Strategy for the Analysis of Lossy Bragg Fibers

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***Resumo*—Este artigo apresenta um modelo matemático para a análise de perdas e dispersão em fibras de Bragg, de núcleo oco, baseado no método da matriz de transferência. O artigo introduz uma estratégia para o eficiente cálculo numérico das características de dispersão de modos TE_{0n} e TM_{0n}. Os resultados são validados por comparação com outros métodos na literatura.**

Palavras-Chave*—*Fibras de Bragg, matriz de transferência, modos transversais TE_{0n} e TM_{0n}.

Abstract— **This paper presents a mathematical model for the analysis of losses and dispersion in Bragg fibers with hollow core, based on the transfer matrix method. The paper introduces a strategy for the efficient numerical calculation of the dispersion characteristics of TE_{0n}, and TM_{0n} modes. The results are validated by comparison with other methods in the literature.**

Keywords*—*Bragg fibers, transfer matrix, transversal modes TE_{0n}, and TM_{0n}.

INTRODUCTION

Bragg fibers began to be studied in the 1970's [1]–[3]. Since then, the literature on the subject has been vast [1]-[11]. Among the several approaches employed in the analysis of Bragg fibers are the asymptotic method [11], Galerkin, finite element methods [8], and transfer matrix method [8], [9].

For a Bragg fiber with hollow core, as considered in this paper, there are losses due to the occurrence of leaky modes. The dispersion curves fall in the complex plane, making the analysis of such structures even harder. This paper employs two approaches for the solution of the dispersion problem in the complex plane: the Fortran routine DZANLY, based on Müller's method, and the well-known regula falsi method. The results are validated thorugh comparison with those obtained in [9]. The paper introduces a strategy for the complex plane calculation of the dispersion characteristics of transversal TE_{0n}, and TM_{0n} modes. The same strategy can also be applied to hybrid modes. The paper discusses the losses, and dispersion properties of the TE_{01}, TE_{02}, and TM_{01} modes. In general, the strategy adopted in this work showed to be reliable, and efficient.

MATHEMATICAL MODEL

A time variation $e^{j\omega t}$ is assumed throughout. The geometry of the Bragg fiber (BF) is illustrated in Figure 1. The refractive index profile in the cross section of the fiber is as shown in Figure 2. The physical characteristics of the fiber are described by the following parameters: core refractive index – $n_0 = 1$; Bragg cells refractive indices – $n_1 = 1.49$, $n_2 = 1.17$; cladding refractive index – $\eta_N = 1.49$; core radius – $r_0 = 1.3278$ µm; Bragg cell widths – $d_1 = 0.2133$ µm, $d_2 = 0.3460$ µm; number of Bragg cell – N = 16.

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Fig. 1. Geometry of Bragg fiber.

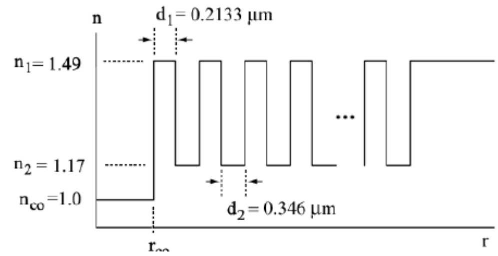

Fig 2. Refractive index profile.

The propagating modes of interest are the transversal TE_{0n}, and TM_{0n} modes, which do not depend on the azimuthal variable θ ($\partial / \partial\theta = 0$).

The transversal components of the TE_{0n}-mode are functions of the longitudinal magnetic field. Considering a region "ℓ" of the fiber, we have:

$$\begin{aligned} \vec{H}_{T\ell}(r,z) &= -\frac{j\beta}{K_{T\ell}^2}\nabla_T H_{z\ell}(r,z) \\ \vec{E}_{T\ell}(r,z) &= \frac{j\omega\mu_0\mu_{r\ell}}{K_{T\ell}^2}\left(\vec{z}\wedge\nabla_T H_{z\ell}(r,z)\right) \end{aligned} \tag{1}$$

$H_{z\ell}$ obeys the Helmholtz's equation:

$$\nabla^2 H_{z\ell}(r,z) + K_\ell^2 H_{z\ell}(r,z) = 0 \tag{2}$$

$$\text{with } K_{T\ell} = \sqrt{K_\ell^2 - \beta^2} = K_0\sqrt{n_\ell^2 - n_{\text{eff}}^2} = K_0 k_\ell \tag{3}$$

n_{eff} is the effective refractive index, a complex quantity.

The equations for the TM-mode are obtained from (1)–(3) with the help of the duality theorem.

TE_{on} MODES:

The external region of the Bragg fiber is characterized by circular waves that propagate radially ($H_0^{(2)}, H_0^{(1)}$). The internal regions (core, and Bragg cells) are modeled by stationary circular waves (J_0, Y_0).

TRANSFER MATRIX METHOD:

– Internal regions (core, and Bragg cells): These regions are modeled by superposition of two stationary radial waves:

($J_0(K_{Tp}r)$ $Y_0(K_{Tp}r)$), $2 \le p \le N-1$, $\ell = p-1$.

where $J_0(x)$, $Y_0(x)$ are the zero-order Bessel's functions of the first and the second kind, respectively.

This formulation includes frustrated internal reflections at the boundaries between the dielectric regions, which are responsible for the radial Poynting vector, and consequently for the leaky loss of the mode.

The solution of Helmholtz's equation (2) for the longitudinal magnetic field yields:

$$H_{zp}(r,z) = \left(A_p J_0\left(K_{Tp}r\right) + B_p Y_0\left(K_{Tp}r\right)\right)e^{-j\beta z} \quad (4)$$

In the core region, $B_1 = 0$. The substitution of (4) in (1) yields the electric field:

$$E_{\varphi p}(r,z) = \frac{j\omega\mu_0\mu_{rp}}{K_{Tp}}\left(A_p J_0'\left(K_{Tp}r\right) + B_p Y_0'\left(K_{Tp}r\right)\right)e^{-j\beta z} \quad (5)$$

Equations (4) and (5) are conveniently written in matrix form:

$$\begin{pmatrix} H_{zp} \\ \dfrac{E_{\varphi p}}{j} \end{pmatrix} = \left[M(p)\right]^{(1)} \begin{pmatrix} A_p \\ B_p \end{pmatrix} \quad (6)$$

where
$$\left[M(p)\right] = \begin{pmatrix} J_0\left(K_{Tp}r\right) & Y_0\left(K_{Tp}r\right) \\ \dfrac{z_0\mu_{rp}}{k_p} J_0'\left(K_{Tp}r\right) & \dfrac{z_0\mu_{rp}}{k_p} Y_0'\left(K_{Tp}r\right) \end{pmatrix} \quad (7)$$

with $K_{TP} = K_0 k_p$; $k_p = \sqrt{n_p^2 - n_{eff}^2}$; $z_0 = \dfrac{\omega\mu_0}{K_0} = 120\pi\,\Omega$. For convenience, the factor $e^{-j\beta z}$ was omitted.

EXTERNAL REGION ($r \ge r_N$)

This region is modeled by two cylindrical waves that propagate radially: one direct wave – $H_0^{(2)}\left(K_{TN}r\right)$ – and a reverse wave – $H_0^{(1)}\left(K_{TN}r\right)$:

$$H_{zN}(r,z) = \left(A_N H_0^{(1)}\left(K_{TN}r\right) + B_N H_0^{(2)}\left(K_{TN}r\right)\right)e^{-j\beta z} \quad (8)$$

with $A_N = 0$.

The substitution of (8) in (1) yields:

$$E_{\varphi N}(r,z) = \frac{j\omega\mu_0\mu_{rN}}{K_{Tp}}\left(A_N H_0^{(1)\prime}\left(K_{TN}r\right) + B_N H_0^{(2)\prime}\left(K_{TN}r\right)\right)e^{-j\beta z} \quad (9)$$

In matrix form, (8)–(9) are written as:

$$\begin{pmatrix} H_{zN} \\ \dfrac{E_{\varphi N}}{j} \end{pmatrix} = \left[M(N)\right] \begin{pmatrix} A_N \\ B_N \end{pmatrix} \quad (10)$$

with

$$\left[M(N)\right] = \begin{pmatrix} H_0^{(1)}\left(K_{TN}r\right) & H_0^{(2)}\left(K_{TN}r\right) \\ \dfrac{z_0\mu_{rN}}{k_N} H_0^{(1)\prime}\left(K_{TN}r\right) & \dfrac{z_0\mu_{rN}}{k_N} H_0^{(2)\prime}\left(K_{TN}r\right) \end{pmatrix} \quad (11)$$

Before we can determine the dispersion equation for the propagating modes, we need to obtain the boundary transfer matrices. These matrices express the continuity conditions the electromagnetic field must obey at the boundary between different regions.

We must consider two boundaries: one between adjacent Bragg cells, one between the last Bragg cell and the external region.

At the boundary between adjacent Bragg cells, $2 \le p \le N-1$, $\ell = p-1$, and:

$$\begin{pmatrix} H_{zp} \\ \dfrac{E_{\varphi p}}{j} \end{pmatrix}_{r=r_p} = \begin{pmatrix} H_{z\ell} \\ \dfrac{E_{\varphi \ell}}{j} \end{pmatrix}_{r=r_p} \quad (12)$$

Using the matrix form obtained earlier, (12) is written as:

$$\begin{pmatrix} A_p \\ B_p \end{pmatrix} = \left(\left[M(p)\right]^{-1}\left[M(\ell)\right]\right)_{r=r_p} \begin{pmatrix} A_\ell \\ B_\ell \end{pmatrix} \quad (13)$$

Inverting the matrix in (13), we obtain the boundary transfer matrix:

$$\begin{pmatrix} A_p \\ B_p \end{pmatrix} = \left\{\left(\frac{\pi K_{Tp} r_p}{2}\right)\left[m(p,\ell)\right]_{r=r_p}\right\} \begin{pmatrix} A_\ell \\ B_\ell \end{pmatrix} \quad (14)$$

with $r = r_p$, $k_{p\ell} = \left(\dfrac{k_p}{k_\ell}\right)$, $\mu_{\ell p} = \left(\dfrac{\mu_{r\ell}}{\mu_{rp}}\right)$, and

$$m_{11} = Y_0'\left(K_{Tp}r\right)J_0\left(K_{Tp}r\right) - \left(k_{p\ell}\mu_{\ell p}\right)Y_0\left(K_{Tp}r\right)J_0'\left(K_{Tp}r\right)$$

$$m_{12} = Y_0'\left(K_{Tp}r\right)Y_0\left(K_{Tp}r\right) - \left(k_{p\ell}\mu_{\ell p}\right)Y_0\left(K_{Tp}r\right)Y_0'\left(K_{Tp}r\right)$$

$$m_{21} = -J_0'\left(K_{Tp}r\right)J_0\left(K_{Tp}r\right) + \left(k_{p\ell}\mu_{\ell p}\right)J_0\left(K_{Tp}r\right)J_0'\left(K_{Tp}r\right)$$

$$m_{22} = -J_0'\left(K_{Tp}r\right)Y_0\left(K_{Tp}r\right) + \left(k_{p\ell}\mu_{\ell p}\right)J_0\left(K_{Tp}r\right)Y_0'\left(K_{Tp}r\right) \quad (15)$$

At the boundary between the last Bragg cell and the external region, $p = N$, $\ell = N-1$, and:

$$\begin{pmatrix} A_N \\ B_N \end{pmatrix} = \left(\left[M(N)\right]^{-1}\left[M(N-1)\right]\right)_{r=r_p} \begin{pmatrix} A_{N-1} \\ B_{N-1} \end{pmatrix} \quad (16)$$

The boundary transfer matrix is obtained as:

$$\begin{pmatrix} A_N \\ B_N \end{pmatrix} = \left\{\left(\frac{\pi K_{TN} r_N}{4}\right)\left[m(N,N-1)\right]_{r=r_p}\right\} \begin{pmatrix} A_{N-1} \\ B_{N-1} \end{pmatrix} \quad (17)$$

with $r = r_N$, $k_{p\ell} = \left(\dfrac{k_N}{k_{N-1}}\right), \mu_{\ell p} = \left(\dfrac{\mu_{r\ell}}{\mu_{rp}}\right)$, and

$$m_{11} = J_0(K_{T\ell}r)H_0^{(2)'}(K_{Tp}r) - (k_{p\ell}\mu_{\ell p})J_0'(K_{T\ell}r)H_0^{(2)}(K_{Tp}r)$$
$$m_{12} = Y_0(K_{T\ell}r)H_0^{(2)'}(K_{Tp}r) - (k_{p\ell}\mu_{\ell p})Y_0'(K_{T\ell}r)H_0^{(2)}(K_{Tp}r)$$
$$m_{21} = -J_0(K_{T\ell}r)H_0^{(1)'}(K_{Tp}r) + (k_{p\ell}\mu_{\ell p})J_0'(K_{T\ell}r)H_0^{(1)}(K_{Tp}r)$$
$$m_{22} = -Y_0(K_{T\ell}r)H_0^{(1)'}(K_{Tp}r) + (k_{p\ell}\mu_{\ell p})Y_0'(K_{T\ell}r)H_0^{(1)}(K_{Tp}r) \tag{18}$$

DISPERSION EQUATION FOR THE TE_{0n} MODES

A Bragg fiber is composed by the core and N cells; therefore, there are $N + 1$ boundaries at which the tangential electric field must be continuous. The enforcement of this continuity condition corresponds to multiplying the boundary transfer matrices obtained earlier.

Starting at the external boundary, and moving inward, we obtain the following 2×2 matrix:

$$\begin{pmatrix} A_N \\ B_N \end{pmatrix} = \begin{bmatrix} M(1,1) & M(1,2) \\ M(2,1) & M(2,2) \end{bmatrix} \begin{pmatrix} A_0 \\ B_0 \end{pmatrix} \tag{19}$$

with

$$[M] = \left\{ \left(\frac{j\pi K_{TN} r}{4} \right) [m(N, N-1)] \right\}_{r=r_N} \cdot \left\{ \prod_{p=N-2}^{1} \left(\frac{j\pi K_{Tp} r}{2} \right) [m(p, p-1)] \right\}_{r=r_p} \tag{20}$$

The initial hypothesis, $A_N = 0$, $B_0 = 0$, decouples equation (20):

$0 = M(1,1)\, A_0$

$$B_N = M(2,1)\, A_0 \tag{21}$$

The term $M(1,1) = 0$ is the dispersion equation for the TE_{on} mode, n – 1,2:

$$M(1,1) = F(\lambda, n_{\mathrm{eff}}(\lambda)) = 0 \tag{22}$$

The solution of equation (22) in the complex plane requires a special strategy, to be considered in Section III.

The formulation outlined above allow for the computation of the electromagnetic field $\{H_z(r), E_\varphi(r)\}$ of TE_{0n} modes. For convenience, the longitudinal field in the core is normalized, so that $A_0 = 1$. The field components in the core are given by equations (4)–(5):

$$H_{z0}(r) = J_0(K_{T0}r) \tag{23}$$

$$E_{\varphi 0}(r) = J_1(K_{T0}r) \tag{24}$$

The electric field has also been normalized, and use was made of the identity: $J_0'(x) = -J_1(x)$.

The transfer matrix is applied at the core boundary, and the field coefficients at the next region are given by:

$$H_{z1}(r) = A_1 J_0(K_{T1}r) + B_1 Y_0(K_{T1}r) \tag{25}$$

$$E_{\varphi 1}(r) = \frac{j\omega\mu_{r1}}{k_1 \mu_{r0}} \left(A_1 J_1(K_{T1}r) + B_1 Y_1(K_{T1}r) \right) \tag{26}$$

The electric field has been normalized again, and use was made of the identity: $Y_0'(x) = -Y_1(x)$.

This process is repeated for all the boundaries.

SOLUTION OF THE DISPERSION EQUATION

There are several alternatives for the numerical solution of the dispersion equation (22) in the complex plane. In this work, we used the well-known regula falsi method, described in [10], and the IMSL DZANLY Fortran routine, based on Müller's method.

All the propagating modes in the Bragg fiber satisfy the transverse resonance condition, that correspond to wavelengths where the propagation losses are minimum. Therefore, at theses wavelengths, the dielectric layers of the Bragg cells reflect the radial energy strongly, and the structure can be modeled by a resonant core with metallic boundary. The electric field E_φ must then go to zero at the core boundary.

For TE_{0n} modes, equation (24) yields

$$J_1(K_{T0}r_0) = 0, K_{T0} = \frac{x_{1,m}}{r_0} \tag{27}$$

where $x_{1,m}$ is the m–th root of the Bessel function of order 1, and r_0 is the core radius.

An estimate of the real part of the effective refractive index is obtained for the TE_{0m} mode as

$$\mathrm{Re}(n_{eff})_{TE_{0m}} = \sqrt{1 - \left(\frac{x_{1,m}}{2\pi} \frac{\lambda}{r_0} \right)^2} \tag{28}$$

It is assumed that $\mathrm{Im}(n_{eff}) \ll \mathrm{Re}(n_{eff})$. Each root $x_{1,m}$ of the Bessel function J_1 correspond to a different TE_{0m} mode.

At and near to the low loss operation point of the Bragg fiber, the results obtained with equation (28) agree with those obtained with the electromagnetic method provided the mode is in the photonic band of the structure. For the Bragg fiber described in Figures 1 and 2, the first two TE modes are characterized by:

$$TE_{01}: x_{11} = 3.832,\ \mathrm{Re}(n_{eff})_{TE_{01}} = \sqrt{1 - 0.211\lambda^2} \tag{29}$$

$$TE_{02}: x_{11} = 0.7016,\ \mathrm{Re}(n_{eff})_{TE_{02}} = \sqrt{1 - 0.7072\lambda^2} \tag{30}$$

The estimates of $\mathrm{Re}(n_{eff})$ is valid solely for the fundamental mode TE_{01}, as the TE_{02} mode is not in the photonic band. But it is sufficient, and allows for the determination of the dispersion characteristics of the other modes.

Figure 3 shows that only the TE_{01} mode is in the photonic band of the Bragg fiber. We can also conclude that the strategy of orienting the curve for $\mathrm{Re}(n_{\mathrm{eff}})$ according the estimate in eq. (29) is recommended for the TE_{0n} modes that fall in the photonic band; in the present case, it is only the TE_{01}mode. The results obtained with the DZANLY routine, and the regula falsi method agree within 2%.

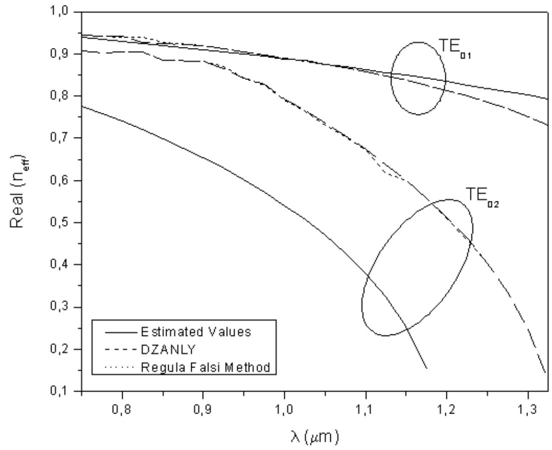

Fig. 3. Real part of the effective refractive index as obtained by equation (29)-(30), DZANLY routine, and regula falsi method.

A mode in the photonic band of the Bragg fiber exhibits a "λ" associated with minimum losses, due to the field reflection by the Bragg cells. As "λ" varies, the reflections become less intense, and the losses increase. Therefore, the losses of a mode in the photonic band will be represented by a parabolic curve; the real part of n_{eff} varies slowly close to minimum loss wavelength λ.

Figures 4–5 show the losses and Re(n_{eff}) for the TE_{02} mode, and we notice that there are no points of minimum losses, and no the variation of Re(n_{eff}) is not slow. This indicates that the TE_{02} mode is not in the photonic band of the Bragg fiber. This explains the large discrepancy between the values estimated for Re(n_{eff}) – equation (30) – and the ones shown in Figure 3.

IV. Results

To validate the model developed here for the hollow core Bragg fiber, we compare our results with other found in the literature [9]. The Bragg fiber is the same as before. The mode losses were calculated as [7,8]:

$$Loss\left(\lambda\right)=\frac{40\pi}{\lambda\, ln\left(10\right)}\operatorname{Im}\left(n_{eff}\right)=\frac{54.57}{\lambda}\operatorname{Im}\left(n_{eff}\right)\cdot 10^{6}\ \left(\text{dB/m}\right) \tag{31}$$

Figure 4 shows Re(n_{eff})for the TE_{01} and TE_{02} modes as calculated by the DZANLY routine, the regula falsi method, and reference [9]. The agreement between the various results is very good.

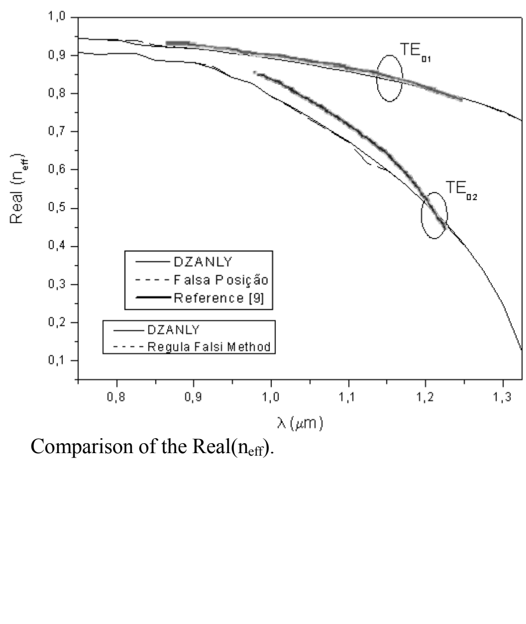

Fig.4. Comparison of the Real(n_{eff}).

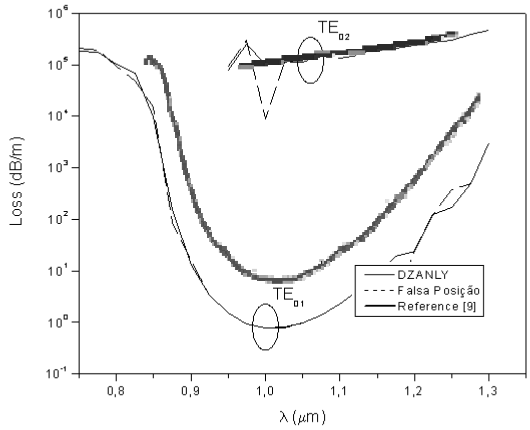

Fig. 5. Comparison of the losses.

Figure 5 shows a comparison of results obtained for the losses of the same modes. For the TE_{02} mode, this figure indicates a difference of about 12% between the results obtained with the DZANLY routine and the regula falsi method, although the curves exhibit the same general behavior. For the fundamental TE_{01} mode, the difference between the results is smaller than 2%.

At the point of minimum loss of the TE_{01} mode, λ = 1 μm, and the effective refractive indices of the two modes are given by:

Mode	Reference [8]	This work
TE_{01}	$0.8910672175 - j1.422605\times10^{-8}$	$0.891067 - j1.4230\times10^{-8}$
TE_{02}	$0.7920859031 - j1.819323\cdot\times10^{-3}$	$0.792975 - j1.8140\times10^{-3}$

The asymptotic method of reference [11] yields, also at λ = 1 μm:

TE_{01}: $0.90919 - j6.86126 10^{-9}$; TE_{02}: $0.7869166 - j3.556\times10^{-3}$

Figures 4 and 5 confirm that the TE_{02} mode is outside the photonic band, and that explains the large losses.

The TM_{01} and TM_{02} modes were also investigated with the DZANLY routine. The results in Figures 6, Re(n_{eff}), and 7, losses, indicate clearly that both modes are outside the photonic band, due to the large losses. Reference [9] does not show results for theses modes, only mentioning that both have large losses.

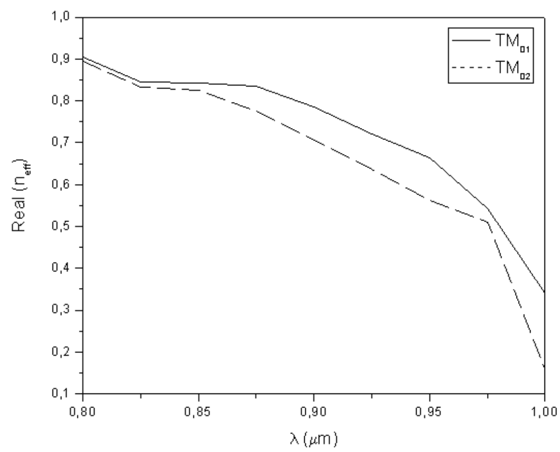

Fig. 6. Re (n_{eff}) of the TM_{01} and TM_{02} modes.

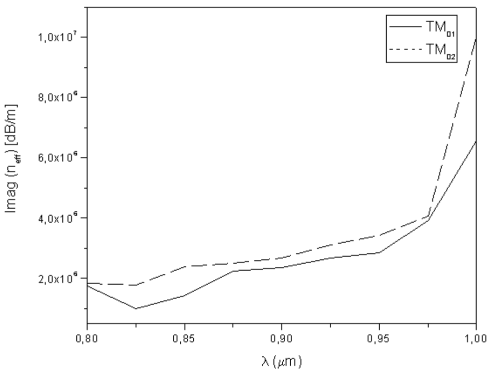

Fig. 7. Imag (n_{eff}) of the TM_{01} and TM_{02} modes.

Figure 7 makes it clear that the TM_{01} and TM_{02} modes are outside the photonic band of the Bragg fiber, as this figure does not show a deep minimum loss at $\lambda = 0.825$ μm. At this wavelength, the curves in Figure 6 exhibit a rapid variation.

The normalized field components (H_z, E_φ) are shown in Figures 8 (TE_{01} mode, $\lambda = 1$ μm), 9 (TE_{02} mode, $\lambda = 1$ μm), and 10 (TM_{01} mode, $\lambda = 0.825$ μm). These figures indicate clearly that the TE_{01} mode is well confined in the Bragg cells, due to the strong reflections by the cells. The hybrid modes exhibit large losses, and we conclude that, at $\lambda = 1$ μm, this Bragg fiber is a single-mode structure.

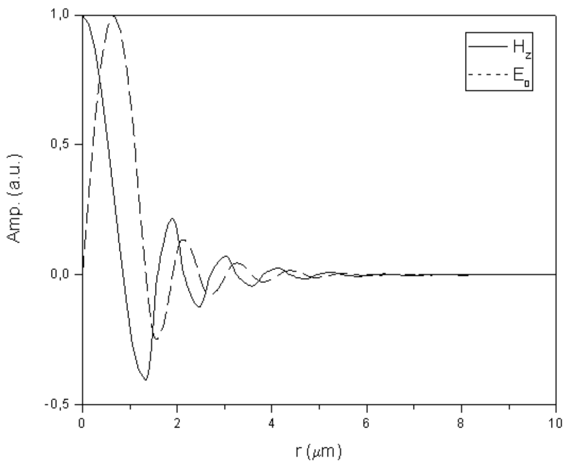

Fig. 8. TE_{01} mode, $\lambda = 1$μm, $n_{eff} = 0{,}8911\text{-}j1{,}423\times10^{-8}$

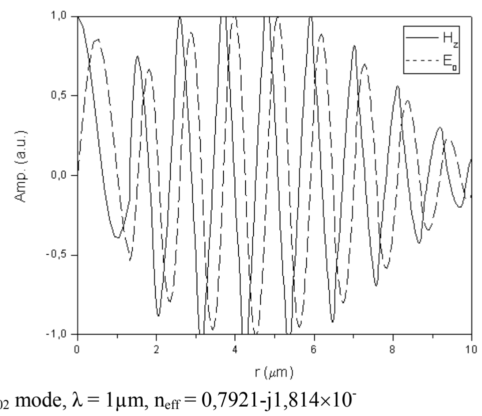

Fig. 9. TE_{02} mode, $\lambda = 1$μm, $n_{eff} = 0{,}7921\text{-}j1{,}814\times10^{-}$

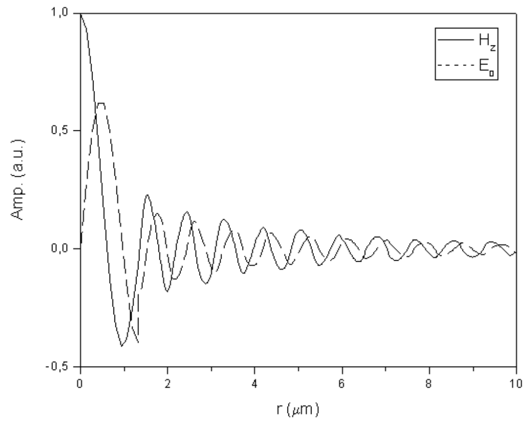

Fig. 10. TM_{01} mode, $\lambda = 0{,}825$μm, $n_{eff} = 0{,}8462\text{-}j2{,}241\times10^{-6}$

V. CONCLUSION

The results obtained with the model developed in this work were validated by comparison with results obtained by other methods [9], [11]. For the Bragg fiber considered, the results obtained with the DZANLY routine and the regula falsi method were more accurate for the TE_{01} mode (discrepancy smaller than 2%) than for the TE_{02} mode, for which the difference between the results was of the order of 12%.

REFERENCES

[1] Kapron F.P., D.B. Keck, R.D. Maurer. "*Radiation losses in glass optical waveguides*". Appl.Phys. Letter", pp. 423-425, 1970.

[2] P. Yeh, A. Yariv. "*Bragg Reflection Waveguides*". Optics Communications, PP. 427-430, 1976.

[3] P. Yeh, A. Yariv, E. Marom. "*Theory of Bragg Fiber*". J. Opt. Soc. Am, pp. 1196-1201, 1978.

[4] E. Yablonovitch. "*Inhibited Spontaneous Emission in Solid State Physics and Electronics*". Phys. Ver. Lett, pp. 2059, 1987.

[5] Jhon. S. "*Strong Localization of Photons in Certain Disordered Dielectric Superlatticest*", Phys. Ver. Lett, pp. 2486, 1987.

[6] E. Yablonovitch, "*Photonic bang gap structures*", Josa B, pp. 283-295, 1993.

[7] Gaponov Dmitry, Philippe Roy and S. Biriukov. "*Optical Properties of Microstructured Optical Fibers and Fiber Based on Large Node Area Bragg Fiber*". Thére Docteur de L'université de Limonges, 2008.

[8] Shangping Guo, Sacharia Albin. "*Comparative Analysis of Bragg* Fibers". Optic Express, vol 12, pp. 198-207, 2004.

[9] Alexander Argyros. "*Guided Modes and Loss in Bragg Fibers*". Optic Express, vol 10, pp. 1411-1417, 2002.

[10] Gardiol F.E. "*Anisotropic Slabs in Rectangular* Waveguides". IEEE Trans. on Microwave theory and techniques, vol. Mtt-18, 1970.

[11] Xu Y., Lee R.K. and Yariv A. "*Asymptotic Analysis of Silicon Based Bragg* Fibers". Opt. Express 11, 2003.

[12] Marinho R. Leonardo. "*Análise Rigorosa das Fibras de Bragg com Núcleo Ôco*". Master's thesis defense, december 2012.