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The Effects of Wavelength Dependence on the Estimation of Blocking Probabilities in Linear All-Optical Networks

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Abstract— **In this paper we investigate the effects of wavelength dependence on the estimation of blocking probabilities in linear all-optical networks without wavelength changers. We have proposed in [1] that the independent link assumption [2] in such networks be replaced by an independent object assumption that takes all active paths, as well as free links, as independent objects on the network topology. The new model was shown to generate estimates that fit exactly the blocking probability for a wavelength plane assuming Poissonian, spatially homogeneous traffic. Nevertheless, for multi-wavelength networks we still had considered the independent wavelength assumption in our analysis. In this work we replace the independent wavelength assumption by a model that takes into account the wavelength dependence in a single hop of the path and that assumes the independent object assumption in the remaining hops. We show that this model improves the estimation of blocking probabilities in multi-wavelength linear all-optical networks.**

Keywords— **blocking probability, all-optical network, WDM.**

Resumo— **Neste trabalho investigamos os efeitos da dependência entre comprimentos de onda na estimação de probabilidades de bloqueio em redes totalmente ópticas lineares sem conversores de comprimento de onda. Nós propusemos em [1] que a suposição de independência entre enlaces [2] seja substituída por uma suposição de independência entre objetos, a qual assume que todos os caminhos ativos, assim como os enlaces livres, possam ser classificados como objetos na topologia da rede. O novo modelo apresentou estimativas que correspondem com exatidão à probabilidade de bloqueio para tráfego Poissoniano e espacialmente homogêneo, considerando um plano de comprimento de onda. Contudo, para redes com múltiplos comprimentos de onda, nós ainda havíamos considerado a independência entre comprimentos de onda em nossa análise. Neste trabalho, substituímos a suposição de independência entre comprimentos de onda por um modelo que leva em conta a dependência entre comprimentos de onda em um único enlace do caminho e que assume a independência entre objetos nos enlaces restantes. Mostramos que este modelo melhora as estimativas de probabilidade de bloqueio em redes totalmente ópticas lineares com múltiplos comprimentos de onda.**

Palavras-Chave— **probabilidade de bloqueio, rede totalmente óptica, WDM.**

I. INTRODUCTION

The blocking performance of all-optical networks has been extensively studied in the last years [1], [3], [4], [5], [6], [7], [8]. Given the existence of previous theory on circuit-switched blocking networks, there was a natural will to reuse its results, focusing on the peculiarities of the all-optical networks: a) the *wavelength continuity constraint*, when there are not wavelength converters at the nodes; and b) the relatively *poor connectivity* of the first optical networks, associated with the high costs of optical ports and high granularity of traffic. The latter peculiarity renders the Lee approximation very inaccurate for use in the optical networks framework, as this approximation was proposed in 1955 [2] with the aim of facilitating the calculation of blocking probabilities in circuit-switched telephone networks, which have many input and output ports at their nodes.

Several works [3], [4], [5], however, still make use of the Lee approximation on the blocking performance of all-optical networks. The authors in [3] estimate the capacity gain generated by the availability of wavelength conversion in all nodes. In [4] the author considers Poisson input traffic and a Markov chain with state-dependent arrival rates, being the idle wavelengths in different links of a route independent, thus also implying the Lee hypothesis. This model is computationally intensive, with the complexity growing with the number of hops, what restricts its application in small, dense networks. The analytical model in [5] also assumes Poisson traffic input and the series link independence assumption, so the authors point out that their model cannot be used in sparse networks due to the load correlation on subsequent hops of the path. The models in [6], [7] and [8], however, do not use the Lee's series link independence assumption, but the tradeoff for accuracy in the estimates of the blocking probabilities is the increasing complexity of these models.

In [1] we have proposed that the independent link assumption be replaced by an independent object assumption that takes all active paths, as well as free links, as independent objects on the network topology. The presented analytical model accurately quantifies the blocking probability for a wavelength plane in sparse networks assuming Poissonian, spatially homogeneous traffic. We have presented closed-form expressions for the blocking probabilities in linear all-optical networks and our analysis has a lower computational complexity compared with all other models that take into account the link correlation. Nevertheless, for multi-wavelength networks, we have used the independent wavelength assumption in our analysis.

We show in this paper that we can improve the estimation of blocking probabilities in such networks by a model that takes

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into account the wavelength dependence in a single hop of the path and that assumes the independent object assumption in the remaining hops. The terms link and hop are used interchangeably in this paper.

The paper is described as follows. Section II outlines the main results of the independent object assumption presented in [1], which will be used in the following sections. Section III presents the independent object assumption in conjunction with independent wavelength assumption. In Section IV, we propose a model that accounts for wavelength dependence in a single hop of the path in combination with the independent object assumption in the remaining hops. We show that this model improves the estimates of blocking probabilities in linear all-optical networks. Finally, Section V summarizes the conclusions of our work.

II. THE INDEPENDENT OBJECT ASSUMPTION

In this section we present the main results of the independent object assumption first proposed in [1]. However, before describing this assumption, it is convenient to state the Lee approximation for circuit-switched networks.

In [10] the Lee hypothesis is presented as comprising two assumptions:

1) the independent path assumption; and
2) the independent link assumption.

The independent *path* assumption says that parallel paths are available or blocked independently. In WDM networks without wavelength changers, a route and a wavelength from a pool define a path. If we assume that routes are predefined for each source-destination pair (as usually done in current protocols), one might then identify this assumption as an independent wavelength assumption.

On its turn, the series independent *link* assumption states that all links are available or blocked independently, i.e. the probability of a link being available at any given wavelength is $(1-\rho)$ regardless of the states of its neighbors. Thus, for one wavelength plane, the probability that this wavelength does not support a request for a path with H hops is estimated as:

$$p_b(\rho) = 1 - (1-\rho)^H, \tag{1}$$

where ρ is the network occupancy at the wavelength under consideration.

We can notice that the independent *link* assumption recognizes the existence of only two kinds of objects in the network: single (available) links, which do support requests for new paths; and single busy links, which do not support them. Actually, however, independent single busy links may occur only in one-hop paths. Any path with $H > 1$ links will give rise to a set of H spatially connected busy links.

It is then possible, without increase in complexity, to replace the independent link assumption by an independent *object* assumption that considers the existence of the real objects that are present in the network: individual *free* links; and *paths*, or groups of successive busy links. This assumption assumes that, when you walk along the linear network, the next found object is independent of the previous ones. So, in order for a H-link request not to be blocked, the following H events must happen:

1) The first link must be free. This will happen with the probability $(1-\rho)$;
2) The next $(H-1)$ links must also be free. Differently from the Lee approximation, the probabilities of these events are *larger* than $(1-\rho)$, as the busy links are grouped in objects.

Given that the linear network has N nodes or links, there are $(1-\rho)N$ free links and $\rho N/H$ paths with size H. Then the probability of finding the next object *free* is $(1-\rho)/(1-\rho+\rho/H)$ for large N. The blocking probability for a H-link request will then be:

$$p_b(\rho) = 1 - (1-\rho)\left(\frac{1-\rho}{1-\rho+\rho/H}\right)^{H-1}. \tag{2}$$

In the case where the paths may have any size up to H_{max}, the maximum allowed path size in the network, the blocking probability for a request for a path with i links will be:

$$p_{bi} = 1 - \frac{(1-\rho)^i}{\left(1-\rho+\sum_k \frac{\rho_k}{k}\right)^{i-1}}, \tag{3}$$

where ρ_i is the occupancy rate for i-link paths and the sum is done over all allowed path sizes in the network. Each i-link path request will have a traffic intensity ν_i in Erlangs per node, $i = 1, 2, 3, \ldots, H_{max}$. Identifying the birth and death rates of i-link paths in the network, we have for the equilibrium in a bidirectional network:

$$\nu_i(1-p_{bi}) = \frac{\rho_i}{i}\frac{\Delta}{2}, \tag{4}$$

where Δ is the nodal out degree. Therefore, the mean active path length is

$$\overline{H} = \frac{\sum_i \rho_i}{\sum_i \frac{\rho_i}{i}} = \frac{\sum_i i\nu_i(1-p_{bi})}{\sum_i \nu_i(1-p_{bi})}. \tag{5}$$

As $\overline{H}$ is a function of ρ, and each partial occupancy rate ρ_i also depends on the p_{bi} and on the spatial profile ν_i, an iteration process is necessary to obtain $\overline{H}$. Plugging (5) in (3), we have the general expression for the blocking probability for i-link paths, on a wavelength plane:

$$p_{bi} = 1 - \frac{(1-\rho)^i}{\left(1-\rho+\frac{\rho\sum_j \nu_j(1-p_{bj})}{\sum_j j\nu_j(1-p_{bj})}\right)^{i-1}}. \tag{6}$$

Once convergence is obtained, the overall blocking probability on a wavelength plane will then be:

$$p_b = \frac{\sum_i \nu_i p_{bi}}{\sum_i \nu_i}. \tag{7}$$

In [1] we have compared the calculation of p_b with the estimates of the blocking probability obtained from simulations and from the Lee model. We have observed that the results fit exactly the simulation results.

Sections III and IV present the extensions of the independent object assumption to multi-wavelength networks. First, in Section III we use our model in conjunction with the independent wavelength assumption. In Section IV we propose a model that accounts for wavelength dependence in a single hop of the path and that assumes the independent object assumption in the remaining hops. We consider the *random* wavelength assignment algorithm in both extensions.

III. THE INDEPENDENT WAVELENGTH ASSUMPTION (WI)

Under the random wavelength assignment algorithm, the assigned wavelength is chosen randomly among all wavelengths where the requested path may be accommodated. If no such wavelength exists, the request is blocked. The network blocking probability for an i-link path request will then be, under the independent wavelength assumption:

$$P_{bi} = p_{bi}^W, \tag{8}$$

where p_{bi} is the per-wavelength blocking probability of i-link path requests. Now the mean path length results from the global blocking probabilities:

$$\overline{H} = \frac{\sum_i i\nu_i(1-P_{bi})}{\sum_i \nu_i(1-P_{bi})} = \frac{\sum_i i\nu_i(1-p_{bi}^W)}{\sum_i \nu_i(1-p_{bi}^W)}. \tag{9}$$

Given this expression to $\overline{H}$, the following expression is obtained for the blocking probabilities for a given occupancy and traffic demand spatial distribution:

$$p_{bi} = 1 - \frac{(1-\rho)^i}{\left(1-\rho+\dfrac{\rho\sum_j \nu_j(1-p_{bj}^W)}{\sum_j j\nu_j(1-p_{bj}^W)}\right)^{i-1}}. \tag{10}$$

As for (6) in the single wavelength case, this expression can now be solved by iteration and convergence. The network blocking probability will then be given by:

$$P_b = \frac{\sum_i \nu_i P_{bi}}{\sum_i \nu_i} = \frac{\sum_i \nu_i p_{bi}^W}{\sum_i \nu_i}. \tag{11}$$

The next section presents the model we propose in this paper for wavelength correlation. It accounts for the wavelength dependence in a single hop of the path and assumes the independent object assumption in the remaining hops.

IV. A MODIFIED ERLANG MODEL (ME)

In this section we present a substitute for the independent wavelength assumption discussed in section III. In fact, even though a multi-wavelength all-optical path network without wavelength changers can be seen as a set of separate subnetworks with a common physical topology, the wavelengths cannot be considered independent. One of the dependencies comes from the fact that the amount of traffic that goes to the available wavelengths depends on the number of busy wavelengths of the wavelength pool. One may notice that this dependence happens even in one-hop paths. Obviously, the more wavelengths are busy in a pool, the more traffic will be submitted to the available ones. These facts make the independent wavelength assumption inaccurate in several situations. In this paper we apply the wavelength dependence only for a single hop of the path and the independent object assumption in the remaining hops of each wavelength plane separately.

Consider a request for a path of size i that contains a given link. Suppose also that this link is free in a given wavelength plane. According to Section II, the probability that this request cannot be accommodated (i.e. be "blocked") on that wavelength plane in the $(i-1)$ remaining steps will be:

$$r_i = 1 - \left(\frac{1-\rho}{1-\rho+\rho/\overline{H}}\right)^{i-1} \tag{12}$$

The probability r_i may be called the "lateral"blocking probability, since it is associated with activity in the vicinity of the reference link.

If we take a given link as the reference link, we can apply a modified version of the classic Erlang's model to a single link. This link will be in state k when it is busy in k wavelengths, $0 \leq k \leq W$. Let $q(k)$ be the temporal steady-state probability of state k. Then, the blocking probability of a request for a path of size i will be:

$$p_{bi} = \sum_{k=0}^{W} q(k) r_i^{W-k} \tag{13}$$

Consider the Erlang's classical model applied to a single isolated link. Each state transits from and to its neighbors only. The birth rate λ is the same for all the states because the link is isolated, so the request will never be blocked due to the occupancy of the adjacent links. However, the death rate $k\mu$ is proportional to the population of active calls k.

For a link in state k inserted in a network topology, the death rate is still $k\mu$. However the birth rate depends on the state and it must take into account the possibility of "lateral"blocking and, therefore, the necessity of the request accommodation in one of the $(W-k)$ free wavelengths found in the first step. Then, if λ_k is the birth rate from the state k to the state $(k+1)$, we have:

$$\lambda_k = \sum_{i=1}^{H_{max}} i\lambda_i(1-r_i^{W-k}), \tag{14}$$

$k = 0, 1, \ldots, W$. Notice that $\lambda_W = 0$ in (14) so that it is not necessary to specify this condition separately.

Therefore, given an initial set of p_{bi}'s (e.g., $p_{bi} = 0$), we begin an iterative process to obtain the overall blocking probability based on the following steps:

1) calculate $\overline{H}$ from (5);
2) plug the value of $\overline{H}$ in (12) to obtain the r_i's;
3) calculate the λ_k's from (14);
4) obtain the $q(k)$'s by the steady-state solution of the Markov system of Fig. 1;

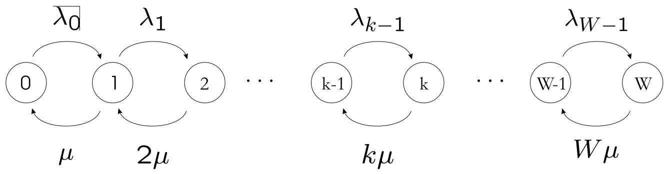

Fig. 1. Modified Erlang Model (ME).

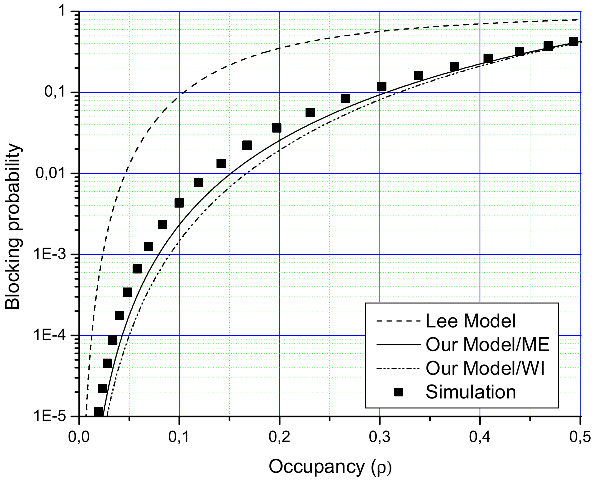

Fig. 2. Multiple wavelengths. Comparison among the Lee model, our model with wavelength dependence and wavelength independence and simulation results for a 24-node ring with $W = 4$ wavelengths and $H_{max} = 12$ hops.

5) obtain the p_{bi}'s from (13);
6) go back to step 1 until convergence is achieved.

Notice that, since Fig. 1 represents a modulated Poisson process, then step 4 may be achieved by the following closed-form [9]:

$$q(k) = \frac{\dfrac{\prod_{j=0}^{k-1}\lambda_j}{k!\mu^k}}{\displaystyle\sum_{u=0}^{W}\dfrac{\prod_{j=0}^{u-1}\lambda_j}{u!\mu^u}}, \qquad (15)$$

where $\lambda_{-1} = 1$.

The overall blocking probability is given by:

$$P_b = \frac{\sum \nu_i p_{bi}}{\sum \nu_i}. \qquad (16)$$

If all the paths in the network have the same size, there is no need of iteration since $\overline{H} = H$.

We have compared both models presented in this section with the Lee Model and the simulation results on a 24-node ring for $W = 4$ and $W = 8$ wavelengths, with traffic given by uniform size distributions with $H_{max} = 12$ hops. The results are shown in Figs. 2 and 3, respectively. We can see that the Modified Erlang model (ME) improves the accuracy of the estimation when compared with the independent wavelength (WI) assumption, although both ME and WI underestimate the blocking probability.

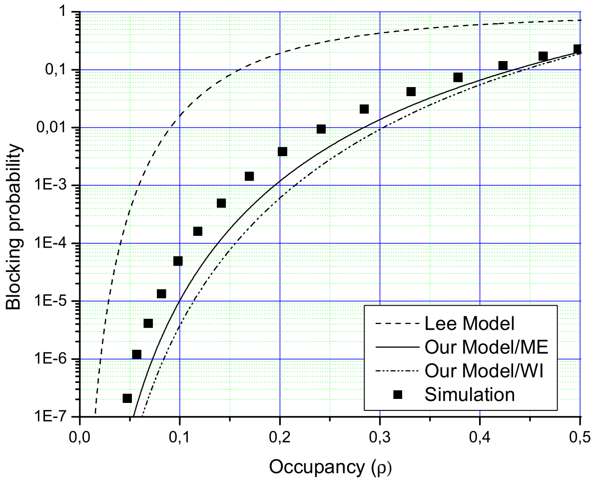

Fig. 3. Multiple wavelengths. Comparison among the Lee model, our model with wavelength dependence and wavelength independence and simulation results for a 24-node ring with $W = 8$ wavelengths and $H_{max} = 12$ hops.

V. CONCLUSION

The Lee model generates two different errors (with opposite signs) for the estimation of blocking probabilities in all-optical path networks: a) the independent link assumption overestimates the blocking probability in such networks; and b) the independent wavelength assumption underestimates it. In [1] we have corrected the first error by the introduction of a new assumption for evaluating blocking probabilities in linear all-optical networks: the independent *object* assumption. We have seen that the new assumption accurately quantifies such estimations for a wavelength plane.

In this paper we have presented a new proposal for replacing the independent wavelength assumption (WI) in the estimation of blocking probabilities in multi-wavelength linear all-optical networks. The Modified Erlang model (ME) takes into account the wavelength dependence in a single hop of the path and considers the independent object assumption in the remaining hops. We have compared the independent object assumption in association with both extensions (WI) and (ME), and with the Lee model. As expected, the results show that ME outperformed WI, but both performed much better than the Lee model, suggesting that the independent wavelength assumption is *not* an accurate approximation in estimation of blocking probabilities in linear all-optical networks.

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