A New Method for Blind Identification and Equalization of Nonminimum Phase Channels

Carlos E. R. Fernandes, Gérard Favier and João C. M. Mota

***Resumo*— Este artigo apresenta um novo algoritmo de identificação cega de canais de fase não-mínima baseados em cumulantes de ordem 4, buscando um melhor compromisso entre qualidade de estimação paramétrica e complexidade computacional. Propõe-se um método que permite a escolha sistemática dos valores mais bem estimados das estatísticas de 4^a ordem do sinal de saída. Com base nesta escolha, um novo estimador paramétrico é desenvolvido a partir de uma técnica muito simples de identificação cega, o clássico algoritmo $C(q,k)$ [1]. Esta proposição é avaliada em termos do erro médio quadrático de estimação contra algoritmos bem mais complexos e robustos que utilizam todos os cumulantes de 4^a ordem possíveis. Através de simulações computacionais verifica-se um significativo ganho de desempenho para sinais complexos do tipo QAM e PSK. Por fim, considera-se uma aplicação em equalização linear cega através do uso dos coeficientes de canal estimados pelo algoritmo proposto.**

***Palavras-Chave*— Identificação, equalização, fatias de cumulantes de ordem 4, estatísticas de ordem superior.**

***Abstract*— In this paper we are interested in improving the performance of blind parametric estimation methods for identification of nonminimum phase channels based on $4th$ order cumulants, keeping a low level of computational complexity. We present a method for systematically choosing the best estimated $4th$ order statistics. A new parameter estimator is then proposed based on a very simple blind identification technique, the classical $C(q,k)$ algorithm [1]. The new method is evaluated in terms of mean squared error (MSE) of estimation and compared with more complex and robust algorithms which make use of all possible $4th$ order cumulant information. Computer simulations indicate great performance gains when using complex signals as QAM and PSK. Finally, we consider an application to linear blind equalization by using the estimated channel coefficients obtained from the proposed estimator.**

***Keywords*— Identification, equalization, $4th$ order cumulant slices, higher-order statistics.**

I. INTRODUCTION

Blind deconvolution of linear systems is of great importance for several problems in different domains, such as mobile communications, speech processing, image restoration, radar and sonar applications and seismic signal processing. The operation basically consists of retrieving unknown information from the output signal only, which means (direct) system identification and input signal estimation (inverse identification or equalization). For more than forty years now, the estimation of the power spectral density of discrete-time deterministic or stochastic Gaussian signals has been a very useful tool in accomplishing this task. However, in such an approach, accurate phase reconstruction can only be achieved if the signal is minimum phase. For nonminimum phase signals, phase relations between frequency components are suppressed by second-order (correlation) statistics (SOS). On the other hand, higher-order spectra have the ability to preserve both magnitude and nonminimum phase information. Moreover, it is well-known that for Gaussian signals only, all cumulant spectra of order greater than 2 are identically zero and, therefore, a transform to a higher-order cumulant domain will automatically eliminate additive Gaussian noise corrupting non-Gaussian signals.

For these reasons, the efforts for implementing blind system identification (BSI) have been concentrated in part on higher-order statistics (HOS)-based algorithms. This family of blind deconvolution algorithms includes *Bussgang* algorithms, which exploit the HOS of the received signal in an implicit sense, and *higher-order cumulants*-based algorithms (*polyspectra* methods, in the frequency domain), which explicitly exploit the higher-order spectra to first determine a channel transfer function estimate and then the input signal sequence. These latter ones are particularly interesting due to theoretical insensitiveness to additive Gaussian noise, phase-preserving feature when dealing with nonminimum-phase signals and ability to process non-gaussian input signals. A vast amount of literature can be found on the subject and numerous methods have been suggested as solutions for the task of identifying linear *autoregressive* (AR), *moving-average* (MA) and ARMA models, exploiting only the cumulants of input/output signals. These earlier approaches include propositions by Giannakis [1], Brillinger and Rosenblatt [2], Mendel and Giannakis [3], Tugnait [4], Stogioglou and McLaughlin [5], Swami and Mendel [6], and more recently, Abderrahim et al [7] and [8] among others. General information on explicit HOS-based methods can be found in tutorial articles, such as [9], [10].

Higher-order cumulants information can be naturally arranged in a multi-dimensional array format. Depending on the order of cumulants involved, these arrays may have big dimensions ($3rd$- or higher order *tensors*). For the sake of simplicity, explicit HOS methods generally make use of just a few of the possible higher-order cumulant information, i.e. some *slices* of cumulants of possibly different orders. Nevertheless, the use of more information about the system should lead to more precise results. Even if closed-form expressions were not established to relate all cumulant slices of all orders, this idea gave rise to a couple of blind identification

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methods which, inspired by blind source separation (BSS) techniques, make use of joint decomposition tools in order to take into account all the higher-order statistical information simultaneously [11], [12]. Another related approach is reported in [13] based on linear combinations of higher-order cumulant slices.

Generally, higher-order cumulants of output signals are not available in BSI. Hence, they need to be estimated from output data and this estimation is more precise for some cumulant than for others. Joint-decomposition based methods as well as linear-combination based ones require the estimation of all the possible higher-order cumulant information, including possibly different orders. Enormous amounts of information may be required to estimate all this statistical information satisfactorily. As a result, despite their good performances, their computational complexity and convergence speed may be a drawback. In this paper we are concerned with this problem and we look for a good trade-off between complexity, performance and robustness. Our main contribution lies on methodizing the choice for the most precisily estimated cumulants. We then propose a novel $4th$ order cumulant selection criterion for the blind identification of nonminimum phase MA models which, despite its simplicity, performs very close to more complex and robust algorithms, sometimes outperforming them.

The rest of the article is organized as follows: section II presents the parametric system model and briefly discusses the idea behind the use of higher-order cumulants in blind identification algorithms; section III describes two well-known identification algorithms that make use of all the possible $4th$ order cumulants; in section IV we introduce our proposition, based on the choice of the best estimated slice of cumulants; simulation results using complex constellation signals are presented in section V, where we evaluate the proposed estimator in terms of MSE as well as performance in channel equalization in terms of bit error rate (BER); conclusions and some perspectives are drawn in section VI.

II. System model and cumulant slices

Let us consider the baseband representation of a digital *single-input single-output* (SISO) communication system as depicted in fig. 1. The output signal $y(n)$ after sampling at the symbol rate is described by a MA process as follows:

$$y(n) = \sum_{l=0}^{q} h(l)s(n-l) + w(n), \quad h(0) = 1, \qquad (1)$$

where $h(l)$ represents the equivalent channel impulse response (i.r.) coefficients, including the pulse shaping filter, the channel itself and the receiving filters; q is the finite channel memory (order), i.e. $h(l) = 0 \, \forall \, l > q$; the input symbol $s(n)$ is taken from a complex and finite alphabet $\mathcal{A} = \{a_0, \ldots, a_{M-1}\}$ and it composes a non-measurable, centered, stationary, discrete sequence which is supposed independent and identically distributed (iid) and has non-Gaussian distribution; the noise $w(n)$ is white, complex, Gaussian and independent from $s(n)$.

Since all odd-order statistics are zero for symmetric input sequences, most of methods to be applied in digital

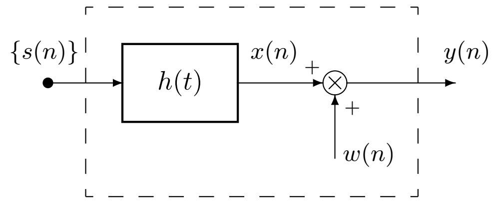

Fig. 1. Baseband representation of a digital communication system.

communication systems make use of $4th$ order cumulants, which is the simplest non-zero higher-order cumulant in this case. From the Brillinger and Rosenblatt relationship [2] between cumulants and system coefficients, we know that the $4th$ order cumulant of the output sequence $y(n)$ can be written as

$$C_{4,y}(\tau_1, \tau_2, \tau_3) = \gamma_{4,s} \sum_{l=l_1}^{l_2} h(l)h(l+\tau_1)h(l+\tau_2)h(l+\tau_3), \quad (2)$$

where $l_1 = \max(0, -\tau_1, -\tau_2, -\tau_3)$, $l_2 = \min(q, q-\tau_1, q-\tau_2, q-\tau_3)$ and $\gamma_{4,s}$ is the *kurtosis* of the *input* signal $s(n)$. Based on this relationship, several methods have been proposed for identifying linear AR, MA and ARMA models. Most of them estimate certain cumulants $C_{4,y}(\tau_1, \tau_2, \tau_3)$ from output data before performing parameter recovery. The classic $C(q, k)$ algorithm [1], for instance, uses a unidimensional (1-D) cumulant slice to compute channel coefficients as follows:

$$h(k) = \frac{C_{4,y}(q, 0, k)}{C_{4,y}(q, 0, 0)}, \quad k = 0, \ldots, q. \qquad (3)$$

Thus, the condition $h(0) = 1$ from (1) is naturally matched.

This very simple method, despite its moderate performance and weak robustness to cumulant estimation errors, has become a reference and sometimes is used to benchmark other approaches. One main drawback is the need for exact a priori knowledge about channel memory q.

III. MA parameter estimation algorithms using all cumulant statistics

A. FOSI algorithm

The *Fourth-Order System Identification* (FOSI) [11] algorithm proposes a solution based on joint diagonalization using Jacobi techniques. Firstly, a set of $4th$ order cumulant matrices is orthonormalized. Then, an unitary matrix factor $\mathbf{Q}$ is used to diagonalise the orthonormalized set $\mathcal{M}$, up to permutation and phase shifts. The orthonormalizing matrix along with the unitary factor $\mathbf{Q}$ identify a channel matrix composed from channel coefficients. This approach, strongly motivated by BSS works, finds its solution in the optimization of the following criterion:

$$\psi(\mathbf{Q}, \mathcal{M}) \stackrel{\text{def}}{=} \sum_{k=1}^{K} | \operatorname{diag}(\mathbf{Q}^H \mathbf{M}(k) \mathbf{Q}) |^2 \qquad (4)$$

where $\mathbf{M}(k)$ are the orthonormalized cumulant matrices. This algorithm makes use of $\sum_{i=1}^{q+1}(i+1)i/2$ sample $4th$ order statistics.

B. W-Slice algorithm

The W-Slice algorithm [13] proposes an approach based on the linear combination of 1-D slices of second- and/or higher-order cumulants, providing a general framework to combine all the statistics. The weights of this linear combination are the solution of an underdetermined linear system which is always well conditioned if *singular value decomposition* (SVD) techniques are used. Besides the use of cumulants of different orders, no a priori knowledge about the channel memory q is required (filter order estimation is not necessary). In this work, the use of the W-Slice algorithm will be limited to the $4th$ order cumulants in such a manner that the amount of sample $4th$ order statistics used is the same as for FOSI algorithm.

IV. The best 1-D slice algorithm

Although very sensitive to cumulant estimation errors, Giannakis' $C(q,k)$ algorithm requires very few statistics estimations and it is very robust to additive Gaussian noise. We will rewrite (3) in a vector notation as follows:

$$\mathbf{h}^{(0)} = \frac{\mathbf{c}_0}{c_0(0)}, \tag{5}$$

where $\mathbf{h}^{(0)} = [h(0)\dots h(k)\dots h(q)]^T$ is the vector containing channel coefficients and $\mathbf{c}_0 = [c_0(0)\dots c_0(k)\dots c_0(q)]^T$ is a 1-D slice of $4th$ order cumulants, defined entrywise as $c_0(k) = C_{4,y}(q,0,k)$, where the delay τ_2 was fixed to *zero*.

In practice, as aforementioned in this paper, $4th$ order cumulants of the output signal are not available and need to be estimated from output data. These estimations are more precise for some cumulants than for others, since their values may differ in some orders of magnitude. Hence, even small signal perturbations play an important role in the estimation of weak cumulant components. On the other hand, strong cumulant components estimates are quite insensitive to small signal perturbations since these perturbations can be considered (almost) negligible compared to cumulant magnitude. For this reason, we have indexed the coefficient vector in (5) according to the respective cumulant slice $\mathbf{c}_0$. The goal is to reformulate the $C(q,k)$ algorithm in order to provide different indexed coefficient vectors $\mathbf{h}^{(j)}$, each one associated to a 1-D slice of $4th$ order cumulants of the form $\mathbf{c}_j = [c_j(0)\dots c_j(k)\dots c_j(q)]^T$, $j = 0,\dots,q$, defined entrywise as $c_j(k) = C_{4,y}(q,j,k)$, thus assuming $\tau_1 = q$, $\tau_2 = j$ and $\tau_3 = k$ with $0 \le j,k \le q$.

Replacing these values in (2) we will always have $l_1 = l_2 = 0$ and, consequently, each element of the cumulant slice $\mathbf{c}_j$ will be written as an ordinary product (no summations) of channel coefficients: $c_j(k) = \gamma_{4,s}h(0)h(q)h(j)h(k)$. This explains why (5) holds even when we replace $\mathbf{c}_0$ by any $\mathbf{c}_j$. So, we can generalize (5) rewriting it as follows:

$$\mathbf{h}^{(j)} = \frac{\mathbf{c}_j}{c_j(0)}, \quad j = 0,\dots,q. \tag{6}$$

The above equation allows us to computate the squared error of parametric estimation, defined as

$$\epsilon_j = \|\mathbf{h} - \hat{\mathbf{h}}^{(j)}\|^2, \tag{7}$$

and leads us to a very important question: which cumulant slice among all $\mathbf{c}_j$, $j = 0,\dots,q$, gives the smaller parametric estimation error? Following our previous reasoning, an intuitive answer to this question lies on the fact that strong cumulant components are more robust to signal perturbations since these perturbations should be considerably smaller than cumulants magnitude. As a result, one may suppose that "strongest" cumulants are the most precisely estimated and consequently they lead to smaller parametric estimation errors. Indeed, this idea can be verified by means of the following claim along with theorem 1 bellow:

Claim 1: Considering the channel estimates $\hat{\mathbf{h}}^{(j)}$, computed as in (6) using estimated cumulants $\hat{\mathbf{c}}_j$, the lowest parametric estimation error is given by (7) with $j = j_0$, where j_0 gives $|h(j_0)| \ge |h(j)|$, $\forall\ 0 \le j \le q$.

In order to justify this claim, we consider that estimated cumulant slices $\hat{\mathbf{c}}_j$ can be written as:

$$\hat{\mathbf{c}}_j = \begin{bmatrix} \hat{C}_{4,y}(q,j,0) \\ \hat{C}_{4,y}(q,j,1) \\ \vdots \\ \hat{C}_{4,y}(q,j,q) \end{bmatrix} = \begin{bmatrix} \gamma_{4,s}h(0)h(q)h(j)h(0) + e_{0,j} \\ \gamma_{4,s}h(0)h(q)h(j)h(1) + e_{1,j} \\ \vdots \\ \gamma_{4,s}h(0)h(q)h(j)h(q) + e_{q,j} \end{bmatrix}, \tag{8}$$

where $e_{k,j}$ are the cumulant estimation errors. From (8) and (6) we have

$$\hat{\mathbf{h}}^{(j)} = \begin{bmatrix} 1 \\ \frac{\gamma_{4,s}h(0)h(q)h(j)h(1)+e_{1,j}}{\gamma_{4,s}h(0)h(q)h(j)h(0)+e_{0,j}} \\ \vdots \\ \frac{\gamma_{4,s}h(0)h(q)h(j)h(q)+e_{q,j}}{\gamma_{4,s}h(0)h(q)h(j)h(0)+e_{0,j}} \end{bmatrix}, \tag{9}$$

The above equation leads to the following expression for the estimated channel coefficients:

$$\hat{h}^{(j)}(k) = \frac{\kappa h(k)}{\kappa + e_{0,j}/h(j)} + \frac{e_{k,j}/h(j)}{\kappa + e_{0,j}/h(j)}, \tag{10}$$

where $1 \le k \le q$, $h(0) = 1$ as in (1) and we assumed $\kappa = \gamma_{4,s}h(q)$. From (10), it is evident that, the larger the magnitude of $h(j)$, the closer $\hat{h}^{(j)}(k)$ will be to $h(k)$.

Theorem 1: Consider the integer j_0 associated with the largest channel coefficient, i.e. $|h(j_0)| \ge |h(j)|$, $\forall\ 0 \le j \le q$. The 1-D slice denoted by $\mathbf{c}_{j_0}$ is the one with maximum norm amongst all $\mathbf{c}_j$, $0 \le j \le q$

Proof:

$$\begin{aligned}\left(\mathbf{c}_j^T\mathbf{c}_j\right)^{1/2} &= \gamma_{4,s}\left(\sum_{i=0}^{q}|h(0)h(q)h(j)h(i)|^2\right)^{1/2} \\ &= \gamma_{4,s}|h(j)h(q)|\left(\sum_{i=0}^{q}|h(i)|^2\right)^{1/2},\end{aligned}$$

where we assumed $h(0) = 1$ as in (1). Note that the left-hand side of the above equation is constant for a fixed

j. Furthermore, we observe that $\|\mathbf{c}_j\|$ is proportional to $|h(j)|$ and it is straightforward to conclude that

$$\arg\max(|h(j)|) = \arg\max(\|\mathbf{c}_j\|).$$

■

From Theorem 1, we conclude that the cumulant slice $\mathbf{c}_j$ with maximum norm indicates the channel coefficient $h(j)$ that has maximum magnitude (actually its index j_0). Furthermore, Claim 1 shows that $\mathbf{c}_{j_0}$ is the one that provides the lowest parametric estimation error. Choosing this slice and using it in (6) for identifying channel coefficients constitutes our proposal, the so-called *Best 1-D Slice* algorithm, which can be briefly resumed by the following steps:

1) Using the output data sequence $y(n)$, compute estimates of $\mathbf{c}_j = [c_j(0) \ldots c_j(k) \ldots c_j(q)]^T$, for $j = 0, \ldots, q$, where $c_j(k) = \hat{C}_{4,y}(q, j, k)$.
2) Determine the number j_0 as the argument that maximizes the norm of $\mathbf{c}_j$, i.e.

$$j_0 = \arg\max\{(\mathbf{c}_j^T \mathbf{c}_j)^{1/2}\}, \quad j = 0, \ldots, q. \qquad (11)$$

3) Compute channel coefficients using $\tau_2 = j_0$ in (3). The result is as follows:

$$h(k) = \frac{\mathbf{c}_{j_0}(k)}{c_{j_0}(0)} = \frac{\hat{C}_{4,y}(q, j_0, k)}{\hat{C}_{4,y}(q, j_0, 0)}, \quad k = 1, \ldots, q. \qquad (12)$$

With this new method we need to compute just $(q+1)(q+2)/2$ sample $4th$ order statistics, although only $(q+1)$ are really employed in the estimation of channel parameters. Comparing these numbers to W-Slice and FOSI algorithms we observe a reduction of $\sum_{i=1}^{q}(i+1)i/2$ in the number of sample $4th$ order statistics estimations, thus providing a reduction rate of $(q/3)+1$ (this means, for instance, that for a channel of memory $q = 3$, W-Slice and FOSI algorithms require twice the amount of cumulant estimations required by Best 1-D Slice). Moreover, implementation of Best 1-D Slice requires only $q+1$ operations while for W-Slice and FOSI algorithms the computational cost is roughly of the order of $O[(2q+1)^5]$ for the former and $O[(2q+1)^4]$ for the latter. Further investigations are needed on the influence of estimating channel memory q and how its precision affects performance results, though we expect *Best 1-D Slice* algorithm to be as sensitive to estimation errors on this parameter as $C(q,k)$ algorithm, as long as both make the same use of it.

V. Simulation Results

In this section we present computer simulations results aiming to compare FOSI, W-Slice and Best 1-D Slice algorithms both in terms of estimation and equalization performance. We start by BSI applications and then we show some results in blind equalization.

A. Blind system identification

The performance of blind identification methods is evaluated by means of the normalized mean squared error (NMSE) of the estimator, which is given by:

$$\texttt{NMSE} = \frac{1}{R}\sum_{r=1}^{R}\frac{\|\hat{\mathbf{h}}^{(r)} - \mathbf{h}\|^2}{\|\mathbf{h}\|^2}, \qquad (13)$$

where R is the number of Monte Carlo runs. The results were obtained using $R = 100$. In all cases the input sequence was centered, with variance $\sigma_s = 1$. Fourth order cumulants were estimated from an output data sequence of length $N = 10000$ for $2nd$ order models and $N = 40000$ for $4th$ order models. Assuming perfect knoledge about channel memory q, data blocks of length N are first used to estimate second- and fourth-order moments of the output sequence at different lags on the range $[-q, q]$. Afterwards, these moments are used to compute fourth-order cumulant estimates associated to the same lags. For each Monte Carlo run, a new data sequence is acquired, cumulant estimates are obtained and then the channel coefficients are computed.

TABLE I

Nonminimum phase channels configuration.

Channel	Order	Coefficients
a	q=2	$\mathbf{h} = [1 \quad -2,3333 \quad 0,6667]^T$
b	q=4	$\mathbf{h} = [1 \quad 2.4 \quad -0,3 \quad -1,45 \quad 1,15]^T$

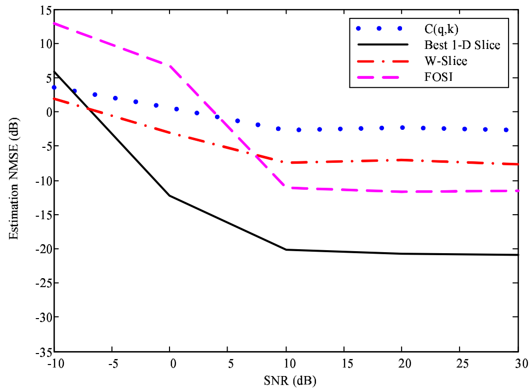

Fig. 2. NMSE versus SNR for a $2nd$ order model (channel a) with a 4-QAM signal.

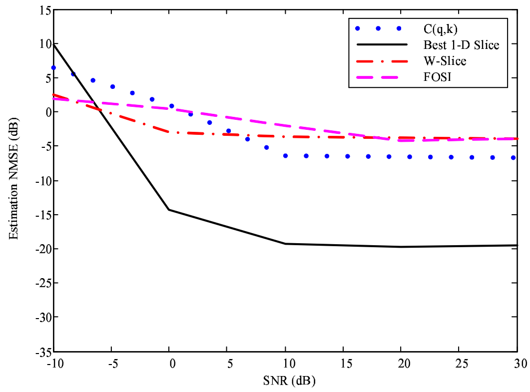

Fig. 3. NMSE versus SNR for a $4th$ order model (channel b) with a 4-QAM signal.

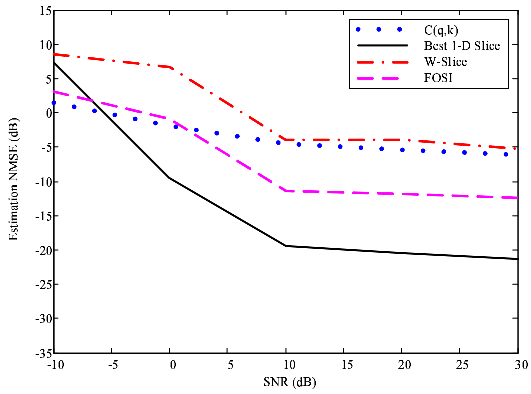

Fig. 4. NMSE versus SNR for a $2nd$ order model (channel a) with an 8-PSK signal.

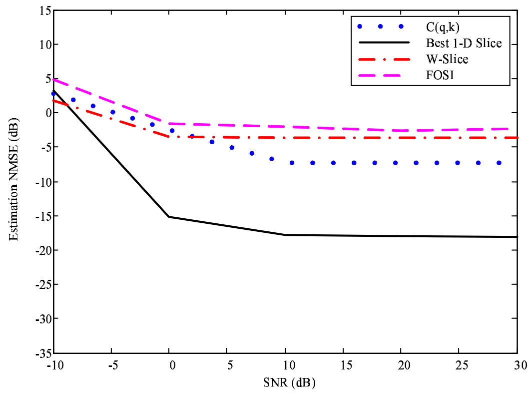

Fig. 5. NMSE versus SNR for a $4th$ order model (channel b) with an 8-PSK signal.

The following two types of input signals were considered:

a) *4-QAM input signal*: The input sequence $s(n)$ was extracted from a 4-QAM constellation, with equiprobable symbols. Nonminimum phase channels are used, described by second ($q = 2$) and fourth ($q = 4$) order MA models, as indicated in the table I. Besides FOSI, W-Slice and Best 1-D Slice algorithms, the curves include the conventional $C(q, k)$ algorithm ($j_0 = 0$). Figure 2 shows the NMSE in dB obtained in estimating channel a for signal-to-noise ratio (SNR) ranging from -10 to 30 dB. The results shown in fig. 3 are for channel b. For both models, we observe that $C(q, k)$ algorithm is considerably boosted by the use of the proposed criterion for the selection of the best 1-D cumulant slice. Moreover, the Best 1-D Slice algorithm outperforms FOSI and W-Slice algorithms, despite the huge difference in the amount of statistical information used.

b) *8-PSK input signal*: For the next results, the input sequence $s(n)$ was extracted from a 8-PSK constellation, with equiprobable symbols. Channels are described as in table I. Figure 4 shows the NMSE in dB versus SNR obtained in estimating channel a from a 8-PSK signal. The plots concerning channel b are depicted in fig. 5. Here again, Best 1-D Slice algorithm provides the best estimation results. On the other hand, from figures 2 to 5 we observe that W-Slice and FOSI algorithms have problems in estimating longer channels. This limitation comes from the fact that they take into account all $4th$ order cumulants, including the least precise ones.

B. Blind channel equalization

Once the channel coefficients are satisfactorily estimated, several equalization methods can be used to recover the input data sequence. The classical Wiener solution is the optimal solution in minimizing the mean squared error and it seems to be the most suitable technique. A linear equalizer $g(n)$ is disposed at channel's output followed by a decision device. Then, after decision, the estimated input sequence $\{\hat{s}(n)\}$ is recovered. The equalizer's weight vector $\mathbf{g}$ of length m is given by:

$$\mathbf{g} = \left(\mathbf{H}^T\mathbf{H} + \hat{\sigma}_w^2\mathbf{I}\right)^{-1}\mathbf{H}^T\mathbf{s}_d, \tag{14}$$

where $\mathbf{H}$ is the $(m+1)\times(m+q+1)$ channel matrix built from the channel coefficients as follows:

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}^T & 0 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & 0 & \cdots & \mathbf{h}^T \end{bmatrix}, \tag{15}$$

and $\mathbf{s}_d = [0 \ldots 0\,1\,0 \ldots 0]^T$, where d indicates the position of the non-zero entry and it is chosen as the communication delay.

Next, in order to illustrate the application of the proposed method in blind channel equalization we will consider channel b (from table I) with both 8-PSK and 16-PSK constellations. The equalizer weight vector is obtained from (14), where d was chosen to be half of the length of the global impulse response. We start by showing, in fig. 6, the constellations of the equalized and the unequalized output signals for channel b with 8-PSK using Best 1-D Slice algorithm for SNR=20dB. Then, figures 7 and 9 show the bit error rate (BER) in dB versus SNR for channel b with 8-PSK and 16-PSK, respectively. These results highlight the superiority of Best 1-D Slice algorithm in equalizing a $4th$ order FIR channel.

Finally, as a last example, we varied the size N of the input data sequence (number of data samples) for a SNR fixed at 20dB. Figure 8 illustrates the BER against the sample size for channels a and b with 8-PSK. The curves evidence significant performance gains when increasing the number of

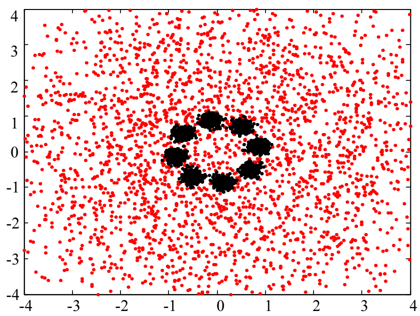

Fig. 6. Equalized and unequalized output constellations for channel b with 8-PSK using Best 1-D Slice algorithm (SNR=20dB).

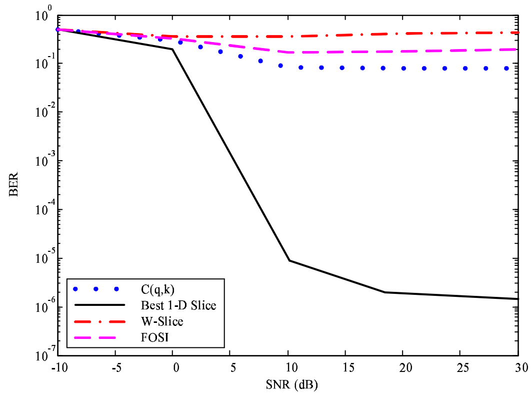

Fig. 7. BER versus SNR for channel b with 8-PSK ($4th$ order model).

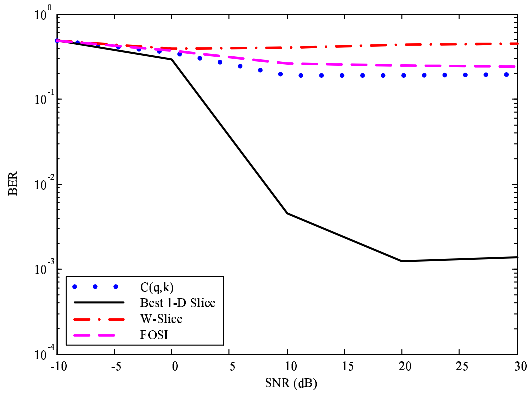

Fig. 9. BER versus SNR for channel b with 16-PSK ($4th$ order model).

output measurements used for estimating $4th$ order cumulants. In addition, note that Best 1-D Slice satisfactorily equalizes (BER $< 10^{-3}$) channel a with input data records as short as 10000 samples and channel b with about 15000 samples.

VI. Conclusions and Perspectives

This paper proposes a criterion for the choice of the best 1-D $4th$ order cumulant slice in order to improve parametric estimation quality for the identification of MA models. We were concerned by the trade-off between estimation errors (performance) and computational complexity. The proposed method, though based on an well-known concept, is original and very simple to implement. Furthermore, it provides very advantageous results in identification and equalization of nonminimum phase channels using complex signals, even when compared to more complex and robust algorithms such as FOSI and W-Slice algorithms, which combine all the possible cumulant information together. One reason for that is the fact that the suggested algorithm discard low precision statistics while FOSI and W-Slice algorithms keep them all, thus limiting their capacities and causing an overhead in cumulant estimation computations. Nevertheless, it is important to note that this work does not concern the development of estimation procedures for HOS and, at a more fundamental level, further investigation is needed on this point. Finally, an extension of the proposed method to the case of complex impulse response channels is under study and higher-level modulations should be considered in the near future in order to observe the robustness of the method.

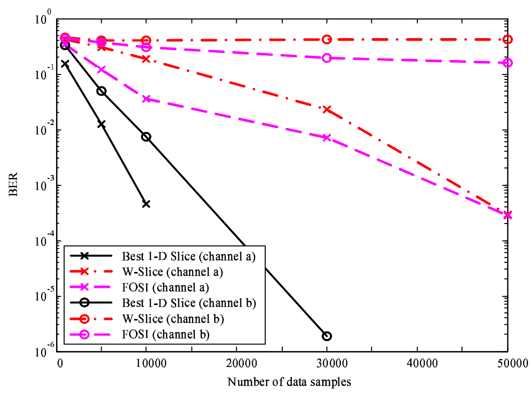

Fig. 8. BER versus sample size for channels a and b with 8-PSK (SNR=20dB).

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