# Evaluation of the Effects of Co-Channel Interference on the Bit Error Rate for QPSK and M-QAM Modulations

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Abstract—In this paper, a performance analysis for QPSK, 16-QAM and 64-QAM, modulations in the presence of co-channel interference is made. At first, an expression of the bit error rate (BER) for *M*-PAM as a function of the  $E_b/N_0$  ratio in the presence of *K* interferers is obtained. The formula from the BER gotten earlier is used to derive a simplified expression for the BER of *M*-QAM modulations. For each modulation scheme, it is shown that there is a minimum signal to interference ratio (SIR) for the system does not present a BER floor. The results obtained show that modulations with higher order alphabets need high SIR in order to maintain the BER at satisfactory levels. This analysis can be applied in the performance evaluation of cellular radio networks, as WiMAX and LTE.

Index Terms—M-PAM, Q-PSK, M-QAM, Co-Channel Interference, SIR, BER.

## I. INTRODUCTION

A correct performance evaluation of digital systems in cellular networks have to contemplate the presence of cochannel interference.

Some papers [3], [4], [5], [6] have evaluated the BER for different digital modulation schemes in the presence of cochannel interference. These analyses are not clear, nor present all results that are presented here.

In this paper, simple expressions of BER are presented for modulations M-PAM, QPSK and M-QAM in the presence of K interferers. The total mean co-channel interference power is maintained constant for different number of interferers.

This paper is organized as follows. Section II shows the system description. Section III provides the BER expressions for M-PAM and M-QAM modulations for K interferers. Section IV shows the results and finally the conclusions are presented in section V.

# **II. SYSTEM DESCRIPTION**

We start by getting the BER expressions for M-PAM modulations and then we will derive the corresponding expressions for M-QAM schemes. Consider the following received signal:

$$r(t) = \sum_{k=0}^{K} s_k(t) + n(t)$$
(1)

where  $s_k(t)$  is a *M*-PAM signal, and *M* is the alphabet size. From these signals,  $s_0(t)$  is the signal of interest,  $s_k(t)$  for k = 1, 2, ..., K is the signal of each one of the interferers and n(t) is additive white gaussian noise with unilateral power spectral density given by  $N_0$ .

The k-th PAM signal for  $k = 0, 1, 2, \dots, K$  is given by:

$$s_k(t) = \sum_{i=-\infty}^{\infty} a_{i,k} g(t - iT_s)$$
<sup>(2)</sup>

where  $a_{i,k}$  is the transmitted symbol at the *i*-th time interval of the *k*-th user, that assumes one of the *M* values of the alphabet with equal probability and g(t) is the pulse format with duration  $T_s$ .

At first, we are going to consider a system with just one interferer. At the *i*-th symbol time interval, that is for  $iT_s \leq t \leq (i+1)T_s$ , excluding the noise and assuming that interest and interference signals are synchronous, the sample at the matched filter output at time  $t = (i+1)T_s$  is given by:

$$y(i+1) = a_{i,0} + \alpha a_{i,1} \tag{3}$$

where  $a_{i,0}$  is the signal of interest symbol at the *i*-th time interval,  $a_{i,1}$  is the interferer symbol and  $\alpha$  is the interference amplitude relative to the signal. In order to obtain (3), we assumed that  $\int_{iT_s}^{(i+1)T_s} g^2(t) dt = 1$ .

The instantaneous power is given by:

$$P = (a_{i,0} + \alpha a_{i,1})^2 \tag{4}$$

Therefore, the mean power is given by:

$$\overline{P} = E\left\{a_{i,0}^2\right\} + \alpha^2 E\left\{a_{i,1}^2\right\} + 2\alpha E\left\{a_{i,0}a_{i,1}\right\}$$
(5)

where the third term of (5) is zero and  $E\left\{a_{i,0}^2\right\} = E\left\{a_{i,1}^2\right\} = P_a$  is the constellation mean power.

Therefore, the mean power is given by:

$$\overline{P} = P_a + \alpha^2 P_a \tag{6}$$

From (6), the SIR is given by:

$$\frac{S}{I} = \frac{P_a}{\alpha^2 P_a} = \frac{1}{\alpha^2} \tag{7}$$

Generalizing for K interferers, we have that:

$$y(i+1) = a_{i,0} + \frac{1}{\sqrt{K}} \sum_{k=1}^{K} \alpha a_{i,k}$$
(8)

where the factor  $\frac{1}{\sqrt{K}}$  maintains the total interference power constant, independent on the number of interference, and equal to  $I = \alpha^2 E_a$ . Therefore the SIR for K interference is also equal to (7).

# III. BIT ERROR RATE

## A. 2-PAM

For 2-PAM modulation, the BER without co-channel interference is given by [1]:

$$P_b = Q\left(\sqrt{2\frac{E_b}{N_0}}\right) \tag{9}$$

For one interferer, we can easily obtain the BER as [2]:

$$P_b = \frac{1}{2}Q\left((1+\alpha)\sqrt{2\frac{E_b}{N_0}}\right) + \frac{1}{2}Q\left((1-\alpha)\sqrt{2\frac{E_b}{N_0}}\right)$$
(10)

Generalizing for K interferers, we can write the BER as [2]:

$$P_b = \sum_{k=0}^{K} \frac{\binom{K}{k}}{2^K} Q\left(\left[1 - \frac{K - 2k}{\sqrt{K}}\alpha\right]\sqrt{2\frac{E_b}{N_0}}\right)$$
(11)

# B. M-PAM

For a M-PAM modulation, the symbol-error probability without co-channel interference is given by [1]:

$$P_M = \frac{2(M-1)}{M} Q\left(\sqrt{6\frac{E_b}{N_0} \frac{\log_2 M}{(M^2 - 1)}}\right)$$
(12)

Therefore, considering the Gray mapping at high  $E_b/N_0$ , the BER for a *M*-PAM modulations is well approximated as:

$$P_b \cong \frac{P_M}{\log_2 M} \tag{13}$$

Generalizing for K interferers, it can be shown that the BER is given by:

$$P_b \cong \sum_{m=0}^{\frac{M}{2}-2} \sum_{k=0}^{K} \frac{\binom{K}{k}}{2^{K-2}M \log_2 M} Q(A) + \sum_{k=0}^{K} \frac{\binom{K}{k}}{2^{K-1}M \log_2 M} Q(B)$$
(14)

where

$$Q(A) = Q\left(\left[1 - \frac{(2m+1)(K-2k)}{\sqrt{K}}\alpha\right]\sqrt{6\frac{E_b}{N_0}\frac{\log_2 M}{(M^2 - 1)}}\right)$$

and

$$Q(B) = Q\left(\left[1 - \frac{(M-1)\left(K - 2k\right)}{\sqrt{K}}\alpha\right]\sqrt{6\frac{E_b}{N_0}\frac{\log_2 M}{(M^2 - 1)}}\right)$$

where the first and second terms in (14) correspond to the error probability for internal and external symbols of the PAM constellation, respectively.

# C. M-QAM

The symbol-error probability of a M-QAM modulation is easily obtained through the cartesian product of two  $\sqrt{M}$ -PAM signals. The symbol error rate for a M-QAM modulation is given by [1]:

$$P_s = 1 - \left(1 - P_{\sqrt{M}}\right)^2 \tag{15}$$

where  $P_{\sqrt{M}}$  is the symbol error probability for a  $\sqrt{M} - PAM$  modulation given by:

$$P_{\sqrt{M}} = \sum_{m=0}^{\frac{\sqrt{M}}{2}-2} \sum_{k=0}^{K} \frac{\binom{K}{k}}{2^{K-2}\sqrt{M}} Q'(A) + \sum_{k=0}^{K} \frac{\binom{K}{k}}{2^{K-1}\sqrt{M}} Q'(B)$$
(16)

where

$$Q'(A) = Q\left(\left[1 - \frac{(2m+1)(K-2k)}{\sqrt{K}}\alpha\right]\sqrt{6\frac{E_b}{N_0}\frac{\log_2\sqrt{M}}{(M-1)}}\right)$$

and

$$Q'(B) = Q\left(\left[1 - \frac{\left(\sqrt{M} - 1\right)\left(K - 2k\right)}{\sqrt{K}}\alpha\right]\sqrt{6\frac{E_b}{N_0}\frac{\log_2\sqrt{M}}{\left(M - 1\right)}}\right)$$

Similarly to (13), the BER for a M-QAM modulation is given by:

$$P_b \cong \frac{P_s}{\log_2 M} \tag{17}$$

## **IV. NUMERICAL RESULTS**

In order to evaluate the co-channel interference effects on the BER of digital modulations, we are going to plot some of the equations obtained in section III.

Fig. 1 presents the BER as a function of  $E_b/N_0$ for BPSK modulation with one interferer, for SIR =-3, 0, 3, 6, 9,  $\infty$  [dB]. For SIR = -3 dB and SIR = 0 dB, we observe BER floor approaches 1/2 and 1/4, respectively. While for SIR = 3, 6, 9 [dB] there is not a BER floor, but the performance, as expected, is inferior to the case without interference. Therefore, it exists a SIRvalue below wich there is always a BER floor regardless of any  $E_b/N_0$  increasing.



Figure 1. BER as a function of  $E_b/N_0$  and SIR for BPSK modulation with one interferer.

Fig. 2 shows the BER as a function of  $E_b/N_0$  for QPSK modulation with one interferer for SIR = -3, 0, 3, 6, 9,  $\infty$  [dB]. The curves and conclusions are very similar to BPSK case.



Figure 2. BER as a function of  $E_b/N_0$  and SIR for QPSK modulation with one nterferer.

Fig. 3 presents the BER as a function of  $E_b/N_0$  for 16-QAM modulation with one interferer for SIR = 9, 12, 15, 18, 21,  $\infty$  [dB]. This modulation is much more susceptible to interference than BPSK and QPSK. There is a BER floor for SIR = 9 dB.



Figure 3. BER as a function of  $E_b/N_0$  and SIR for 16-QAM modulation with one interferer.

Fig. 4 shows the BER as a function of  $E_b/N_0$  for 64-QAM modulation with one interferer for  $SIR = 15, 17, 20, 23, 26, \infty$  [dB]. The conclusions are similar to 16-QAM modulation, except that the SIR relations are higher.



Figure 4. BER as a function of  $E_b/N_0$  and SIR for 64-QAM modulation with one interferer.

For *M*-PAM modulation, using (14), it is easy to show that for high  $E_b/N_0$  and  $SIR = (M-1)^2 K$ , the BER floor approaches  $1/(2^K M \log_2 M)$ . On the other hand, when  $SIR < (M-1)^2 K$  the BER floor approaches  $1/(2^{K-1}M \log_2 M)$ . Using a similar reasoning, we conclude that for M - QAMmodulation when  $SIR = (\sqrt{M} - 1)^2 K$  the BER floor, considering (15) approaches  $1/(2^{K-1}\sqrt{M} \log_2 \sqrt{M})$ , and when SIR is lightly lower than  $(\sqrt{M} - 1)^2 K$  the BER floor

approaches  $1/\left(2^{K-2}\sqrt{M}\log_2\sqrt{M}\right)$ 

Fig. 5, Fig. 6, Fig. 7 and Fig. 8 presents the BER as a function of  $E_b/N_0$  for BPSK, QPSK, 16-QAM and 64-QAM, respectively. The number of interferers assumes the values from 0 to 5 and  $\infty$ , while the mean interference power remains constant. For BPSK SIR = 10 dB, for QPSK 15 dB, for 16-QAM 20 dB and for 64-QAM 25 dB. In these curves, we can observe the system performance when the number of interferers is varied. Observe that the performance decreases as the number of interferers is infinity the interference is gaussian. Observe that in this case, there is always a BER floor, independently on the  $E_b/N_0$  value. It is a common mistake in the literature to suppose that the co-channel interference is gaussian. In fact, in a cellular network the number of strong interferers is in general one or two.



Figure 5. BER as a function of  $E_b/N_0$  for BPSK modulation with 0 to 5 and  $\infty$  interferers and SIR = 10 dB.



Figure 6. BER as a function of  $E_b/N_0$  for QPSK modulation with 0 to 5 and  $\infty$  interferers and SIR = 15 dB.



Figure 7. BER as a function of  $E_b/N_0$  for 16-QAM modulation with 0 to 5 and  $\infty$  interferers and SIR = 20 dB.



Figure 8. BER as a function of  $E_b/N_0$  for 64-QAM modulation with 0 to 5 and  $\infty$  interferers and SIR = 25 dB.

Fig. 9 presents a comparison among the modulations analyzed in this paper for SIR = 20 dB, with one interferer.

## V. CONCLUSIONS

In this paper closed expressions of the BER for QPSK and *M*-QAM modulations, as a function of the SIR and the number of interferers were obtained. Observe that when  $SIR \leq (\sqrt{M} - 1)^2 K$  there is a BER floor, so increasing the  $E_b/N_0$  ratio is worthless to improve the system performance. On the other hand, when  $SIR > (\sqrt{M} - 1)^2 K$  the system performance increases with  $E_b/N_0$ . When the number of interferers increases, while the SIR constant, the system performance worsens. Modulations with higher order alphabets are much more susceptible to co-channel interference than lower order one. The expressions presented in this work are a powerful tool to evaluate the performance of digital systems in cellular networks, such as WiMAX and LTE.



Figure 9. BER comparison for BPSK, QPSK, 16-QAM and 64-QAM for SIR = 20 [dB] with one interferer.

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