H_2 and H_∞ Approaches for Time-Variant Multipath Channel Equalization

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***Abstract*— The H_∞ filters are interesting alternatives to the famed Kalman filter in most estimation problems. They give hard upper bounds on the estimation errors, independently of the disturbances distributions, while the optimality of the Kalman filter relies on the knowledge of statistical properties of the disturbances. H_2 and H_∞ equalizers were implemented in order to deal with time-variant channels and their behavior was investigated.**

***Keywords*— Linear equalization, H_∞ filtering, wireless communications.**

I. INTRODUCTION

IN a typical wireless environment, a transmitted signal often reaches a receiver via multiple propagation paths. In high-bit-rate transmission, the propagation delay spread of the time-dispersive (or frequency-selective) multipath fading channel results in intersymbol interference (ISI), which dramatically increases the transmission Bit Error Rate (BER).

Digital communication systems designed to perform the data transmission in short time blocks may prevent another aspect which degrades the communication performance: the time-variance of the channel, common aspect of mobile radio environments. Data blocks with time duration smaller than the coherence time of the channel, time interval where the impulse response of the channel can be considered constant, are free from the Doppler spread effect. However, if the time required to transmit a data block overcomes the coherence time of the multipath channel, the receiver must deal with ISI and Doppler (or time-varying channel) effects.

Channel equalization is an efficient technique to compensate ISI. A generic adaptive equalizer is a time-varying filter which must be constantly returned. Adaptive algorithms perform the task of equalizing a channel in a step-by-step fashion. A standard approach to this problem is to minimize some quadratic criterion involving estimation errors.

Wiener filter theory provides the optimum solution to the problem whose optimization criterion is the minimization of the variance of the equalizer error. For time-invariant channels, the error-performance surface is fixed and the essential requirement is to seek its minimum point (Wiener solution), and thereby assure optimum or near-optimum performance [1]. However, time-varying channels imply time-varying error-performance surfaces. In this case, the equalizer is also required to continuously track the channel variations.

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The general operating modes of an adaptive equalizer include training and tracking. In the training mode, a known, fixed-length training sequence is sent by the transmitter so that the receiver's equalizer may average to a proper setting through a recursive algorithm. Immediately following the training sequence, the data sequence is sent and the equalizer switches to direct-decision operating mode, where it also utilizes a recursive algorithm to evaluate the channel and estimate its coefficients to compensate for the channel. However, in the direct-decision mode, the equalizer utilizes the decided sequence in the place of a training one.

The formulation of the channel equalization problem in a state space model provides a close connection with the field of control and estimation theory, which have been intensively studied over the last decades. Thus, some of the most celebrated recursive algorithms used in channel equalization may be represented in a concise manner, clarifying their own characteristics and interrelations, and making possible new approaches.

The exponentially weighted Recursive Least Squares (RLS) algorithm is one of the most used algorithms in channel equalization and is based on the minimization of the variance of the equalizer error, i.e., the H_2 estimation. The well known Kalman filter is the optimum recursive estimator in the least squares (H_2) sense. The connection between the standard RLS and Kalman filtering theory was shown in [2]. Using Kalman filtering algorithm, it is assumed that the receiver knows the statistical properties of the additive white Gaussian noise (AWGN) and of the uncertainties of the model. Then, it is possible that the Kalman filter is not robust against any uncertainty of channel models, that means, small modeling errors may result in large equalization errors.

Recently, another approach to the estimation problem has been considered, the H_∞ filtering. The aim in H_∞ filtering is to minimize the maximal energy gain from the modeling errors and noise to the estimation errors. The H_∞ criterion can thus be understood as a worst-case criterion: the estimator will be robust against the worst possible disturbances. This is a completely different approach to the estimation problem compared to the least squares, or H_2, approaches that are the standard tools today [3].

In this work we investigate the behavior of H_2 and H_∞ equalizers for time-variant channels. Section II gives a brief discussion on H_2 estimation. In Section III we deduce a state space formulation of the equalization problem and present the standard (Kalman-based) RLS algorithm. Section IV presents a brief introduction on H_∞ estimation and an H_∞ equalizer model. Simulation results are discussed in Section V and the conclusions of this work are given in Section VI.

II. H_2 OPTIMAL ESTIMATION

The Kalman filter is known to be the best linear estimator in the least squares (H_2) sense and has been the subject of extensive research and application for the last decades. It addresses the general problem of trying to estimate the state of a discrete-time process that is governed by the linear stochastic difference equation:

$$\mathbf{x}_{k+1} = A_k \mathbf{x}_k + B_k \mathbf{p}_k \tag{1}$$

using the measure given by the following equation:

$$\mathbf{y}_k = C_k \mathbf{x}_k + \mathbf{v}_k \tag{2}$$

where k is a discrete-time index, and $\mathbf{x}_k \in \mathbb{R}^n$, $\mathbf{y}_k \in \mathbb{R}^m$, $\mathbf{p}_k \in \mathbb{R}^n$, $\mathbf{v}_k \in \mathbb{R}^m$ are, respectively, the state vector, the measurement, the process disturbance and the measurement noise. The initial state $\mathbf{x}_0$ is a random variable with mean $\bar{\mathbf{x}}_0$ and covariance $\prod$, and A_k, B_k and C_k are known matrices of appropriate sizes.

The random variables $\mathbf{p}_k$ and $\mathbf{v}_k$ are assumed to be independent, white, and with zero-mean normal probability distributions:

$$p(\mathbf{p}) \sim N(0, Q) \tag{3}$$

$$p(\mathbf{v}) \sim N(0, R) \tag{4}$$

It follows a brief discussion about the Kalman filtering theory that can be found in [1], [2], [3] and [4] with more details. The Kalman filter estimates a process by using a feedback control: the filter estimates the process state at some time and then obtains feedback in the form of (noisy) measurements. Thus, the equations of Kalman filter fall in two groups: time update equations and measurement update equations. The time update equations are responsible for obtaining the *a priori* estimates for the next time step. The measurement update equations provide the feedback, i.e., the incorporation of new measurements into the *a priori* estimates to obtain improved *a posteriori* estimates.

Consider the process to be estimated represented by (1-4). In practice, the process disturbance covariance Q and the measurement noise covariance R matrices might change with each time step, however here we assume they are constant. Other definitions are made in Table I.

In deriving the equations for the Kalman filter, we begin with the goal of finding an equation which computes an *a posteriori* state estimate $\hat{\mathbf{x}}_k$ as a linear combination of an *a priori* estimate $\hat{\mathbf{x}^{\mathbf{a}}}_k$ and the weighted measurement *innovation*:

$$\begin{aligned} \hat{\mathbf{x}}_k &= \hat{\mathbf{x}^{\mathbf{a}}}_k + K_k \left[\mathbf{y}_k - C_k \hat{\mathbf{x}^{\mathbf{a}}}_k \right] \\ \hat{\mathbf{x}^{\mathbf{a}}}_k &= A_{k-1} \hat{\mathbf{x}}_{k-1} \end{aligned} \tag{5}$$

where the *innovation* or *residual* reflects the discrepancy between the predicted measurement and the actual measurement.

The matrix $K_k \in \mathbb{R}^{n \times m}$ is chosen to be the gain factor (Kalman gain) that minimizes the *a posteriori* error covariance

TABLE I

DEFINITION OF PARAMETERS

$\mathbf{x}_k$	state to be estimated
$\hat{\mathbf{x}^{\mathbf{a}}}_k$	*a priori* state estimate
$\hat{\mathbf{x}}_k$	*a posteriori* state estimate
$\mathbf{e^a}_k = \mathbf{x}_k - \hat{\mathbf{x}^{\mathbf{a}}}_k$	*a priori* estimate error
$\mathbf{e}_k = \mathbf{x}_k - \hat{\mathbf{x}}_k$	*a posteriori* estimate error
$P_k^a = E\{\mathbf{e^a}_k \mathbf{e^a}_k^T\}$	*a priori* estimate error covariance
$P_k = E\{\mathbf{e}_k \mathbf{e}_k^T\}$	*a posteriori* estimate error covariance

P_k. Then, using (5) and the definitions of Table I, after some work we may write P_k as a function of K_k:

$$\begin{aligned} P_k = P_k^a - P_k^a C_k^T K_k^T - K_k C_k P_k^a \\ + K_k C_k P_k^a C_k^T K_k^T + K_k R K_k^T \end{aligned} \tag{6}$$

where R is the m x m measurement noise covariance matrix.

Taking the derivative of the trace of (6) with respect to K_k, setting that result equal to zero and solving for K_k, we have the expression of the Kalman gain:

$$K_k = P_k^a C_k^T \left[C_k P_k C_k^T + R \right]^{-1} \tag{7}$$

Then, substituting (7) into (6), we obtain also a recursive expression for the *a posteriori* estimate error covariance:

$$P_k = [I - K_k C_k] P_k^a \tag{8}$$

which obeys a Discrete Riccati Equation (DRE).

The *a priori* estimate error covariance P_k^a can be determined starting from its definition (Table I):

$$P_k^a = A_{k-1} P_{k-1} A_{k-1}^T + B_{k-1} Q B_{k-1}^T \tag{9}$$

which is a recursive expression and Q is the n x n process disturbance covariance matrix.

A summary of the Kalman filter equations is given in Table II.

TABLE II

SUMMARY OF THE DISCRETE KALMAN FILTER ALGORITHM

Time Update Equations
$\hat{\mathbf{x}^{\mathbf{a}}}_k = A_{k-1} \hat{\mathbf{x}}_{k-1}$ $P_k^a = A_{k-1} P_{k-1} A_{k-1}^T + B_{k-1} Q B_{k-1}^T$
Measurement Update Equations
$K_k = P_k^a C_k^T \left[C_k P_k C_k^T + R \right]^{-1}$ $\hat{\mathbf{x}}_k = \hat{\mathbf{x}^{\mathbf{a}}}_k + K_k \left[\mathbf{y}_k - C_k \hat{\mathbf{x}^{\mathbf{a}}}_k \right]$ $P_k = [I - K_k C_k] P_k^a$

The Kalman filter provides an efficient computational recursive solution of the least-square methods, since this procedure corresponds to the minimization the following cost-function:

$$J = ||\mathbf{x}_0 - \bar{\mathbf{x}}_0||^2_{\prod^{-1}} + \sum_{i=0}^{N-1} \left(||\mathbf{p}_i||^2 + ||\mathbf{v}_i||^2 \right) \qquad (10)$$

where $||\mathbf{x}_0 - \bar{\mathbf{x}}_0||^2_{\prod^{-1}} = (\mathbf{x}_0 - \bar{\mathbf{x}}_0)^T \prod^{-1} (\mathbf{x}_0 - \bar{\mathbf{x}}_0)$.

III. State Space Formulation of the Equalization Problem and Kalman Equalizer

The state space approach to the equalization problem consists on to estimate the state vector of tap coefficients of the equalizer.

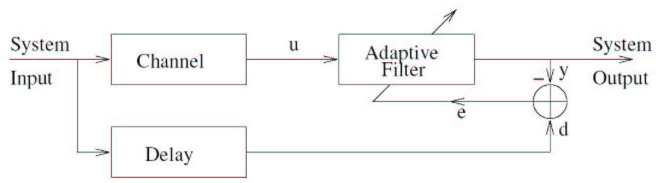

Fig. 1. Baseband communication scheme.

A block-diagram of the baseband communication scheme is shown in Fig. 1. In the following, no delay is assumed for simplicity. The transmitted message $\mathbf{d}$ at time step k is composed by a sequence of symbols that belong to a finite set, $\mathbf{d}_k = [d_k, \cdots, d_{k-M+2}, d_{k-M+1}]^T$, where M is the number of tap coefficients of the equalizer. We consider a time-varying channel whose impulse response is modeled by a tapped-delay line with c components, $\mathbf{h}_k = [h_k^1, h_k^2, \cdots, h_k^c]^T$. Each tap corresponds to an independent random process.

The optimum equalizer for time-varying channels is also time-varying. Its impulse response at time step k is given by $\mathbf{w}_k = [w_k^1, w_k^2, \cdots, w_k^M]^T$. We model the optimum equalizer as a first-order Markov process:

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \mathbf{q}_k \qquad (11)$$

where $\mathbf{q}_k$ is the process noise vector due to the uncertainty relative to the time-variance of the channel. The process noise vector is assumed to be a zero mean, white, Gaussian random variable, with correlation matrix Q.

The output signal of the equalizer at instant k, $\hat{d}_k$, is the estimate of the transmitted signal d_k. It is a result of the convolution between the input signals vector of the equalizer and its tap coefficients:

$$\hat{d}_k = \mathbf{u}_k^T \mathbf{w}_k \qquad (12)$$

where the input signals vector of the equalizer is $\mathbf{u}_k = [u_k, \cdots, u_{k-M+2}, u_{k-M+1}]^T$.

We can rearrange (12) in order to express the transmitted signal d_k as a function of the optimum equalizer tap coefficients. Indeed, the signal input of the equalizer at instant i, u_i, for $i = k - M + 1, \cdots, k$, is the convolution between the transmitted message $\mathbf{d}_i$ and the channel impulse response $\mathbf{h}_i$ added by a sample of a zero-mean white Gaussian noise, n_i:

$$u_i = \mathbf{d}_i^T \mathbf{h}_i + n_i \qquad (13)$$

Then, since the optimum equalizer removes ISI completely, from (12) and (13) we have as output of the optimum equalizer the transmitted signal d_k added by a sum r of M random variables, which are scaled versions of the AWGN at the receiver:

$$r_k = \mathbf{n}_k^T \mathbf{w}_k \qquad (14)$$

where $\mathbf{n}_k = [n_k, \cdots, n_{k-M+2}, n_{k-M+1}]^T$.

Thus:

$$d_k = \mathbf{u}_k^T \mathbf{w}_k - r_k \qquad (15)$$

The Kalman equalizer is composed by the process equation (11) and the measurement equation (15). Kalman filtering theory assumes zero-mean white Gaussian noises, what is assured for the process noise in (11) by the adopted model itself, but not for the measurement noise in (15).

Using (14), the measurement noise can be rewritten as:

$$r_k = n_k w_k^1 + \cdots + n_{k-M+1} w_k^M \qquad (16)$$

Invoking the Central-Limit Theorem, the density of a random variable composed by a sum of n random variables, independently of any probabilistic considerations, approaches a normal curve as n increases. If the densities are reasonably concentrated, then a normal curve is a close approximation to the density even for moderate values of n [5].

Thus, we conclude that r_k can be considered a Gaussian random variable. Furthermore, since n_i is a sample of a zero-mean white Gaussian noise and is independent from w_k^{k-i+1}, for $i = k - M + 1, \cdots, k$, each term on the right side of (16) is a zero-mean white random variable. Then, r_k is a zero-mean white Gaussian random variable, even for moderate number of tap coefficients of the equalizer.

Therefore, we write the process equation (17) and the measurement equation (18) of the Kalman equalizer:

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \mathbf{q}_k \qquad (17)$$

$$d_k = \mathbf{u}_k^T \mathbf{w}_k + r_k \qquad (18)$$

The standard RLS algorithm is a special case of Kalman filter application, as shown in [2], with measurement noise variance R set 1, and with the following process and measurement equations:

$$\mathbf{w}_{k+1} = \lambda^{-1/2} \mathbf{w}_k \qquad (19)$$

$$d_k = \mathbf{u}_k^T \mathbf{w}_k + r_k \qquad (20)$$

where $\mathbf{w}$ is the vector coefficients of the equalizer, $0 \ll \lambda \leq 1$ is the forgetting factor, d is the transmitted signal and $\mathbf{u}$ is the input signals vector of the equalizer.

Note that the process disturbance covariance matrix Q is assumed zero. Then, the forgetting factor in the standard RLS algorithm is the unique responsible for tracking the variations of the channel, which are reproduced in the tap coefficients of the equalizer.

IV. H_∞ Optimal Estimation

The H_∞ filters are interesting alternatives to the famed Kalman filter in most estimation problems. As we shall see, the filter equations are very similar despite that the underlying ideas are completely different.

The optimality of the Kalman filter relies on the knowledge of the covariance matrices Q and R. In most real-world applications this kind of *a priori* information is not available and one has to use *ad hoc* choices of Q and R. Then, this filter is not guaranteed to achieve a certain level of performance. The H_∞ filters, on the other hand, gives hard upper bounds on the estimation errors, independently of the disturbances distributions. It follows an introductory discussion about H_∞ estimation that can be found in [3], [6] and [7] with more details. We define $\| \mathbf{g} \|_S = \left(\mathbf{g}^T S \mathbf{g}\right)^{\frac{1}{2}}$ for a vector $\mathbf{g}$ and a symmetric matrix S.

Consider the linear discrete-time system described by:

$$\mathbf{x}_{k+1} = A_k \mathbf{x}_k + B_k \mathbf{p}_k \tag{21}$$

$$\mathbf{y}_k = C_k \mathbf{x}_k + D_k \mathbf{v}_k \tag{22}$$

$$\mathbf{z}_k = L_k \mathbf{x}_k \tag{23}$$

where $\mathbf{x}_k \in \mathbb{R}^n$, $\mathbf{y}_k \in \mathbb{R}^m$, $\mathbf{z}_k \in \mathbb{R}^n$, $\mathbf{p}_k \in \mathbb{R}^n$ and $\mathbf{v}_k \in \mathbb{R}^m$ are, respectively, the state vector, the measurement, the vector to be estimated, the process disturbance and the measurement noise. The initial state $\mathbf{x}_0$ is an unknown quantity, and A_k, B_k, C_k, D_k and L_k are known matrices of appropriate sizes. Moreover, we assume that $R_k := D_k D_k^T > 0$ holds for any k and that A_k is nonsingular.

Note that $\mathbf{p}_k$ and $\mathbf{v}_k$ are unknown and arbitrary $L_2[0, N]$ signals. Further, an *a priori* estimate of the initial state $\mathbf{x}_0$ is given by $\bar{\mathbf{x}}_0$.

Optimal H_∞ Problem

Find the estimator $\hat{\mathbf{z}}_i$ that minimize the H_∞ norm of the transfer operator from disturbances to prediction error, and obtain:

$$\gamma_o^2 = \inf_{\hat{\mathbf{z}}_i} \sup_{x_0,p,v} \frac{\sum_{i=0}^{N} ||\mathbf{z}_i - \hat{\mathbf{z}}_i||^2}{||\mathbf{x}_0 - \bar{\mathbf{x}}_0||^2_{\prod^{-1}} + \sum_{i=0}^{N-1} \left(||\mathbf{p}_i||^2 + ||\mathbf{v}_i||^2\right)} \tag{24}$$

where $\prod$ is a positive definite weighting matrix which represents the uncertainty of the initial state.

Closed form solutions to the optimal H_∞ problem are available only in some special cases and it is common to settle for a sub-optimal solution.

Sub-optimal H_∞ Problem

Given $\gamma > 0$, find estimation strategies that achieve

$$\sup_{x_0,p,v} \frac{\sum_{i=0}^{N} ||\mathbf{z}_i - \hat{\mathbf{z}}_i||^2}{||\mathbf{x}_0 - \bar{\mathbf{x}}_0||^2_{\prod^{-1}} + \sum_{i=0}^{N-1} \left(||\mathbf{p}_i||^2 + ||\mathbf{v}_i||^2\right)} < \gamma^2 \tag{25}$$

The H_∞ filtering problem has been solved from different viewpoints. The minimaximization problem can be solved by using a game theory approach, as shown in [6] and [8].

The solution to the sub-optimal H_∞ problem is stated in the following theorem, whose proof is given in [6]:

Theorem 1: There exist $\hat{\mathbf{x}}_k$ and $\hat{\mathbf{z}}_k$ satisfying (25) if, and only if,

(i) There exists a matrix $P_k > 0$ satisfying

$$P_{k+1} = A_k P_k \Sigma_k^{-1} A_k^T + B_k B_k^T \tag{26}$$

$$\Sigma_k = I_n + \left(C_k^T R_k^{-1} C_k - \gamma^{-2} L_k^T L_k\right) P_k \tag{27}$$

where $P_0 = \prod$.

$$(ii) V_k := \gamma^2 I_n - L_k P_k \left(I_n + C_k^T R_k^{-1} C_k P_k\right)^{-1} L_k^T > 0 \tag{28}$$

If this is the case, an H_∞ filter achieving (25) is given by

$$\hat{\mathbf{z}}_k = L_k \hat{\mathbf{x}}_k \tag{29}$$

$$\hat{\mathbf{x}}_{k+1} = A_k \hat{\mathbf{x}}_k + K_k \left[y_k - C_k \hat{\mathbf{x}}_k\right] \tag{30}$$

where

$$K_k = P_k C_k^T \left[R_k + C_k P_k C_k^T\right]^{-1} \tag{31}$$

This filter is the central level-γ H_∞ *a priori* filter. Statistical properties of noise and uncertainties of the model are not known here. Although the performance of an H_∞ equalizer may be worse than that of Kalman filtering-based ones for some scenarios, its robustness is an interesting characteristic for hostile environments.

The factor γ determines the upper bound of the transfer function from noise and uncertainties to the estimate error, as stated by (25). Note that if γ tends to infinite, (25) tends to the cost-function (10), that is, the H_∞ filter becomes a Kalman filter. On the other hand, low values of γ correspond to hard restrictions on the gain of the estimate error due to noise and uncertainties. Therefore, the tradeoff between error performances, as the Mean Squared Error (MSE) and BER, and robustness in the H_∞ filtering is represented by the factor γ.

V. SIMULATION RESULTS

Using a linear equalizer, the standard RLS algorithm is implemented as described in Section III, with process and measurement equations (19) and (20), respectively, while the design of the H_∞ equalizer follows Section IV statements. MSE and BER performances are evaluated.

We consider the frequency-selective (two paths) Rayleigh (worst-case) fading channel in a mobile wireless network, encountered when there is not a strongly dominant path and the communication link is heavily shadowed. Typical values of rms delay spread for urban outdoor environments and of symbol time are chosen, 4 μs and 10 μs. These settings imply in a channel coherence bandwidth of approximately 47 kHz. Signal bandwidth, W (kHz) equals bit rate $R_b = 100$ kbps, surpasses the channel coherence bandwidth. This indicates the need for combating ISI.

In order to simulate the channel, an independent Rayleigh fading gain for each of both channel taps is generated. BPSK is the assumed modulation and we used a 2 GHz carrier frequency. A vehicular speed of 30 m/s is considered, with the corresponding Doppler shift $f_d = 200$ Hz. Thus, approximately 250 bits are received in the coherence time interval. Furthermore, fast-fading is simulated following the Jakes' model [9].

A signal-to-noise ratio (SNR) of 40 dB at the receiver input is considered. The high SNR level is chosen in order to make the thermal noise effect on the estimation errors worthless, giving place to effects of the uncertainties of the model on the estimation errors. We perform the transmission/reception of 3000 bits corresponding to 12 coherence times. The equalizers initialize in training mode with their 11 tap coefficients set to zero and switch to the direct decision mode after the first 250 bits, tracking the channel. The training mode period is sufficient to convergence of the algorithms and the replica of the desired response is generated locally in the receiver.

Considering the same state space model for both approaches, it holds the following settings in (21-23) for the H_∞ one:

$$\begin{aligned} A_k &= \lambda^{-1/2} I_M \\ B_k &= \mathbf{0} \\ D_k &= 1 \\ L_k &= I_M \end{aligned} \qquad (32)$$

where I_M is the M x M identity matrix, with M as the number of tap coefficients of the equalizer.

Setting p_k equal to zero and λ equal to 1 in (19) and (21), respectively, is a good assumption, since it can be considered that the tap coefficients of the equalizers do not change between two consecutive time instants for slow fading channels.

Fig. 2 shows the BER performance of the standard RLS algorithm and of the H_∞ algorithm for some different values of the parameter γ. It is clearly observable that the higher the parameter γ is, the nearer is its performance from that of the RLS algorithm. Thus, the RLS algorithm outperforms the different H_∞ equalizers in terms of BER.

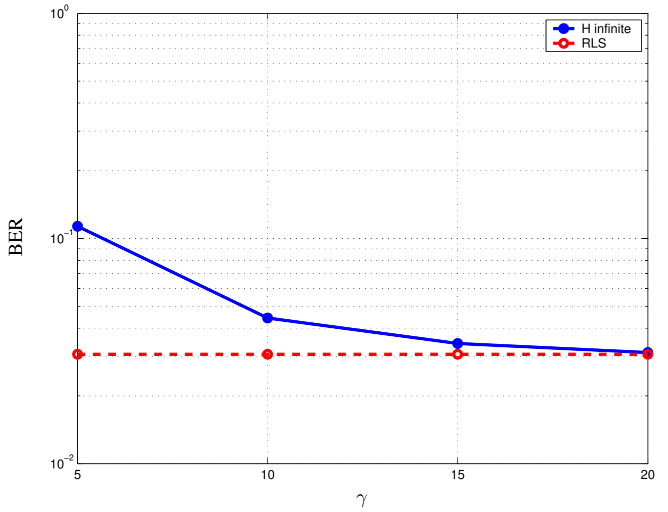

Fig. 2. Average BER for RLS and (noise sensitive) H_∞ filtering algorithms.

It is also important to emphasize the hard conflict between robustness and error performance for the experimented H_∞ equalizer. Its robustness, i.e., low γ values, implies in a poor BER performance. Moreover, another undesirable characteristic of this H_∞ equalizer is its very poor performance in terms of MSE.

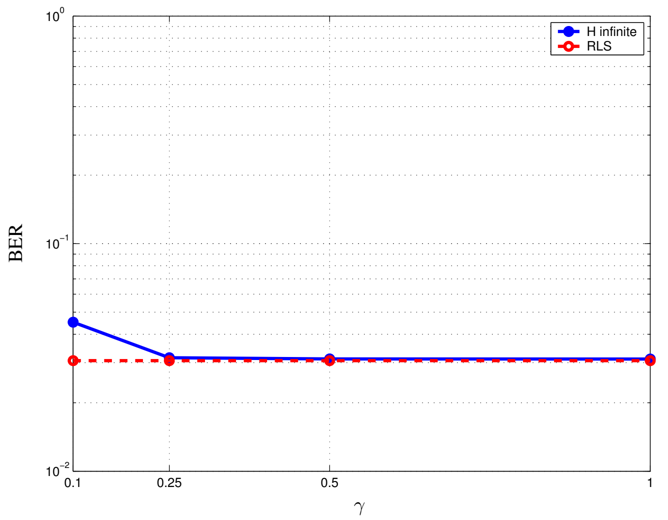

Fig. 3. Average BER for RLS and (noise insensitive) H_∞ filtering algorithms.

In order to improve the error performances of the H_∞ equalizers, we assume a different value for D_k in the state space model presented in (32). We set D_k equal to 0.01. This new approach for the H_∞ equalizers is less sensitive with respect to the measurement noise. Fig. 3 illustrates the new BER curves.

In addition, the MSE behavior for the new H_∞ equalizer is comparable to that of the H_2 approach, as shown in Fig. 4. Therefore, in this case, the H_∞ equalizer provides good performance in terms of estimation errors and guarantees high level robustness, due to low γ values.

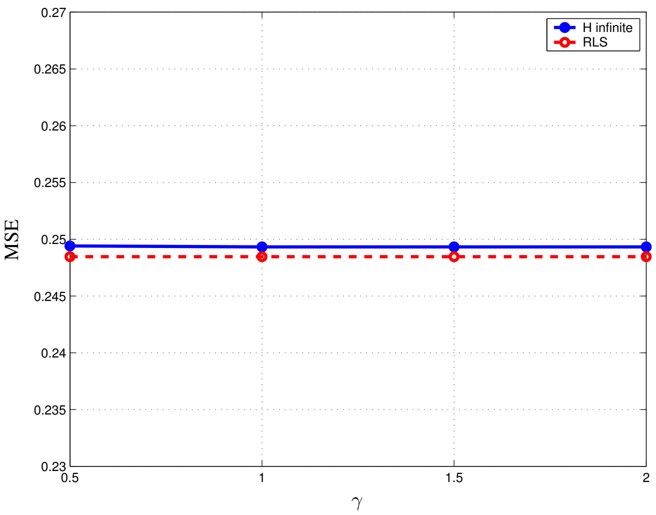

Fig. 4. Average MSE for RLS and (noise insensitive) H_∞ filtering algorithms.

Some aspects in the presented computational simulations deserve comments, since so high values of error performances (MSE and BER) for an SNR of 40 dB must have been not expected and could suggest the algorithms are not appropriate. First, a hard scenario was considered, with high Doppler shift due to high receiver speed, where a long sequence of bits is transmitted and only at the beginning a training sequence is used. Furthermore, the simplified communication scheme has not considered channel coding and we have used linear equalizers, which are simple, but not the best ones in terms of performance on mitigating ISI. For both approaches, non-linear equalizers, as the Decision-Feedback Equalizers (DFEs), could be used in order to obtain better error performances.

VI. Conclusions

The classical Kalman filtering theory (H_2 estimation) and the H_∞ estimation theory were briefly discussed and their main results were presented. A state space formulation to the equalization problem was deduced and the structure of the standard (Kalman-based) RLS algorithm and of the H_∞ algorithm were presented in order to equalize outdoor dispersive fading channels.

The H_∞ filtering algorithm minimizes the effect of the worst disturbance (both measurement noise and modeling error) on the estimation error. It has been shown through simulations that as γ approaches infinite, the H_∞ filtering algorithm converges to the RLS algorithm. The state space model of the H_∞ filter was modified in order to make its error performances comparable with those of the RLS algorithm, while providing a high level of robustness with low values of the parameter γ.

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