Iterative Water-Filling for Digital Subscriber Lines

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Abstract— **Digital Subscriber Lines (DSL) technology are of great importance nowadays especially due to the demand for higher speeds in Internet access. New spectrum management techniques are under study today so the binder environment can be coordinated and hence more cleverly used. These new techniques are called Dynamic Spectrum Management (DSM). This paper analyzes the first level of DSM coordination, in which power control is performed in a distributed and autonomous fashion by each modem. More specifically, this work concerns the Iterative Water-Filling algorithm, which has been shown to considerably improve the performance of DSL systems.**

Keywords— **Digital Subscriber Line, Dynamic Spectrum Management, Iterative Water-Filling, Power Control.**

I. Introduction

Digital Subscriber Line (DSL) is a transmission method that uses frequency bands above the one used by traditional voice channel to carry high-speed data in twisted copper pairs loops. The technology has experienced an important increase in number of users in the past few years. And as costumers become more and more enthusiastic for higher-speed transmission, new solutions are under research for dealing with the current deployment limitations. The major problem that limits the capacity of DSL systems is *crosstalk*, provoked by the electromagnetic coupling between loops bundled together in the same binder. The operation in high frequencies in a medium originally designed for voice-band transmission leads to levels of crosstalk that are about 10-20 dB higher than background noise.

The issue of crosstalk had been dealt with in a rather conservative way in early DSL practice: in order to achieve spectral compatibility and thus a greater number of users in the system, service providers made use of fixed spectral masks that were the result of the analysis of a worst-case situation – the DSL binder was engineered to withstand the highest possible number of crosstalkers of the worst possible interference type in a cable with worst-case electromagnetic couplings [1].

A new standard for DSL practice has been under extensive research for a few years now. The novel systems attempt to make better use of the binder capabilities by basically introducing two concepts:

a) the interference DSL channel is modeled in a more mature and realistic way – the situation of a worst-case scenario is rarely observed in practice and the spectral masks do not take under consideration the geographical differences and levels of interference between users;

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b) some level of coordination between the lines is applied so that crosstalk can be avoided or mitigated and spectral compatibility achieved in a less restrictive way. As the level of coordination increases, a better system performance will be found (most current DSL practice shows no coordination at all).

The new approach has been named Dynamic Spectrum Management (DSM). DSM treats crosstalk as the time-variable and manmade interference it is and hence jointly optimizes the DSL environment. Each modem can dynamically adapt its spectrum in order to maximize its own data rate and minimize interference for other users. Fixed spectral masks are no longer needed in the new standard. Gains in transmission speeds and service range are significant [2], [3].

This paper focuses on the Iterative Water-Filling (IWF) algorithm, which performs the lowest level of coordination DSM, called level-1 DSM.

II. DSM Level 1 Principles

DSM at level 1 coordination addresses the problem of spectral compatibility in an autonomous fashion. Each transceiver has access only to its own power spectrum, noise levels and data rate. Adequate processing of this information leads to the dynamic adaptation of the transmission spectrum in order to avoid crosstalk couplings. This distributed computation of each user power spectral densities (PSD) provides levels of spectral compatibility that are virtually impossible to achieve in a fixed spectral mask-based design – a.k.a. Static Spectrum Management (SSM). Interesting features are that DSM at level 1 is immediately practical and deployable, as well as beneficial to all DSL lines in a binder. The latter is reached since no loops will perform worse than if no DSM was implemented [2].

The IWF algorithm applies DSM at level 1 coordination. It is based on game-theory [2] and models the DSL interference channel as an uncooperative game. The IWF aims at reaching equilibrium of optimal performance in the multi-user DSL environment by allocating power in a greedy way. This means that each player should make choices that privilege only its own performance in spite of all others. The coordination lies in the fact that the algorithm will try to achieve a given target data rate by using the minimum amount of power possible – excessive power in the line causes excessive interference to other users. Given the channel frequency-dependent channel-to-noise-plus-interference ratio (CNIR), the IWF works by allocating more power in frequency bands that experience less significant noise-plus-interference levels and less power in frequency bands with higher noise-plus-interference levels. The scheme in a single-user case can be viewed as the "pouring" of power in the bowl of the inverse CNIR curve. That means that each user "moves away" from those frequency bands that experience great noise disturbance, focusing on transmission in bands it can use more efficiently use. As for DSL modems the amount of power is restricted, each user should dynamically allocate its PSD over frequency selfishly regarding its own functioning. The IWF algorithm makes continuous iterations and uses the non-cooperative game scheme to get to a point of equilibrium (Nash Equilibrium) in which each player strategy in power allocation is optimal considering other players' strategies [2].

For a $i = 1, 2, ..., N$ user DSL channel the rules of a IWF game are:

a) each user i should create a strategy based on the allocation of its PSD $P_i(f)$. This strategy is based on the water-filling equation

$$P_i(f) = \begin{cases} K_i - N_i(f), & N_i(f) \leq K_i \\ 0, & N_i(f) > K_i \end{cases}, \tag{1}$$

where K_i is the water level, a constant chosen so the power constraint is met, and $N_i(f)$ is the total noise-to-channel ratio of user i, where noise includes the background noise's PSD σ_i and crosstalk;

b) each user should have a maximum power available $\mathbf{P_i}$, which must satisfy the following constraint:

$$\mathbf{P_i} \geq \int P_i(f) df; \tag{2}$$

c) each line only has access to its own channel transfer function and noise profile (no central coordination);

d) the objective is for each user to attempt to achieve its target data rate by selectively allocating a minimum amount of power in the available spectrum. The practical data rate for each user i can be obtained by

$$R_i = \log_2 \int \left[1 + \frac{P_i(f)|H_{ii}(f)|^2}{\Gamma\left(\sigma_i + \sum_{j \neq i}^{N} P_j(f)|H_{ji}(f)|^2\right)} \right] df, \tag{3}$$

where Γ is the Shannon Gap to capacity and $H_{ij}(f)$ is the channel transfer function from Tx i to Rx j. Note that users do not have access to each coupling transfer function (where $i \neq j$), but they have access to their total noise profiles, which are influenced by the coupling transfer functions.

III. Implementation

We will focus on one of the standard simulation scenarios for DSL, depicted in figure 1. We are interested in downstream asymmetrical DSL (ADSL). The situation consists of a 5 Km Central Office (CO) line and a 3 Km Remote Terminal (RT) line sharing the same binder for 1 Km. This scenario is becoming very usual, since service providers are deploying fiber-fed remote transmission stations to be closer to potential new users. Levels of crosstalk from the RT user to the CO user can be prohibitively high due to the initial strength of the RT signal. We are interested in evaluating the performance of the IWF algorithm compared to one of the previous strategies for dealing with the problem, the flat Power Back-Off (PBO). With flat PBO, each modem transmits the minimum possible flat PSD to support its desired data rate. We set the maximum

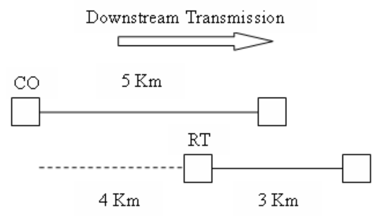

Fig. 1. Loop topology for DSL transmission

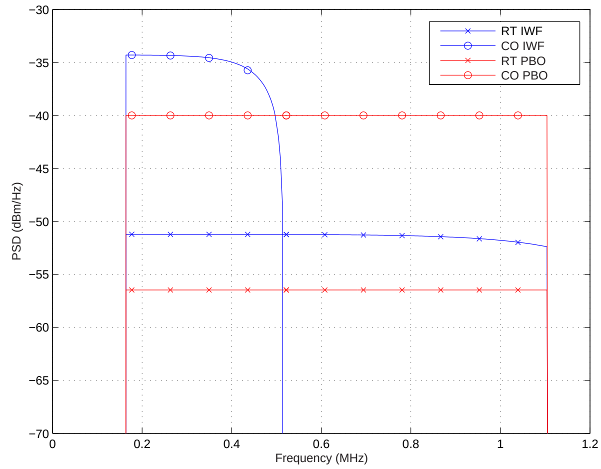

Fig. 2. Power allocated with IWF and flat PBO

power to 20.4 dBm/Hz for each modem, we are interested in a symbol error rate of 10^{-7} or less and we use 0.5 mm (24 AWG) lines. While maintaining a 1.3 Mbps service in the longer line, it is possible to achieve 4.3 Mbps with flat PBO and 5.8 Mbps with IWF in the shorter line. This implies in a gain of over 30% – in real scenarios, where there can be about 30 users in the same binder, this gain can be superior to 100% [2], [3]. As can be seen in Figure 2, the IWF algorithm intrinsically takes into account the interaction between the lines and converges to a situation where each user avoids crosstalk by selectively allocating power in frequency bands that best suit its conditions.

IV. Conclusion

This paper focused on an algorithm that applies DSM level 1 in a DSL binder, the Iterative Water-Filling. The IWF performs the simplest level of coordination DSM, where power control is made in an autonomous and distributed way. It has been studied its principles, seen how the algorithm performs power allocation and investigated what are the gains achieved with its use.

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