The Mobile Cellular Network as a Set of Voronoi Diagrams

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Abstract

It is common place to represent the mobile cellular network by means of hexagonal topology. This structure is useful for planning frequency reuse but not appropriate for the analysis of other functions including handoff, cell breathing and coverage. This work presents the mobile cellular network as a multiplicatively weighted, directional, truncated and dynamic Voronoi diagram. In order to generate this diagram, the proximity rule is based on radio parameters such as antenna height, transmission power, receiver sensitivity and specific-environment propagation characteristics. The cell boundaries are the edges of the Voronoi diagram. They are defined by comparison of the radii of adjacent cells. The proximity between a mobile and a base station is determined by means of an Euclidean distance weighted by propagation parameters. Each cell is represented by a Voronoi region of the diagram.

Index-Terms: Cell planning, mobile cellular network, Voronoi diagram, path loss prediction.

I. INTRODUCTION

In a mobile cellular network, a mobile (MS) is connected to the closest base station (BS) according to the system quality requirements. The connections are made taking into account the envelope received power, the signal to noise ratio (SNR) and the bit error rate (BER). These parameters are determined by the transmission power, antenna heights, path loss and specific-environment propagation characteristics. The network solves a closest-point problem [1] when connecting a mobile to a base station. The BS radio signal power decreases to a minimum level which determines the cell size. The resulting tesselation of the service area can be represented by a Voronoi diagram.

This work presents a coverage prediction by means of a Voronoi diagram, where the cells are the partitions, called Voronoi regions, defined by a proximity rule [2]. The cellular mobile system analyzes a number of probable MS-BS connections and chooses those with best quality. In a general way, the connection quality decreases with distance. Consequently, the connections are made on a "nearest neighbor" basis. The cell boundary is determined by a closest-point searching and connecting, and the mobiles lie in a Voronoi region. The several features of the network can be represented by various characteristics of the Voronoi diagram: generalized, truncated, weighted, directional and dynamic [3].

II. THE VORONOI DIAGRAM

The Voronoi diagram is a geometric construct that assumes the proximity (nearest neighbor) rule in associating each point, in the R^k space, to a site point closest to it. This diagram is a partition set, generated by site points located inside each partition. Any point in the R^k space is associated to a site point by a proximity rule. Each space partition is called a Voronoi region. Let $\mathbf{x}$ be a point in the R^k space, $\mathbf{c}_i$ the i-th site point and V_i the Voronoi region generated by $\mathbf{c}_i$. The association $\mathbf{x} \in V_i$ occurs according to the proximity rule:

If $\mathbf{x} \in R^k$ *is closer to* $\mathbf{c}_i$ *than any other site point* $\mathbf{c}_j, i \neq j$, *then* $\mathbf{x} \in V_i$.

The distance used as a proximity metric can be simply the Euclidean distance, but it can be also the Euclidean distance additively or multiplicatively weighted. The several features of the Voronoi diagrams are extensively described in [4]. This structure is appropriate to represent the cells, since the association mobile-base station is made according on a proximity rule. The base station works as a site point, connecting mobile stations in its coverage area. The cells can be seen as partitions of a Voronoi diagram.

III. PATH LOSS PREDICTION MODELS

The power loss between a transmitter and a receiver is called path loss. Several models aim at predicting this loss through analytic and measurement based methods. The appropriate, environment-specific, model must be chosen for accurate coverage prediction.

There are several path loss prediction models. For macro-cells, the most common are: Lee, Okumura-Hata, Xia-Bertoni, Cost-Hata and Cost-Walfish-Ikegami [5]. For micro- and pico-cells, the Xia-Bertoni model and ray-tracing method are useful. The predicted path loss depend on frequency, antenna heights, distance and propagation environment characteristics. Generically, the path loss can be expressed as

$$L = a + b\log(d) \tag{1}$$

where a and b are parameters dependent on the model.

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IV. The Mobile Cellular Network as a Voronoi Diagram

The coverage is determined by the received radio signal at the mobile. The received field strength becomes more attenuated far from the BS. Thus, there is a maximum accepted path loss. When the limit is achieved, the current link is broken and a handoff or blocking operation is executed.

A. Cell Radius

If a uniform propagation environment is assumed, the cell is circular. Its radius depend on the propagation parameters and is obtained from the downlink received power equation

$$p_r = p_t + G_t + G_r - L, \tag{2}$$

where p_t is the BS transmitter power, G_t and G_r are the BS and MS gain, respectively, and L is the link MS-BS path loss. The minimum accepted received power is the receiver sensitivity z, thus the condition $p_r > z$ defines the cell. It is assumed 0 dBi for G_t and G_r. Inserting (1) into (2), it yields

$$p_r = p_t - a - b\log(d). \tag{3}$$

At the cell border $p_r = z$ and the distance d corresponds to the cell radius

$$r = 10^{\frac{p_t - a - z}{b}}. \tag{4}$$

B. The Two-Cell Model

Consider two adjacent cells as seen in Fig. 1 and a mobile connected to the first one. When the mobile is far from BS_1 and approaches BS_2, the system analyzes the current, and the alternative, connection quality. If the current connection quality degenerates below a certain threshold, the system changes the connection to BS_2. The proximity rule is

If $p_{r1} \geq p_{r2}$, the mobile is closest to BS_1.
Else, the mobile is closest to BS_2,

where p_{rj} is the mobile received power which is transmitted by the j-th BS. The condition $p_{r1} = p_{r2}$ defines a locus shown in Fig. 1 as the dashed circumference which determines the border of two cells. This border is a circular arc inside the overlapping cell area. It is illustrated also in three dimensions in Fig. 2 where the BS transmission power decreases with distance according to the Okumura-Hata model. If $r_1 = r_2$, the locus is a straight line. Else, it is a circumference. The radius r_0 and location (x_0, y_0) of the dashed line circumference can be given as

$$r_0 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 \frac{W^2}{(W^2 - 1)^2}}, \tag{5}$$

$$x_0 = x_1 + (x_2 - x_1)\frac{W^2}{W^2 - 1}, \tag{6}$$

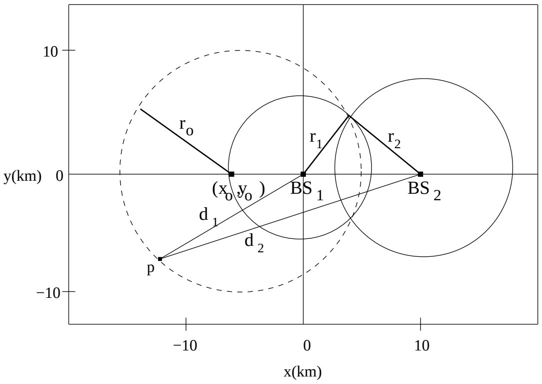

Fig. 1. Two adjacent cell model. The locus of the condition $p_{r1} = p_{r2}$ is shown as dashed line.

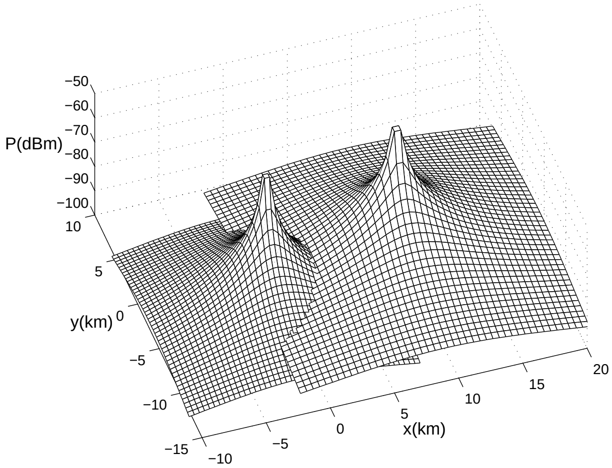

Fig. 2. Two adjacent cell model in a three-dimension diagram. The locus of the border condition $p_{r1} = p_{r2}$ can be seen as a circular arc between the cells.

$$y_0 = y_1 + (y_2 - y_1)\frac{W^2}{W^2 - 1}, \tag{7}$$

where (x_i, y_i) is the BS location and W is the distance ratio d_1/d_2, d_j is the distance of a point on the circumference to the j-th BS. Along the circumference, the distance ratio W is verified. Fig. 1 shows that the $p_{r1} = p_{r2}$ condition occurs at the points where the cells circumference intersect. Hence, the radius ratio r_1/r_2 becomes equivalent to the distance ratio d_1/d_2, such that

$$W = \frac{r_1}{r_2}. \tag{8}$$

The principle of the multiplicatively weighted Voronoi diagram is [6]

$$\frac{d_i}{w_i} = \frac{d_j}{w_j},$$

where w is the weight attributed to the site point and the locus where the equality is verified defines the edge separating two adjacent Voronoi regions. The distance ratio can be obtained as

$$\frac{d_i}{d_j} = \frac{w_i}{w_j}. \tag{9}$$

Taking cells pairwise, the distance ratio w_{ij} is

$$w_{ij} = \frac{r_i}{r_j}, \tag{10}$$

where w_{ij} represents the neighboring relationship of the two stations. For two adjacent stations, the proximity rule is

If $d_i \leq w_{ij} d_j$, the mobile is closest to BS$_i$.
Else, the mobile is closest to BS$_j$.

The Voronoi diagram splits the service area into partitions according to the proximity rule. The radio signal, on the other hand, is not confined to one cell, but penetrates the adjacent cells establishing overlapping areas (See Fig. 5). However, the radio receiver imposes a limit to the reception. When the radio signal power achieves the receiver sensitivity, the receiver blocks the connection. These radio propagation characteristics are represented by the various types of the Voronoi diagram:

- **Generalized** – If two adjacent cells have the same radius, they have also the same weight. Their border is a straight line and the resulting Voronoi diagram is of the generalized type. The Euclidean distance weight is unitary.
- **Multiplicatively weighted** – Two cells with different radius have different weights. Their border is a circular arc and the resulting Voronoi diagram is of the multiplicatively weighted type. An example is illustrated in Fig. 1 and Fig. 2, having the following data

BS	Location (km)	Power (dBm)	Antenna height (m)	Radius (km)
1	$(0, 0)$	40	60	4.6454
2	$(10, 0)$	45	75	7.4877

The edge between the two site points is a circumference described as

Center(km)	Radius (km)	w$_{12}$ (distance ratio)
$(-6.2, 0)$	10.0861	0.6204

- **Directional** – Some path loss prediction models are direction dependent. It means that a weight is obtained for each direction (See Fig. 3). This is also the case of sectorized cells.
- **Truncated** – The radio signal has a reception limit, a minimum power level that blocks the receiver, called receiver sensitivity. This radio signal bounding corresponds to a distance. The resulting Voronoi diagram is called truncated [7].
- **Dynamic** – The network is not static. The traffic demand requires BS insertions and exclusions that alters the diagram. The cell dimensions and shapes are interdependent. The cell breathing mechanism reduces the coverage and alters the diagram. The *ad hoc* networks have no centralized administration and the hosts can be represented by a Voronoi diagram where the site points move.
- **Power diagram** – If two adjacent cells do not overlap, the resulting diagram is clearly a power diagram [8], a generalization of Voronoi diagrams (See cells of BS$_1$ and BS$_3$ in Fig. 5).

C. Directional Models

The path loss prediction models of Xia-Bertoni, Lee and Walfish-Ikegami, are dependent on direction. A cell has a number of weights, one for each direction.

Fig. 3 shows mobiles MS$_1$ and MS$_2$ next to BS$_1$ and BS$_2$. The connections are made based on a proximity rule. The direction are represented by the angles θ and ϕ. Taking MS$_1$ as an example, the proximity rule is

If $d_1 \leq w_{12} d_2$, the mobile is closest to BS$_1$.
Else, the mobile is closest to BS$_2$,

where the distance ratio w_{12} is calculated as

$$w_{12} = \frac{r_1(\phi_1)}{r_2(\theta_1)}, \tag{11}$$

where r_i is the radius of the i-th cell in the direction $(\cdot)$. Omnidirectional models, e.g., Okumura-Hata and Cost-Hata, provides only a weight to the cell.

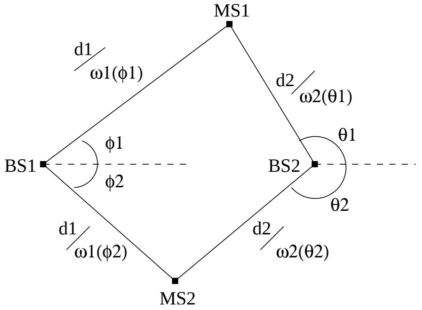

Fig. 3. The path loss direction dependence. A weight for each direction is obtained.

D. Sectorized Cells

If a cell is sectorized, the sector can be represented by a "sector of a circle" whose radius is determined by (4). A weight is assigned for each sector. An example is shown in Fig. 4, where BS_1 and BS_3 are three-sectorized and BS_2 is omnidirectional. The borders can be of the following types

1. Cell-cell;
2. Sector-cell; or
3. Sector-sector.

The neighboring relationship is analysed pairwise. As an example, consider the BS$_1$(sector-III)-BS$_3$(sector II) border and a mobile in its vicinity. The proximity rule is

If $d_1 \leq w_{13} d_3$, the mobile is closest to BS$_1$.
Else, the mobile is closest to BS$_3$,

where the distance ratio w_{13} is calculated as

$$w_{13} = \frac{r_1(III)}{r_3(II)}, \tag{12}$$

where r_i is the radius of the i-th BS, sector $(\cdot)$.

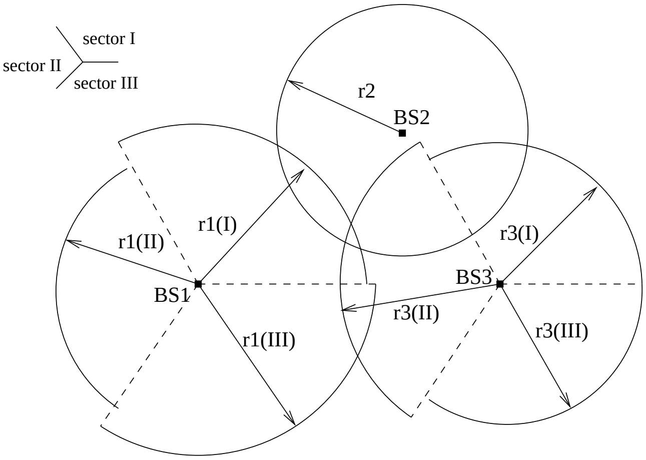

Fig. 4. Each sector has a radius. A distance ratio for each border. BS$_1$ and BS_3 are sectorized. BS$_2$ is omnidirectional.

V. SIMULATION

The network illustrated in Fig. 5 has been simulated. The Okumura-Hata model bas been used for path loss prediction in a plain terrain, medium sized city with medium trees density, BS heights in the 30 to 200 m range above rooftop, mobile height between 1 and 10 m, a 150-1000 MHz frequency range and a link maximum lenght of 20 km. 0 dBi gain was assumed for all the antennas, mobile antenna height of 3 m and a frequency of 850 MHz. The BS location is a technical and financial decision that aims at maximizing the BS antenna height and minimizing the site installation cost. The number of BS is a traffic engineering matter. The simulated network is based on realistic data.

A. Calculation of the Weights

The distance ratio is obtained by using (8) and the weights from (10). The cell radius is obtained from (4). The parameters a and b in (1) are obtained from the Okumura-Hata path loss prediction formula

$$\begin{aligned} L = 69.55 + 26.16\log(f) - 13.82\log(h_b) - a(h_m) \\ + (44.9 - 6.55\log(h_b))\log(d), \end{aligned} \tag{13}$$

where $a(h_m) = (1.1\log(f) - 0.7)h_m - (1.56\log(f) - 0.8)$,

$$a = 69.55 + 26.16\log(f) - 13.82\log(h_b) - a(h_m),$$

$$b = 44.9 - 6.55\log(h_b),$$

h_b is the BS antenna height, h_m is the mobile antenna height and f is frequency. The input data are shown in Table I, the distance ratio in Table II and the weights in Table III. The network in Fig. 5 presents nine borders. The BS$_1$-BS$_3$ cells do not overlap, consequently, the corresponding distance ratio w_{13} is not computed. The cells are represented by the multiplicatively weighted Voronoi diagram shown in Fig. 5 by thick lines. The diagram is truncated by the circles. Thus, a point outside the ciecles do not belong to the diagram. The Voronoi region contours depend on the radius of neighbor cells, according to (8), (9) and (10), where the relationship between distance ratio and the BS weights are given.

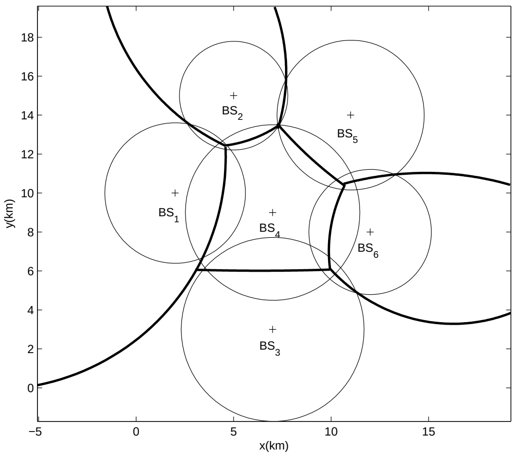

Fig. 5. A six-BS service area. The Voronoi diagram is composed of circular arcs, shown as thick line. The cell border truncates the diagram and confines it into a subspace of the R^2.

w_{12}	w_{13}	w_{14}	w_{24}	w_{34}
1.2973	no overlapping	0.8058	0.6212	1.0477
w_{25}	w_{45}	w_{56}	w_{36}	w_{46}
0.7364	1.1856	1.2010	1.4918	1.4239

TABLE II

THE DISTANCE RATIO BETWEEN TWO OVERLAPPING CELL

w_1	w_2	w_3	w_4	w_5	w_6
1	0.7708	1.3001	1.2410	1.0467	0.8715

TABLE III

THE BS WEIGHTS FOR THE MULTIPLICATIVELY WEIGHTED VORONOI DIAGRAM

VI. CONCLUSIONS

In a mobile cellular network, the connections are made on a proximity rule basis, which takes on account the connection quality. This rule can be expressed in terms of a weighted distance which depend on the path loss, BS power, antenna heights and cell radius. The weighted distances generate a multiplicatively weighted Voronoi diagram. This representation provides also the truncated, directional and dynamic features of the network. The resulting tesselation of the service area is useful to plan coverage, handoff, outage, frequency reuse and cochannel interference. The Voronoi diagram does not show the signal penetration from one cell to a neighbor. The amount of

BS	Location (km)	Power (dBm)	Antenna height (m)	Cell radius (km)	Okumura-Hata parameters	
1	(2,10)	37	55	3.6054	a_1=118.34	b_1=33.50
2	(5,15)	32	65	2.7792	a_2=117.33	b_2=33.02
3	(7,3)	40	61	4.6876	a_3=117.72	b_3=33.20
4	(7,9)	40	56	4.4743	a_4=118.23	b_4=33.44
5	(11,14)	37	60	3.7740	a_5=117.81	b_5=33.25
6	(12,8)	35	55	3.1423	a_6=118.34	b_6=33.50

TABLE I

Input data: BS location, power and antenna heights; Okumura-Hata parameters

this penetration has influence in handoff and cell breathing. The transmission power is directly related to the cell radius and can be used to balance coverage and traffic. As the cell radius is a function of BS power, antenna heights and propagation mechanisms, an optimization method can be applied, using these parameters as input data, in order to optimize the coverage.

VII. Acknowledgments

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