Studies of the performance of a nonlinear directional fiber coupler with periodically modulated dispersion: Analytical Results

M. G. da Silva, E. F. de Almeida, C. S. Sobrinho, J. L. S. Lima and A. S. B. Sombra

***Abstract*— Optical fiber couplers have been studied for their potential applications to ultrafast all optical switching processing, like optical switch. In this work, we report numerical and analytical studies of the propagation of ultrashort optical soliton pulses in two-core nonlinear fiber couplers constructed with periodically modulated dispersion fiber (PMDF). Using the technique developed by Chu, we convert the coupled nonlinear Schrödinger equations (NLSE's) into a variational problem with a corresponding Lagrangian equations of motion for a finite number of degrees of freedom and obtain the switching characteristics of the device. The transmission characteristics, the critical energy, extinction ratio (Xratio) and the crosstalk level (Xtalk) for first order solitons were studied for low to high pump powers.**

***Keywords*— Optics Fiber, nonlinear Couplers, dispersion, Solitons, kerr effect.**

I. Introduction

Optical fiber couplers have been studied for their potential applications to ultrafast all optical switching processing, like optical switch [1-4]. In a nonlinear coupler constructed from a Kerr type medium the dependence of the nonlinear refractive index n on the beam intensity is given by the expression $n = n_o + n_2 I$, where n_o is the refractive index at low intensity and n2 is the Kerr nonlinear coefficient [5]. Jensen showed that varying the input light in the nonlinear coupler could lead to pulse switching between the two cores[6]. He therefore foresaw the possible use of a nonlinear directional coupler as an optical switch. A nonlinear coupler consists of two closely spaced, parallel, single mode waveguides in a material with an intensity dependent index of refraction. The crosstalk level has always been the focus of many investigations because of its importance in device applications and the difficulty of achieving a good level, research of crosstalk in directional coupler and their effects have evaluated [7-11]. In particular the analysis of soliton propagation in nonlinear inhomogeneous waveguides is an important topic with big possibilities of applications. The study of the propagation of fundamental solitons in a waveguide with a periodically modulated dispersion coefficient is a current topic of interest for optical communication systems [12-13]. We will examine the propagation and the switching of fundamental solitons in a two-core nonlinear fiber coupler constructed with periodically modulated dispersion fiber (PMDF). The transmission characteristics and the critical energy of first order solitons obtained by the analytical procedure have agreed well with the results from numerical analysis.

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A. Theoretical Model

We will consider picosecond pulses propagating in the anomalous dispersion regime in an nonlinear directional coupler with twin fibers with second-order dispersion coefficient β_2. The propagating of ultrashort solitons through the PMDF coupler is described by the nonlinear Schrödinger equation with periodically modulated dispersion fiber (PMDF) with profile $p(\xi)$:

$$j\frac{\partial u_1}{\partial \xi} + \frac{1}{2}p(\xi)\frac{\partial^2 u_1}{\partial \tau^2} + |u_1|^2 u_1 + K u_2 + j\alpha u_1 = 0 \quad (1)$$

$$j\frac{\partial u_2}{\partial \xi} + \frac{1}{2}p(\xi)\frac{\partial^2 u_2}{\partial \tau^2} + |u_2|^2 u_2 + K u_1 + j\alpha u_2 = 0 \quad (2)$$

$$p(\xi) = |\beta_2|(1 + Acos(\omega\xi)) \quad (3)$$

where u_1 and u_2 are the modal field amplitudes in soliton units, $p(\xi)$ is the PMDF profile of the GVD, α is the fiber loss. ξ and τ are the normalized length and time in soliton units with $\xi = z/L_D$ and $\tau = t/T_o$. Here $L_D = T_o^2/|\beta_2|$, with pulse width $T_o(T_o = 1.13459ps)$,$|\beta_2| = 20ps2/km$. The varying GVD parameter $p(\xi)$ is a periodic function defined by equation 3 ($|\beta_2| = 20ps2/km$).

B. Numerical procedure

Taking equations 1-2 with no coupling or profile (p=1) and no loss one has the well-known soliton solutions. We have analyzed numerically the soliton transmission of first order solitons through the two core nonlinear directional fiber coupler (equations 1-2) with dispersion profiles given by equations 3 . The initial pulse at the input core is given by:

$$u_1(0,\tau) = asech(a\tau) \quad (4)$$

$$u_2(0,\tau) = 0 \quad (5)$$

This system of nonlinearly coupled NLSE's (equations 1-3) was solved numerically using the split-step method with 1024 temporal grid points taking in account the initial conditions given by equations (4-5). We can define the transmission T_i as a function of the pulse energies:

$$T_i = \frac{\int_{-\infty}^{\infty} |u_i(L_c,\tau)|^2 d\tau}{\int_{-\infty}^{\infty} |u_i(0,\tau)|^2 d\tau} \tag{6}$$

With i=1,2 and an PMDF with length of L_c The crosstalk level of the device was studies considering the input energies (Equations 4 and 5). Xtalk is defined as the ratio of light power in the unwanted output port to the power in the desired output port.

$$XTalk = \frac{\int_{-\infty}^{\infty} |u_2(L_c,\tau)|^2 d\tau}{\int_{-\infty}^{\infty} |u_1(0,\tau)|^2 d\tau} \tag{7}$$

The extinction ratio of an on-off switch is the ratio of the output power in the ´on´ state to the output power in the ´off´ state. This ratio should be as high as possible. For our PMDF it is expressed by:

$$XRatio = \frac{\int_{-\infty}^{\infty} |u_1(L_c,\tau)|^2 d\tau}{\int_{-\infty}^{\infty} |u_2(L_c,\tau)|^2 d\tau} \tag{8}$$

II. Analytical analysis

We introduce the Lagrangian density formulation following reference [14]. For a periodically modulated dispersion fiber (PMDF) with profile $p(\xi)$ we assume the pulse profiles $u1(\xi,\tau)$ and $u2(\xi,\tau)$ take the forms

$$u_1(\xi,\tau) = \frac{A}{\sqrt{p(\xi)}} sech \frac{A\tau}{p(\xi)} cos(\theta) exp(j\varphi + j\psi) \tag{9}$$

$$u_2(\xi,\tau) = \frac{A}{\sqrt{p(\xi)}} sech \frac{A\tau}{p(\xi)} sin(\theta) exp(j\varphi - j\psi) \tag{10}$$

$\theta(\xi)$ is the coupling angle that gives the power coupling between the two cores, $\psi(\xi)$ is the relative phase. For these trial functions, A is a constant of motion. However the parameter ψ have no influence on the other parameters. We expect to have a nontrivial dynamics for $\theta(\xi)$ and $\psi(\xi)$. However our main choice, for the initial solutions given by equations 9-10, is associated to the fact that the value of the total intensity of the solitons $(|u1|^2 + |u2|^2)$ has no dependence on $\theta(\xi)$. It is a constant of motion during the coupler propagation. The obtained Lagrangian for the PMDF coupler is given by:

$$L = -2Acos(2\theta)\frac{\partial\psi}{\partial\xi} - \frac{A^3}{3p}Sin^2(2\theta) + 2AKsin(2\theta)cos(2\psi) \tag{11}$$

We can obtain the Hamiltonian H for the system:

$$H = -\frac{A^3}{3p}sin^2(2\theta) + 2AKsin(2\theta)cos(2\psi) \tag{12}$$

In our case we consider that the soliton is launched in channel 1. The initial condition is then $\theta = 0$ in Equations 9-10. The Hamiltonian will stay in the entire trajectory with this value, $H(\xi) = H(0)$. To obtain the transmission characteristics for each coupler one can compare the PMDF coupler with the reference coupler and considering $p(\xi) = \overline{p}$:

$$\int_0^{\theta} \frac{d\theta}{\sqrt{1 - msin^2(2\theta)}} = -\int_0^{L_c} K d\xi \tag{13}$$

where

$$m = \left(\frac{A^2}{6K\overline{p}}\right)^2 \tag{14}$$

When the solitons are input at core 1 only, we can have the energy transmission for nonlinear PMDF coupler in terms of Jacobian Elliptic functions:

$$T_1 = \begin{cases} \frac{1}{2}[1 + cn(2K\xi|m)], & m < 1 \\ \frac{1}{2}[1 + sech(2K\xi)], & m = 1 \\ \frac{1}{2}[1 + dn(2K\sqrt{m}\xi|\frac{1}{m})], & m > 1 \end{cases} \tag{15}$$

Using equations (7) and solving equation (13) we can calculate the analytical cross talk and extinction ratio for nonlinear PMDF coupler , given:

$$XTalk = \begin{cases} \frac{1}{2}[1 - cn(2K\xi|m)], & m < 1 \\ \frac{1}{2}[1 - sech(2K\xi)], & m = 1 \\ \frac{1}{2}[1 - dn(2K\sqrt{m}\xi|\frac{1}{m})], & m > 1 \end{cases} \tag{16}$$

Using equations (8) and solving equation (13) we can calculate the analytical the extinction ratio of the device,

$$XRatio = \begin{cases} \frac{1+cn(2K\xi|m)}{1-cn(2K\xi|m)}, & m < 1 \\ \frac{1+sech(2K\xi)}{1-sech(2K\xi)}, & m = 1 \\ \frac{1+dn(2K\sqrt{m}\xi|\frac{1}{m})}{1-dn(2K\sqrt{m}\xi|\frac{1}{m})}, & m > 1 \end{cases} \tag{17}$$

III. Results and discussion

In figure 1 one has a plot of the critical energy of the coupler as a function of the amplitude of the PMDF. One can notice that for high modulation frequencies, the critical energy is almost constant. However for low modulation frequencies ($\omega < 5$) the critical energy is quite dependent on the modulation amplitude of the. For $\omega \approx 1$, one can have the critical energy changing from 1,05 to 1,9 for modulation amplitude changing from 0,1 to 1,0 respectively. This strong nonlinearity observed in the transmission of the coupler at high pump power and high modulation amplitude should lead to deformations of the switched pulse.

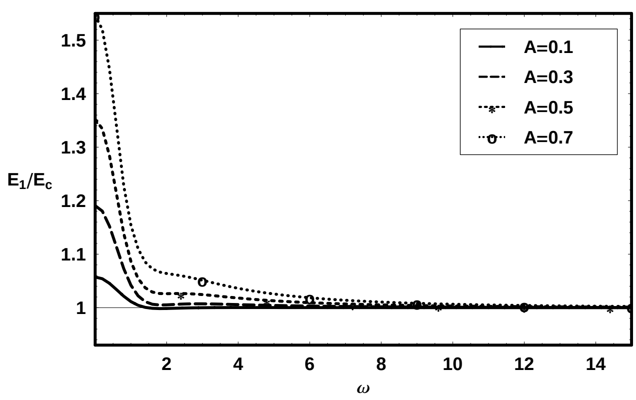

Fig. 1. Critical energy as a function of the modulation frequency and for different modulation amplitude, obtained numerically for the PMDF coupler.

In Figure 2 one has the critical energy of the same coupler, studied in figure 1, now obtained from the analytical procedure. The critical energy of all couplers as function of the modulation frequencies were achieved. The curve of the critical energy obtained from the analytical procedure is in a good agreement with the numerical results, where the critical energy decrease with the increase of the modulation frequency.

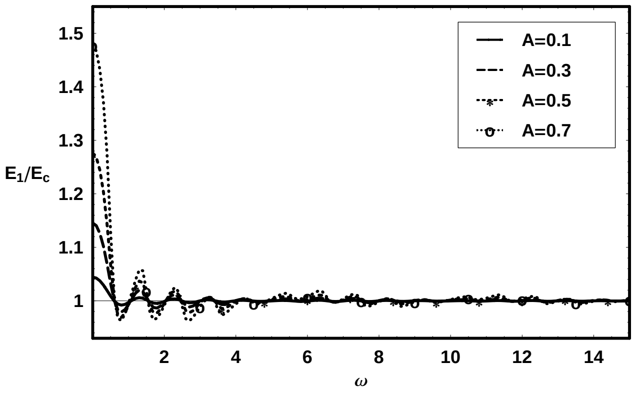

Fig. 2. Critical energy as a function of the modulation frequency and for different modulation amplitude, obtained analytical for the PMDF coupler.

In figure 3 one has the transmission for the coupler with a higher frequency of modulation $\omega = 10$. One can notice that the numerical and analytical solutions are presenting almost the same critical energy. However for high pump power the discrepancy between the two solutions are more dramatic.

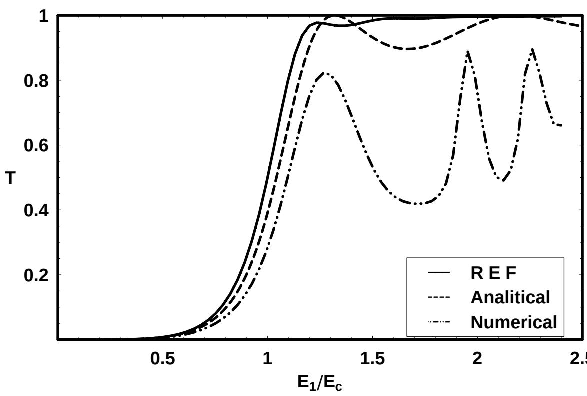

Fig. 3. Switching characteristics of channel (1) obtained from the numerically and analytically, with K=1, $L_C = \pi/2$, $\alpha = 0$, $\omega = 10$ and A=0.7. The case REF identifies the reference nonlinear coupler (p=1).

In figure 4 one has the Xtalk level for high frequency modulation ($\omega = 10$). For this frequency the first minimum is almost constant around 1,23EC. For amplitude modulation A=0.7 the Xtalk level is presenting first minimum around 1,23EC in both techniques, one can conclude that the increase modulation frequency leads to decrease of first minimum.

In figure 5 one has the Xratio at high modulation frequencies ($\omega = 10$). For pump energies around 2.3EC the Xratio decreases from 16.7dB to 5.1dB. One can conclude that the periodically modulated dispersion fiber coupler is extremely dependent on the amplitude and frequency modulation. In both techniques, the Xratio level is presenting first maximun around

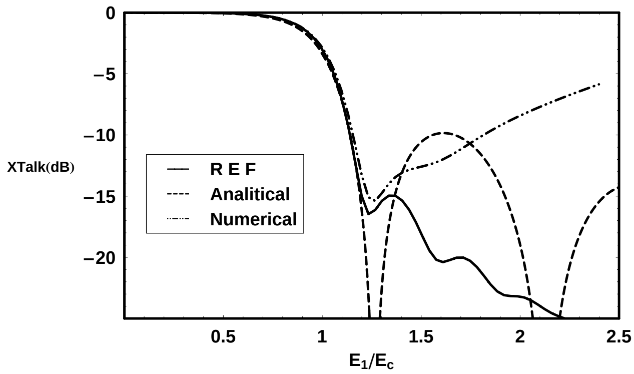

Fig. 4. Xtalk level as a function of the pump energy and modulation amplitude ($\omega = 10$) and amplitude A=0.7, obtained from the numerically and analytically. The case REF identifies the reference nonlinear coupler (p=1).

1,23EC in, one can conclude that the increase modulation frequency leads to decrease of first maximum.

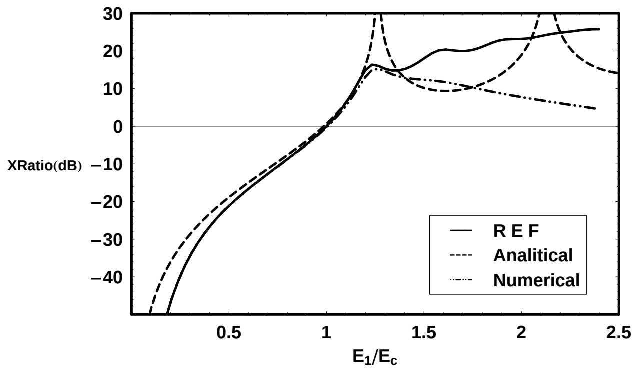

Fig. 5. Xratio level as a function of the pump energy and modulation amplitude ($\omega = 10$) and amplitude A=0.7, obtained from the numerically and analytically. The case REF identifies the reference nonlinear coupler (p=1).

The transmission characteristics of the device as well as the Xtalk and Xratio levels are extremely dependent on the parameters of the modulation and the pump energy of the device. At low and high modulation frequencies the degradation of the coupler operation are less severe at low amplitude modulations and pump energies. Another interesting extension of this study would be to consider with a random inhomogeneity along the fiber, that is under study now in our group now.

IV. CONCLUSIONS

We present an analytical and numerical investigation of the propagation and the switching of fundamental solitons in a two-core nonlinear fiber couplers constructed with periodically modulated dispersion fiber (PMDF) with a variational method using the Lagrangian density formulation. The analytical solutions were directly obtained from the coupled nonlinear Schrödinger equations. Our simulations is taking into account different amplitudes and frequency modulations of the PMDF. It was observed that for low modulation frequencies, the increase of pump power lead to an increase of the critical energy and decrease of the transmission efficiency. In summary we have shown that the performance of the nonlinear directional coupler constructed with periodically modulated dispersion profile fibers is leading the coupler to strong variations in the transmission efficiency, Xtalk and Xratio level as a function of the modulation amplitude and frequency as well as pump energy. Comparing both techniques we can say that the Lagrangian formulation provide very good description of soliton switching in the PMDF coupler.

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