(t) = \begin{bmatrix} \mathbf{I}_N & \mathbf{0}_{N\times(L-N)} \end{bmatrix}.$$

Solving this system for $\mathbf{W}^H\,(t)$ we get

$$\mathbf{W}^H\,(t) = \begin{bmatrix} \mathbf{I}_N & \mathbf{0}_{N\times(L-N)} \end{bmatrix}\mathbf{A}\,(t)\,\left[\mathbf{A}^H\,(t)\,\mathbf{A}\,(t)\right]^{-1}\mathbf{D}^{-1}. \tag{4}$$

III. Minimum Mean-Square Error Solution

In order to obtain the minimum mean-square error solution, we define the mean-square error as

$$\begin{aligned} J\,(\mathbf{W}\,(t)) &= E\left\{\sum_i e_i^*\,(t)\,e_i\,(t)\right\} \\ &= E\left\{\mathbf{e}^H\,(t)\;\mathbf{e}\,(t)\right\}, \end{aligned}$$

where $\mathbf{e}\,(t)$ is the error between the desired signal $\mathbf{d}\,(t)$ (a pilot signal in WCDMA) and the received signal $\mathbf{y}\,(t)$, that is,

$$\mathbf{e}\,(t) = \mathbf{d}\,(t) - \mathbf{y}\,(t)\,. \tag{5}$$

The minimum value of $J(\mathbf{W}(t))$ is obtained when

$$\frac{\partial J(\mathbf{W}(t))}{\partial \mathbf{w}_i} = E\left\{2\left(\mathbf{e}^H(t)\,\mathbf{q}_i\right)\,\mathbf{x}(t)\right\} = \mathbf{0}, \qquad (6)$$

where

$$\mathbf{q}_i = [q_j] \text{ and } q_j = \begin{cases} 1 & \text{if } j = i \\ 0 & \text{if } j = 1, \cdots, N \text{ and } j \neq i. \end{cases}$$

By direct substitution of Eq. 5 in Eq. 6, we get

$$E\left\{\left[\left(\mathbf{d}(t) - \mathbf{W}^H(t)\mathbf{x}(t)\right)^H \mathbf{q}_i\right]\,\mathbf{x}(t)\right\} = \mathbf{0},$$

which results in

$$E\left\{\left[\left(\mathbf{d}^H(t) - \mathbf{x}^H(t)\,\mathbf{W}(t)\right)\mathbf{q}_i\right]\,\mathbf{x}(t)\right\} = \mathbf{0}\;.$$

Since $\left(\mathbf{d}^H(t) - \mathbf{x}^H(t)\,\mathbf{W}\right)\mathbf{q}_i$ is a scalar, we obtain

$$E\left\{\mathbf{x}(t)\left[\left(\mathbf{d}^H(t) - \mathbf{x}^H(t)\,\mathbf{W}(t)\right)\mathbf{q}_i\right]\right\} = \mathbf{0}\;.$$

Therefore,

$$\mathbf{R}_{xx}(t)\,\mathbf{W}(t)\,\mathbf{q}_i = \mathbf{R}_{xd}(t)\,\mathbf{q}_i\;.$$

For all i, the solution to this equation is

$$\mathbf{W}(t) = \mathbf{R}_{xx}^{-1}(t)\,\mathbf{R}_{xd}(t)\;. \qquad (7)$$

Since

$$\mathbf{R}_{xx}(t) = \mathbf{D}\,\mathbf{R}_{vv}(t)\,\mathbf{D}^H + \mathbf{R}_{nn}\;,$$

and

$$\mathbf{R}_{xd}(t) = \mathbf{D}\,\mathbf{R}_{vd}(t)\;,$$

we get the MMSE solution

$$\mathbf{W}(t) = \left(\mathbf{D}\,\mathbf{R}_{vv}(t)\,\mathbf{D}^H + \mathbf{R}_{nn}\right)^{-1}\mathbf{D}\,\mathbf{R}_{vd}(t)\;. \qquad (8)$$

It is interesting to observe what happens with this solution in the case of high signal-to-noise ratio. In this situation the noise correlation matrix $\mathbf{R}_{nn}$ is negligible compared to $\mathbf{D}\,\mathbf{R}_{vv}(t)\,\mathbf{D}^H$, and the MMSE solution becomes

$$\mathbf{W}(t) \cong \left(\mathbf{D}^{-1}\right)^H \mathbf{R}_{vv}^{-1}(t)\;\mathbf{R}_{vd}(t)\;.$$

This result, together with Eqs. 2 and 1, implies that

$$\begin{aligned}\mathbf{y}(t) &= \mathbf{W}^H(t)\;\mathbf{D}\,\mathbf{v}(t) + \mathbf{W}^H(t)\;\mathbf{n}(t)\\ &\cong \mathbf{W}^H(t)\;\mathbf{D}\,\mathbf{v}(t)\\ &= \mathbf{R}_{vd}^H(t)\;\mathbf{R}_{vv}^{-H}(t)\,\mathbf{v}(t)\,.\end{aligned}$$

Since $\mathbf{y}(t) = \mathbf{R}_{vd}^H(t)\;\mathbf{R}_{vv}^{-H}(t)\;\mathbf{v}(t)$ is independent of $\mathbf{D}$, we conclude that for high signal-to-noise ratio the spatial combiners for the MMSE solution are able to compensate for the mutual coupling and amplitude/phase mismatch in the array.

To analyze the channel effects in the MMSE solution, notice that

$$\mathbf{R}_{vv}(t) = E\left\{\mathbf{A}^H(t)\;\mathbf{u}(t)\;\mathbf{u}^H(t)\;\mathbf{A}(t)\right\}.$$

For simplicity, it is assumed that the channel has a deterministic time-varying behavior, that is,

$$\begin{aligned}\mathbf{R}_{vv}(t) &= \mathbf{A}^H(t)\;E\left\{\mathbf{u}(t)\;\mathbf{u}^H(t)\right\}\mathbf{A}(t)\\ &= \mathbf{A}^H(t)\;\mathbf{R}_{uu}(t)\,\mathbf{A}(t)\;,\end{aligned}$$

and

$$\mathbf{R}_{vd}(t) = \mathbf{A}^H(t)\;\mathbf{R}_{ud}(t)\;.$$

The MMSE solution is then given by

$$\mathbf{W}(t) = \left(\mathbf{D}\,\mathbf{A}^H(t)\;\mathbf{R}_{uu}(t)\;\mathbf{A}(t)\;\mathbf{D}^H + \mathbf{R}_{nn}\right)^{-1}\mathbf{D}\,\mathbf{A}^H(t)\;\mathbf{R}_{ud}(t)\,.$$

Assuming high signal-to-noise ratio, that is,

$$\mathbf{R}_{nn} \approx \mathbf{0}\;,$$

and that in the downlink channel for WCDMA the transmitted signals are orthogonal, i.e.,

$$\mathbf{R}_{uu}(t) = \mathbf{I}_L, \qquad (9)$$

$$\mathbf{R}_{ud} = \begin{bmatrix} \mathbf{I}_N \\ \mathbf{0}_{(L-N)\times N} \end{bmatrix}, \qquad (10)$$

then

$$\mathbf{W}(t) = \mathbf{D}^{-H}\left(\mathbf{A^H(t)}\;\mathbf{A(t)}\right)^{-1}\mathbf{A}^H(t)\begin{bmatrix} \mathbf{I}_N \\ \mathbf{0}_{(L-N)\times N} \end{bmatrix}$$

which is the same Zero-Forcing solution given in Eq. 4.

IV. SIMULATION RESULTS WITH COST-259

The previous solutions for the Zero-Forcing and MMSE receiver implicitly takes into account the effects of the combined matrix $\mathbf{D}$. We have assumed that the channel is known and given by $\mathbf{A}(t)$. However, in this section we present simulation results for an adaptive LMS receiver that is more suitable for channel traces obtained with COST-259. We consider a macrocell, typical urban environment, with vehicle speed of 50 km/h. The spreading factor is 16 and the number of users per cell was set to 5 and 10. We assume a 3-cell scenario with soft handoff, as described in [4]. The results presented here try to answer some questions on how the non-ideal effects impact the bit error rate performance of the MT-STR receiver.

In the following figures we show the bit error rate for the MT-STR and Rake receivers, as a function of E_b/N_0, for co-linear antenna arrays with $\lambda/2$ and $\lambda/4$ spacings. We assume that the MT-STR has 6 antenna elements and 6 spatial combiners, and that the Rake receiver is also implemented with 6 fingers.

In Figure 5 we take into account the load matrix but neglect the mutual coupling and amplitude/phase mismatch, i.e., $\mathbf{D} \simeq 0.5\,\mathbf{I}$. In Figure 6 we repeat the simulations for a non-ideal array with $\lambda/2$-spacing with mutual coupling. These results lead us to conclude that the coupling effect is negligible for $\lambda/2$-spacing when compared with the ideal case without coupling. Besides, as in the MMSE analytical solution, the receiver was not able to fully compensate for the non-ideal effects of coupling for low signal-to-noise ratio.

The effect of decreasing the array spacing is quite visible in Figure 7, which is the result obtained for a non-ideal array with $\lambda/4$-spacing and mutual coupling. In fact, differently from the $\lambda/2$-spacing, the coupling effect is no longer negligible.

In Figure 8 we show the results for a non-ideal array with $\lambda/2$-spacing with mutual coupling and amplitude/phase mismatch. Notice that for this case the performance of the MT-STR, as opposed to the Rake receiver, is improved when mismatch is taken into account. This effect may be a consequence of assuming that $\sum_{i=0}^{i=M-1}(b_i)^2 = M$, which guarantees that the overall input signal power remains the same at the output, but further investigation is required.

Finally, in Figure 9 we collect the previous results for a non-ideal antenna array with mutual coupling, either for $\lambda/4$ and $\lambda/2$ antenna spacings, together with the results for an array with only amplitude/phase mismatch for $\lambda/2$-spacing. Note that for BER $= 10^{-2}$ the antenna array with $\lambda/2$-spacing provides a gain of about 5 dB when compared to the case of $\lambda/4$-spacing. On the other hand, the array with $\lambda/4$-spacing occupies only half the space occupied by the array with $\lambda/2$-spacing. This tradeoff may be an important aspect to be considered in the implementation of advanced receivers for the user terminal.

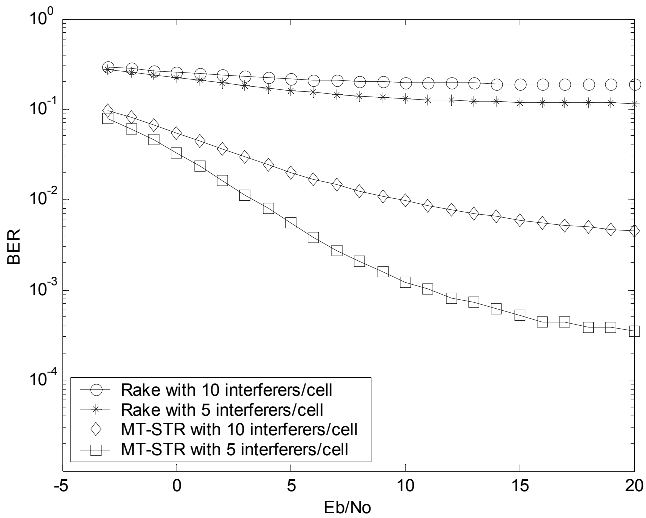

Fig. 5. Bit Error Rate for the MT-STR and Rake receivers, as a function of E_b/N_0, for ideal array without mutual coupling and amplitude/phase mismatch. Antenna spacing is $\lambda/2$ and load impedance is 50 ohms.

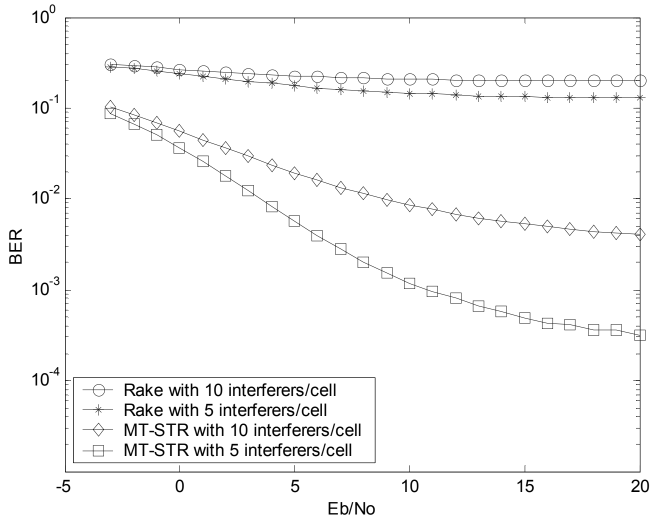

Fig. 6. Bit Error Rate for the MT-STR and Rake receivers, as a function of E_b/N_0, for non-ideal array with mutual coupling. Antenna spacing is $\lambda/2$.

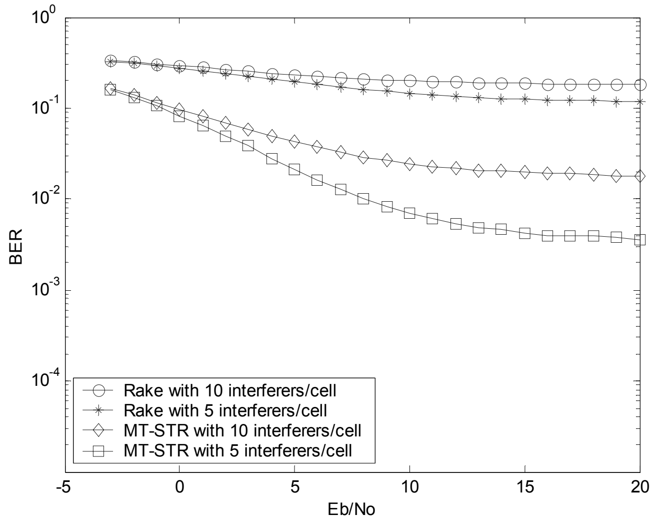

Fig. 7. Bit Error Rate for the MT-STR and Rake receivers, as a function of E_b/N_0, for non-ideal array with mutual coupling. Antenna spacing is $\lambda/4$.

V. Conclusions

Previous works on the effects of mutual coupling and amplitude/phase mismatches in an array of *transmitting* antennas indicate that these impairments are a source of problems in the process of downlink beamforming in a base station. However, the results presented in this paper indicate that mutual coupling and amplitude/phase mismatches in a co-linear *receiving* array usually have negligible effects on a WCDMA *downlink receiver* for $\lambda/2$ antenna spacing. For $\lambda/4$-spacing these impairments may have non-negligible effects on receiver performance.

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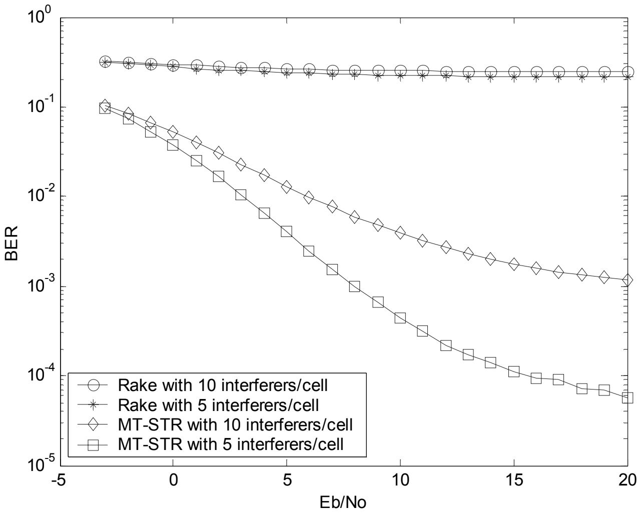

Fig. 8. Bit Error Rate for the MT-STR and Rake receivers, as a function of E_b/N_0, for non-ideal array with mutual coupling and amplitude/phase mismatch. Antenna spacing is $\lambda/2$.

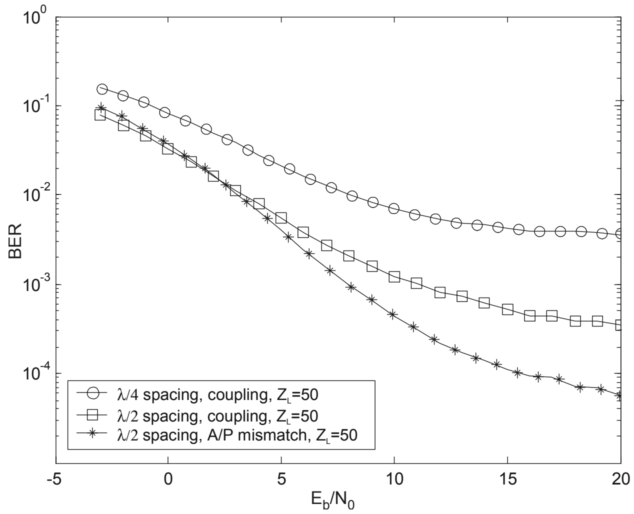

Fig. 9. Bit Error Rate for the MT-STR, as a function of E_b/N_0, for non-ideal array with mutual coupling and with $\lambda/4$ and $\lambda/2$ antenna spacings. Results for array with only amplitude/phase mismatch is also shown for $\lambda/2$-spacing.

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