Closed-form Generalized Power Correlation Coefficient of Ricean Channels

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***Resumo*— Expressões *exatas*, *fechadas*, e *gerais* dos momentos conjunto e marginal e do coeficiente de correlação das potências instantâneas de dois sinais Ricianos são deduzidas. Todas as estatísticas são expressas como somas *finitas* de funções *simples* dos parâmetros do modelo. O modelo inclui ambientes onde o fator de Rice e a potência média do sinal são diferentes de suas contrapartidas do outro sinal. Alguns gráficos ilustram o coeficiente de correlação de potência fornecido neste trabalho. Parâmetros de coerência são deduzidos, e valores práticos para projetos de sistemas são sugeridos.**

***Palavras-Chave*— Canal de desvanescimento, coeficiente de correlação, distribuição de Rice, potência do sinal.**

***Abstract*— *Exact*, *closed-form*, and *general* expressions of the marginal and joint moments as well as of the correlation coefficient of the instantaneous powers of two Ricean signals are derived. All statistics are expressed as *finite* sums of *simple* functions of the model parameters. The model includes environments where the Ricean factor and the signal mean power of one signal are different from their counterparts of the other signal. Some plots illustrate the generalized power correlation coefficient provided in this work. Coherence parameters are derived, and practical values for system design are suggested.**

***Keywords*— Correlation coefficient, fading channel, Rice distribution, signal power.**

I. INTRODUCTION

In wireless communications, the signal envelope fluctuates randomly throughout the propagation environment in a fast fading condition. This fluctuation is caused essentially by the multipath phenomenon, in which the signal reaching the receiver is composed of a large number of scattered waves. The classical distribution used to describe the envelope of the multipath signal is the Rayleigh one [1]. For some physical configurations, besides the scattered waves, the signal envelope is also influenced by a line-of-sight (or direct) wave. In these cases, the Rice distribution [2] constitutes the appropriate model [1].

Different statistics concerning the Ricean model have already been reported in the literature. In particular, [3] and [4] present the correlation coefficient of two instantaneous powers (or squared envelopes). In [4], both wide-band and narrow-band signals are analyzed, and it is observed that the narrow-band model is sufficient for computing the space correlation coefficient within the range of 20% of the carrier frequency. We note that this is the most common situation found in wireless communication scenario. In this work, we provide the space-frequency correlation coefficient of two *generalized* instantaneous powers of narrow-band signals. We consider both stationary[1] and nonstationary environments. As an intermediate step, marginal and joint moments of arbitrary positive integer orders of the instantaneous powers are derived.

As is widely known, the envelope correlation coefficient plays a crucial role in attaining the coherence distance (or time) and the coherence bandwidth of the signal envelope. These coherence parameters are used as reference, respectively, for the space (or time) separation and for the frequency separation in diversity systems. For Ricean signals, these parameters will be evaluated through the power correlation coefficient, which constitutes an accurate approximation to the envelope correlation coefficient [3].

This work is structured as follows. In Section II, the Ricean model is introduced. In Section III, the generalized power statistics of two signals are derived. In Section IV, some applications of the results provided in this work are carried out. In Section V, the main conclusions are summarized.

II. SIGNAL MODEL

Consider two narrow-band signals, S_1 and S_2, transmitted at different frequencies and detected at distinct points. The complex representation of each signal S_i is

$$Z_i = R_i \exp(I\Psi_i) = X_i + IY_i \qquad i = 1,2 \qquad (1)$$

where I is the imaginary unit, R_i is the signal envelope, Ψ_i is the signal phase, and X_i and Y_i are, respectively, the in-phase and quadrature signal components. In the Ricean model, X_i and Y_i, $i = 1, 2$, are uncorrelated variates with identical variances (σ_i^2), X_1, Y_1, X_2, and Y_2 are jointly Gaussian, and the mean of Z_i is

$$m_{Z_i} = m_i \exp(I\varphi_i) = m_{X_i} + Im_{Y_i} \qquad i = 1,2 \qquad (2)$$

The parameter σ_i^2 stem from the multipath waves of S_i, whereas m_{Z_i}, from the direct wave of S_i. Finally, we define

$$\mu_1 = \frac{Cov\{X_1, X_2\}}{\sigma_1\sigma_2} = \frac{Cov\{Y_1, Y_2\}}{\sigma_1\sigma_2} \qquad (3a)$$

$$\mu_2 = \frac{Cov\{X_1, Y_2\}}{\sigma_1\sigma_2} = -\frac{Cov\{Y_1, X_2\}}{\sigma_1\sigma_2} \qquad (3b)$$

where $Cov\{\cdot,\cdot\}$ is the covariance operator. The coefficients μ_1 and μ_2 usually depend on the distance between the reception points, on the frequency difference between the transmitted

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[1] In this work, the term *stationary environment* designates the environment where the Ricean factor and the signal mean power of a signal are equal to their counterpart of the other signal.

signals, and on the statistical behavior of the angles of arrival and the times of arrival of the scattered waves [5]–[7].

In the present model, X_i and Y_i, $i = 1, 2$, have arbitrary means, namely m_{X_i} and m_{Y_i}. However, in calculating the joint moment of the instantaneous powers, it is more appropriate to define new Gaussian random variables, namely $\hat{X}_i$ and $\hat{Y}_i$, $i = 1, 2$, such that $\hat{Y}_i$ has zero mean. To this end, we define $\hat{X}_i$ and $\hat{Y}_i$ as

$$\hat{X}_i = X_i \cos(\varphi_i) + Y_i \sin(\varphi_i) \qquad i = 1, 2 \tag{4a}$$

$$\hat{Y}_i = Y_i \cos(\varphi_i) - X_i \sin(\varphi_i) \qquad i = 1, 2 \tag{4b}$$

where φ_i is the phase of the direct wave of S_i (equation (2)).

Because $\hat{X}_i$ and $\hat{Y}_i$ are linear combinations of X_i and Y_i, also the variates $\hat{X}_1$, $\hat{Y}_1$, $\hat{X}_2$, and $\hat{Y}_2$ are jointly Gaussian. Moreover, using (4) and the statistics of the original Gaussians, the means, variances, and covariances of the new Gaussians are

$$E\{\hat{X}_i\} = m_i \qquad i = 1, 2 \tag{5a}$$

$$E\{\hat{Y}_i\} = 0 \qquad i = 1, 2 \tag{5b}$$

$$Var\{\hat{X}_i\} = Var\{\hat{Y}_i\} = \sigma_i^2 \qquad i = 1, 2 \tag{5c}$$

$$Cov\{\hat{X}_i, \hat{Y}_i\} = 0 \qquad i = 1, 2 \tag{5d}$$

$$Cov\{\hat{X}_1, \hat{X}_2\} = Cov\{\hat{Y}_1, \hat{Y}_2\} = \mu_c \sigma_1 \sigma_2 \tag{5e}$$

$$Cov\{\hat{X}_1, \hat{Y}_2\} = -Cov\{\hat{Y}_1, \hat{X}_2\} = \mu_s \sigma_1 \sigma_2 \tag{5f}$$

where $E\{\cdot\}$ is the expectation operator, $Var\{\cdot\}$ is the variance operator, and[2]

$$\mu_c = \rho \cos(\phi + \varphi_1 - \varphi_2) \tag{5g}$$

$$\mu_s = \rho \sin(\phi + \varphi_1 - \varphi_2) \tag{5h}$$

$$\rho = \sqrt{\mu_1^2 + \mu_2^2} \tag{5i}$$

$$\phi = \arg\{\mu_1 + I\mu_2\} \tag{5j}$$

Thus, the joint probability density function (JPDF) of $\hat{X}_1$, $\hat{Y}_1$, $\hat{X}_2$, and $\hat{Y}_2$ is

$$\begin{aligned} f_{\hat{X}_{1,2}\hat{Y}_{1,2}}(\hat{x}_1, \hat{y}_1, \hat{x}_2, \hat{y}_2) &= \frac{1}{4\pi^2(1-\rho^2)\sigma_1^2\sigma_2^2} \\ \cdot \exp\Bigg\{ -\frac{1}{2(1-\rho^2)} \Bigg[& \frac{(\hat{x}_1 - m_1)^2 + \hat{y}_1^2}{\sigma_1^2} + \frac{(\hat{x}_2 - m_2)^2 + \hat{y}_2^2}{\sigma_2^2} \\ & - 2\mu_c \frac{(\hat{x}_1 - m_1)(\hat{x}_2 - m_2) + \hat{y}_1\hat{y}_2}{\sigma_1\sigma_2} \\ & - 2\mu_s \frac{(\hat{x}_1 - m_1)\hat{y}_2 - \hat{y}_1(\hat{x}_2 - m_2)}{\sigma_1\sigma_2} \Bigg] \Bigg\} \end{aligned} \tag{6}$$

The present model is general and encompasses as special cases: stationary environments, for which $m_1 = m_2$ and $\sigma_1 = \sigma_2$; and the Rayleigh distribution, for which $m_i = 0$, $i = 1, 2$.

III. Generalized Power Statistics

In this section, the marginal and joint moments of arbitrary positive integer orders of the instantaneous powers are provided. With the purpose of expressing these statistics in more compact forms, we shall present them in terms of the normalized instantaneous powers $\hat{W}_i$ (or normalized squared envelopes $\hat{R}_i^2$), which are given by

$$\hat{W}_i = \hat{R}_i^2 = \frac{R_i^2}{E\{R_i^2\}} = \frac{X_i^2 + Y_i^2}{m_i^2 + 2\sigma_i^2} \qquad i = 1, 2 \tag{7}$$

A. Marginal Power Moment

The marginal moment $E\{\hat{W}_i^\nu\}$ of the Ricean model, as well-known in the literature, is given by

$$E\{\hat{W}_i^\nu\} = \frac{\exp(-k_i)\Gamma(\nu+1)\,{}_1F_1(\nu+1, 1, k_i)}{(1+k_i)^\nu} \tag{8}$$

where $\Gamma(\cdot)$ is the gamma function [8, Eq. 8.310.1], ${}_1F_1(\cdot)$ is the Kummer confluent hypergeometric function [8, Eq. 9.14.1], and $k_i = \frac{m_i^2}{2\sigma_i^2}$ is the Ricean factor.

In this work, as our interest are the cases in which ν is an integer n, we provide an alternative expression for that statistic, such that

$$E\{\hat{W}_i^n\} = \frac{1}{(1+k_i)^n} \sum_{j=0}^{n} \sum_{l=0}^{j} \left[\frac{(-1)^{j-l}(n+l)!}{(j-l)!(l!)^2} k_i^j \right] \tag{9}$$

The main advantage of (9) with respect to (8) is the absence of the hypergeometric function, which is generically expressed as an infinite sum of terms. Indeed, (9) is computationally more efficient.

B. Joint Power Moment

Using (7), the joint moment of the instantaneous powers can be expressed as

$$\begin{aligned} E\{\hat{W}_1^{n_1}\hat{W}_2^{n_2}\} =& \frac{1}{2^{n_1+n_2}(1+k_1)^{n_1}(1+k_2)^{n_2}} \\ & \sum_{j_1=0}^{n_1} \sum_{j_2=0}^{n_2} \left[\binom{n_1}{j_1} \binom{n_2}{j_2} C_R \right] \end{aligned} \tag{10a}$$

where n_1 and n_2 are positive integers, and the coefficient C_R is conveniently defined as

$$C_R = \frac{E\{\hat{X}_1^{2j_1}\hat{Y}_1^{2n_1-2j_1}\hat{X}_2^{2j_2}\hat{Y}_2^{2n_2-2j_2}\}}{\sigma_1^{2n_1}\sigma_2^{2n_2}} \tag{10b}$$

From (10b) and the definition of joint moment

$$\begin{aligned} C_R = \frac{1}{\sigma_1^{2n_1}\sigma_2^{2n_2}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \hat{x}_1^{2j_1}\hat{y}_1^{2n_1-2j_1} \\ \hat{x}_2^{2j_2}\hat{y}_2^{2n_2-2j_2} f_{\hat{X}_{1,2}\hat{Y}_{1,2}}(\hat{x}_1, \hat{y}_1, \hat{x}_2, \hat{y}_2) d\hat{x}_1 d\hat{y}_1 d\hat{x}_2 d\hat{y}_2 \end{aligned} \tag{11}$$

The JPDF $f_{\hat{X}_{1,2}\hat{Y}_{1,2}}(\hat{x}_1, \hat{y}_1, \hat{x}_2, \hat{y}_2)$ is given in (6) and can be substituted into (11). For n_1, n_2, j_1 and j_2 positive integers satisfying $j_i \le n_i$, $i = 1, 2$, the authors have solved such an integral in an exact manner, so that

$$\begin{aligned} C_R =& 2^{j_1+j_2} k_1^{j_1} k_2^{j_2} \sum_{j_3=0}^{2j_1} \sum_{j_4=0}^{2j_2} \Bigg[\binom{2j_1}{j_3} \binom{2j_2}{j_4} \\ & \left(2^{j_3+j_4} k_1^{j_3} k_2^{j_4}\right)^{-1/2} C_G \Bigg] \end{aligned} \tag{12a}$$

[2]In this work, $\arg\{\cdot\}$ denotes the argument of the complex number enclosed within.

where[3]

$$C_G = \frac{1+(-1)^{j_3+j_4}}{2} \cdot \sum_{l_1=0}^{\lfloor j_3/2 \rfloor} \sum_{l_2=0}^{j_3-2l_1} \sum_{l_3=0}^{n_1-j_1} \sum_{l_4=\lceil (j_4+l_2)/2 \rceil}^{n_1-j_1-l_3+\lfloor (j_4+l_2)/2 \rfloor} \Bigg[\binom{j_3}{2l_1} \cdot \binom{j_3-2l_1}{l_2} \binom{2n_1-2j_1}{2l_3} \binom{2n_1-2j_1-2l_3}{2l_4-j_4-l_2} \cdot [2(n_1+n_2-j_1-j_2-l_1-l_3-l_4)+j_3+j_4-1]!! \cdot (-1)^{j_4+l_2}(2l_1-1)!!(2l_3-1)!!(2l_4-1)!! \left(1-\rho^2\right)^{l_1+l_3} \cdot \mu_c^{j_4+2(n_1-j_1+l_2-l_3-l_4)} \mu_s^{j_3-j_4+2(l_4-l_1-l_2)} \Bigg] \quad (12b)$$

C. Power Correlation Coefficient

By definition, the correlation coefficient of $\hat{W}_1^{n_1}$ and $\hat{W}_2^{n_2}$ is

$$\delta_{n_1,n_2} = \frac{E\{\hat{W}_1^{n_1}\hat{W}_2^{n_2}\} - E\{\hat{W}_1^{n_1}\}E\{\hat{W}_2^{n_2}\}}{\sqrt{Var\{\hat{W}_1^{n_1}\}Var\{\hat{W}_2^{n_2}\}}} \quad (13a)$$

where

$$Var\{\hat{W}_i^{n_i}\} = E\{\hat{W}_i^{2n_i}\} - E^2\{\hat{W}_i^{n_i}\} \quad i=1,2 \quad (13b)$$

In (13), the joint moment $E\{\hat{W}_1^{n_1}\hat{W}_2^{n_2}\}$ is found through (10) and (12), whereas the marginal moments are obtained directly from (9). Since n_1 and n_2 are arbitrary positive integers, the power correlation coefficient provided here is rather general. For the particular case in which $n_1 = n_2 = 1$, (13) simplifies to

$$\delta_{1,1} = \frac{\rho^2 + 2\mu_c\sqrt{k_1k_2}}{\sqrt{(1+2k_1)(1+2k_2)}} \quad (14)$$

IV. Applications

In this section, we first provide expressions for μ_c and μ_s. Next, we investigate, in both space domain and frequency domain, the power correlation coefficient of the Ricean model for $k_1 = k_2 = k$ (stationary environments) and $n_1 = n_2 = n$. Then, we analyze the coherence parameters of the Ricean model. Finally, we propose approximations for the correlation coefficient of two power signals with non-integer orders.

A. Correlation Parameters μ_c and μ_s

For the multipath phenomenon, we shall assume the physical model described by Jakes [5], which provides

$$\mu_1 = \frac{E\{D(\Theta)\cos(\beta d\cos(\Theta) - \Delta\omega T)\}}{E\{D(\Theta)\}} \quad (15a)$$

$$\mu_2 = \frac{E\{D(\Theta)\sin(\beta d\cos(\Theta) - \Delta\omega T)\}}{E\{D(\Theta)\}} \quad (15b)$$

where $D(\cdot)$ is the horizontal directivity pattern of the receiving antenna, β is the phase constant, d is the distance between the reception points, $\Delta\omega$ is the frequency difference between the transmitted signals, and Θ and T are random variables that designate, respectively, the angles of arrival and the propagation delay times of the multipath waves. For a mobile receiver, $d = v\tau$, where v is the mobile velocity, and τ is the time.

In the Jakes' model [5], the variate $\beta d\cos(\Theta) - \Delta\omega T$ represents the phase difference between each multipath wave of S_1 and its counterpart of S_2. Similarly, we express the phase difference between the direct waves of S_1 and S_2 as

$$\varphi_2 - \varphi_1 = \beta d\cos(\theta_d) - \Delta\omega t_d \quad (16)$$

where θ_d is the angle of arrival of the direct wave, and t_d is the propagation delay time of the direct wave. Both θ_d and t_d are assumed deterministic.

From (5g)-(5j), (15), and (16)

$$\mu_c = \frac{E\{D(\Theta)\cos[\beta d(\cos(\Theta) - \cos(\theta_d)) - \Delta\omega \mathrm{T}]\}}{E\{D(\Theta)\}} \quad (17a)$$

$$\mu_s = \frac{E\{D(\Theta)\sin[\beta d(\cos(\Theta) - \cos(\theta_d)) - \Delta\omega \mathrm{T}]\}}{E\{D(\Theta)\}} \quad (17b)$$

where $\mathrm{T} = T - t_d$. Taking the instant of arrival of the direct wave as time reference, T is the time of arrival of the scattered waves. As the direct wave travels through the shortest path between the transmitter and the receiver, $T \geq t_d$, and hence $\mathrm{T} \geq 0$. The expressions in (17) can be applied to any $D(\cdot)$ and any JPDF of Θ and T.

B. Numerical Results

In this subsection, we shall investigate the space correlation coefficient $\delta_{n,n}(d)$ and the frequency correlation coefficient $\delta_{n,n}(\Delta\omega)$ for stationary environments ($k_1 = k_2 = k$). With the intention of maintaining compatibility with the results already available for the Rayleigh case [5], we shall consider

$$D(\theta) = 1 \quad (18a)$$

$$p_{\Theta,\mathrm{T}}(\theta,\mathrm{t}) = p_\Theta(\theta)p_\mathrm{T}(\mathrm{t}) \quad (18b)$$

$$p_\Theta(\theta) = \frac{1}{2\pi} \qquad 0 < \theta \leq 2\pi \quad (18c)$$

$$p_\mathrm{T}(\mathrm{t}) = \frac{1}{\overline{\mathrm{T}}}\exp\left(-\frac{\mathrm{t}}{\overline{\mathrm{T}}}\right) \qquad \mathrm{t} > 0 \quad (18d)$$

where $\overline{\mathrm{T}}$ is the time delay spread.

Replacing (18) into (17) yields

$$\mu_c = \frac{J_0(\beta d)\left[\cos[\beta d\cos(\theta_d)] - \Delta\omega\overline{\mathrm{T}}\sin[\beta d\cos(\theta_d)]\right]}{1 + (\Delta\omega\overline{\mathrm{T}})^2} \quad (19a)$$

$$\mu_s = -\frac{J_0(\beta d)\left[\Delta\omega\overline{\mathrm{T}}\cos[\beta d\cos(\theta_d)] + \sin[\beta d\cos(\theta_d)]\right]}{1 + (\Delta\omega\overline{\mathrm{T}})^2} \quad (19b)$$

Throughout the following analysis, we shall denote

- $\delta_{n,n}(d)$ ($\Delta\omega = 0$): generalized space correlation coefficient of the instantaneous powers (or squared envelopes);
- $\delta_{n,n}(\Delta\omega)$ ($d = 0$): generalized frequency correlation coefficient of the instantaneous powers (or squared envelopes).

Fig. 1 illustrates the influence of k on $\delta_{1,1}(d)$ for $\theta_d = 90°$. It can be noted that, in general, the values of the modulus

[3] In this work, $\lfloor \nu \rfloor$ is the greatest integer less than or equal to ν, and $\lceil \nu \rceil$ is the smallest integer greater than or equal to ν.

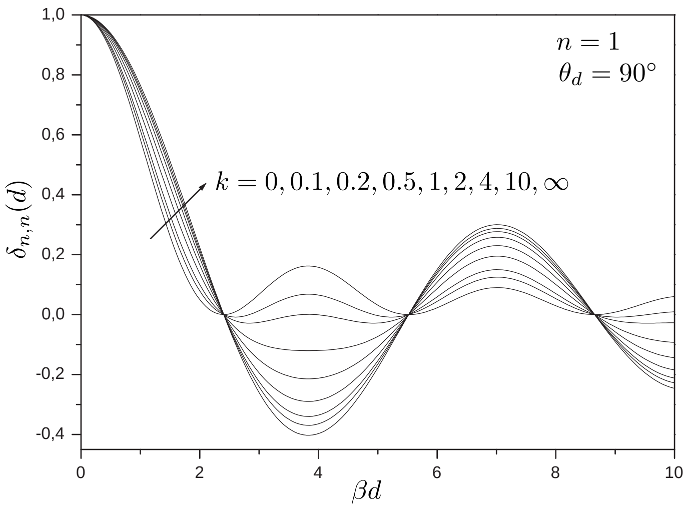

Fig. 1. Influence of k on the space correlation coefficient for $n = 1$ and $\theta_d = 90°$.

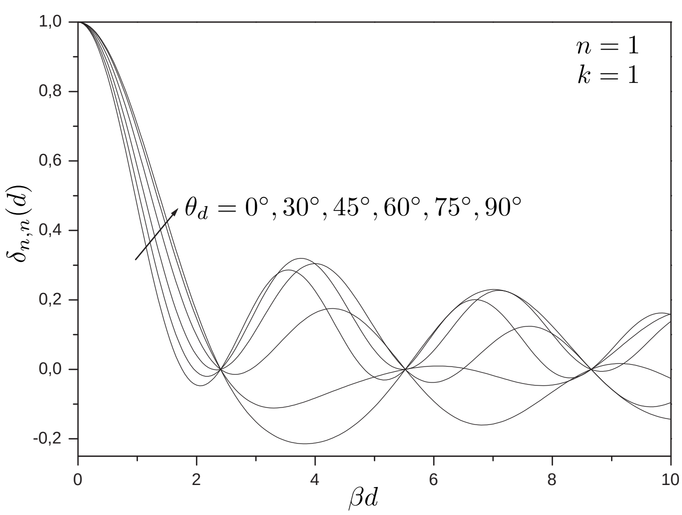

Fig. 2. Influence of θ_d on the space correlation coefficient for $n = 1$ and $k = 1$.

of $\delta_{1,1}(d)$ increases with k. This shows that the line-of-sight wave strengthens the space dependency of the two signals. Fig. 2 presents the influence of θ_d on $\delta_{1,1}(d)$ for $k = 1$. As it can be seen, θ_d affects significantly the behavior of the space correlation coefficient. In both Fig. 1 and Fig. 2, $\delta_{1,1}(d)$ assumes null values at the same points.

Concerning the frequency correlation coefficient, from $d = 0$, (5i), and (19), it follows that $\mu_c = \rho^2$. In this case, substituting this relation and $k_1 = k_2 = k$ into (14), $\delta_{1,1}(\Delta\omega) = \rho^2$. Therefore, the frequency correlation coefficient $\delta_{1,1}(\Delta\omega)$ is independent of k. Furthermore, since θ_d appears in the coefficients μ_c and μ_s only when there is a space separation ($d \neq 0$), θ_d has no effect on $\delta_{n,n}(\Delta\omega)$ $\forall n$.

For different values of n, Fig. 3 and Fig. 4 show $\delta_{n,n}(d)$ ($k = 1$ and $\theta_d = 90°$) and $\delta_{n,n}(\Delta\omega)$ ($k = 1$), respectively. Clearly, the correlation coefficients increases with decreasing the integer n.

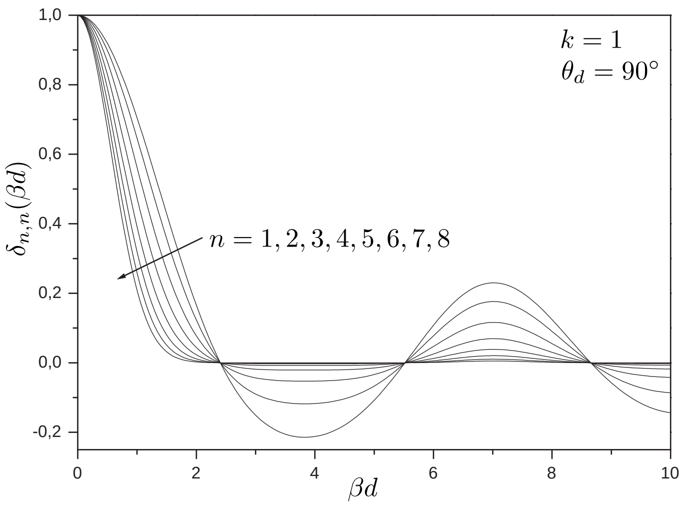

Fig. 3. Influence of n on the space correlation coefficient for $k = 1$ and $\theta_d = 90°$.

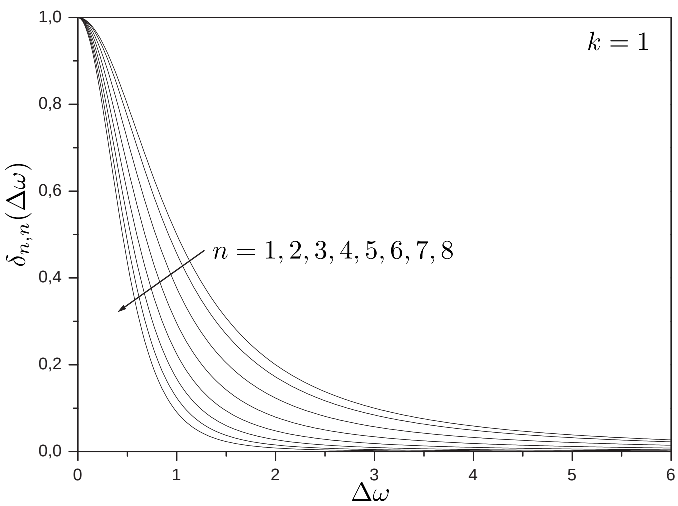

Fig. 4. Influence of n on the frequency correlation coefficient for $k = 1$.

C. Coherence Parameters

It has been shown in [3] that, for the Ricean model, the power correlation coefficient $\delta_{1,1}$ is an accurate approximation to the envelope correlation coefficient $\delta_{0.5,0.5}$, statistic from which the coherence distance (or time) and the coherence bandwidth of the signal envelope are extracted. Based on this, the coherence parameters of the Ricean model can be well-evaluated directly from $\delta_{1,1}$.

1) Coherence Distance (or Time): The coherence distance d_c (or time τ_c) is defined as the space (or time) separation above which the envelope correlation coefficient is below a certain value. For the Rayleigh model, a safe choice for the coherence distance is $d_c = 0.5\lambda$ (λ is the wavelength) [5], since $\forall d \geq 0.5\lambda$, $\delta_{0.5,0.5}(d) < 0.2$. Now, turning our attention to Fig. 1, it can be seen that, for the Ricean case with $k \geq 1$, such a property no longer holds: above $d = 0.5\lambda$ ($\beta d = \pi$), the correlation coefficient still assumes significant values. If the 0.2 threshold is used, then the safest assumption is to have

βd_c greater than several units of π, say $\beta d_c = 6\pi$. This is a very interesting result, which shows that, in order to ensure a reasonable decorrelation between two Ricean signals, the distance between their reception points must exceed 3λ (and not 0.5λ as for the Rayleigh case).

2) Coherence Bandwidth: The coherence bandwidth $\Delta\omega_c$ is defined as the frequency separation above which the envelope correlation coefficient is below a certain value. For the Rayleigh case, $\Delta\omega_c$ is chosen so that $\forall \Delta\omega \geq \Delta\omega_c$, $\delta_{0.5,0.5}(\Delta\omega) < 0.5$ [5]. Since $\delta_{1,1}(\Delta\omega) \approx \delta_{0.5,0.5}(\Delta\omega)$ is independent of k, the coherence bandwidth of the Ricean signal is identical to the coherence bandwidth of the Rayleigh signal.

D. Non-integer Orders of the Instantaneous Powers

Next, we propose an approximation to the correlation coefficient of non-integer orders of the instantaneous power (or squared envelope).

From Fig. 3 and Fig. 4, it can be seen that $\delta_{n,n}$ is close to $\delta_{n+1,n+1}$. Thus, for a non-integer ν satisfying $n < \nu < n+1$ ($n \geq 1$ integer), the correlation coefficient of W_1^ν and W_2^ν, namely $\delta_{\nu,\nu}$, can be well-approximated by the interpolation

$$\delta_{\nu,\nu} \doteq (\nu - n)(\delta_{n+1,n+1} - \delta_{n,n}) + \delta_{n,n} \qquad \nu > 1 \qquad (20)$$

V. Conclusion

In this work, we have derived *exact* and *closed-form* expressions for the marginal and joint moments and for the correlation coefficient of arbitrary positive integer orders of the instantaneous powers (or squared envelopes) of two Ricean signals. All provided statistics have been expressed as *finite* sums of *simple* functions of the model parameters.

For fading environments, the correlation parameters of the Ricean model have been expressed in terms of the distance between the receptions points and of the frequency difference between the transmitted signals. Then, the generalized power correlation coefficient, as well as the coherence parameters, has been investigated in both space domain and frequency domain. It has been observed that the coherence distance (or time) increases with k, and the coherence bandwidth is independent of k. Finally, approximations to the correlation coefficient for non-integer orders of the instantaneous powers have been proposed.

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