Orthogonal Metrics for GRWA Optical Networking

Leandro C. Resendo, Luiz de C. Calmon and Moises R. N. Ribeiro

***Resumo*—Neste artigo foi usado uma formulação de Programação Linear Inteira (ILP) para o Problema de *traffic grooming*, roteamento e alocação de comprimento de onda (GRWA). São apresentadas métricas para avaliar centenas de configurações de redes com 5, 6 e 7 nós, otimizadas para o mínimo número de transceptores. Um estudo estatístico das configurações ótimas, via matriz de correlação, é proposto para avaliar o relacionamento entre as métricas estudadas. Resultados mostram a formação de grupos ortogonais; onde algumas métricas destacam-se como dominantes em seu grupo. O uso de tais métricas são sugeridas para a avaliação de projetos que integram engenharia de tráfego em redes eletrônicas e ópticas, como cenários IP sobre WDM.**

***Palavras-Chave*—Métricas, ILP, GRWA.**

***Abstract*—This paper uses an Integer Liner Programming (ILP) formulation for traffic grooming, routing and wavelength assignment (GRWA) for finding minimal number of transceivers in hundreds of network instances with 5, 6, and 7 nodes. Different metrics are used to evaluate these random scenarios. A statistical study, based on cross correlation matrix, is presented to find out relationships between the diverse metrics. Results highlight few orthogonal groups of metrics, that are well represented by their dominant components. These representative metrics are paramount to joint traffic engineering in scenarios such as IP-over-WDM networks.**

***Keywords*—Metrics, ILP, GRWA.**

I. INTRODUCTION

Traffic grooming tackles the problem of how low bandwidth traffic is assigned to wavelengths in such way that minimizes an objective function, such as number of transceivers. In addition to cost reduction in optical transceivers, IP routers and Add and Drop Multiplexers (ADMs) will also benefit from integrated optimal solutions to Grooming, Routing, and Wavelength Assignment (GRWA) due to the use of fewer ports in such equipment. GRWA approach is needed in joint electronic and optical traffic engineering, given that it optimally determines routes and wavelenghts to be utilized by diverse traffic segments.

GRWA has been studied through heavy ILP formulations ([2],[1]) and use of heuristical approaches ([5]-[9]). However, most ILP models and heuristics proposed until now, as seen in [11] and [2], analyze their outcomes based on a set with few metrics, such as network utilization and congestion, disregarding correlations that might exist between themselves as well as relationships with the objective function. Therefore, it is not clear what kind of metric should be used to evaluate other aspects of network configurations that meet the objective function.

This work proposes the use of statistical tools to GRWA performance metrics. We use a ILP formulation to GRWA [11] which receives a network topology and a traffic matrix supplying the network configuration (traffic partition, their routing, and wavelength assignment) that uses the minimal number of trasceivers. This ILP formulation has been chosen for it solves an network instance with 7 nodes in just few minutes, allowing statistical analysis to be performed over hundreds of random scenarios. Metrics, besides the number of transceivers, such as *Use of resources*, *Electronic Processing*, *Congestion*, *Criticality*, among others, are then taken from each network configuration. Therefore, the outcomes are random variables whose relationships can be unveiled through correlation analysis. The remainder of this paper is as follows. Next Section presents ILP formulation and constraints for the GRWA problem. In Section III the metrics under consideration are defined and characterized. In Section IV correlations are obtained between the metrics as well as orthogonal groupings. Finally, some conclusions are drawn in Section V.

II. PROBLEM FORMULATION

The ILP model shown in this paper is one of the ILP presented in [11] and is here reproduced for the sake of completeness. Let an irregular mesh network with two links (fibers) between each node pair i and j (represented by ij and ji) and assume that all links in the network support the same maximum number $Wmax$ of wavelengths. A unified approach to grooming, routing, and wavelength assignment is aimed at meeting the traffic demands with the minimal use of wavelengths. Once a given a physical topology is defined, traffic demands should be accommodated so that the number of transceivers (i.e. devices in charge of electro-optical conversion at nodes) are kept to a minimum. Presently, transceivers are responsible for much of the network cost, but other metrics are worth considering to evaluate other aspects (as distribution and loading of traffic, and use of resources). In the likeness that bandwidth requests may use only a fraction of the wavelength capacity, properly combining low capacity demands at intermediate nodes enables the sought reduction of transceivers. In order to add flexibility in the grooming process, a request from source to destination can be divided into several lower bandwidth segments (limited to the system granularity) and routed separately. No wavelength continuity constraint is imposed and, as a consequence, bandwidth segments can use different wavelengths along its route.

A. ILP Model to GRWA

The traffic grooming problem in a mesh network under static traffic used here, is presented in a simple formulation with a reduced computational cost. Figure 1 shows a node i that adds

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its traffic demand (s), drops traffic headed to itself (d) and also performs grooming for transit traffic.

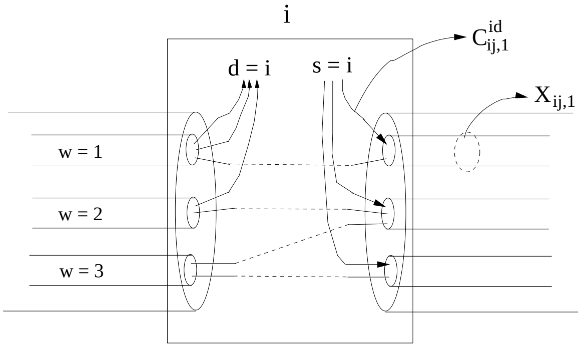

Fig. 1. An illustration for traffic grooming

The following notation is used in our mathematical model.

i and j: are network nodes, bearing link ij .
s and d: are source and destination nodes respectively.
w: is a wavelength in a link ij.

Given:

E[i][j]: the adjacency matrix.
N: number of nodes in the network.
$Wmax$: maximum number of wavelengths per link.
traf[s][d]: traffic matrix.
CW: Capacity of a wavelength.

Variables:

$XB_{ij,w}$: is a binary 1 if the wavelength w is used in link ij, and zero otherwise.
$X_{ij,w}$: is the amount of the traffic in link ij on wavelength w.
$CB^{sd}_{ij,w}$: is a binary 1 if the wavelength w is used in link ij from source s to destination d, and zero otherwise.
$C^{sd}_{ij,w}$: is the amount of traffic (on wavelength w which is used in link ij) due to source s to destination d.
$D_{sd,w}$: is a fraction of the traffic (traf[s][d]) on a wavelength w.
W_{ij}: the number of wavelengths in each link

Objective Function: Minimize the total number of transceivers used by reducing the number of wavelengths interconnecting i to j.

$$min : \sum_{ij} W_{ij} \tag{1}$$

Constraints:

$$XB_{ij,w} \leq X_{ij,w} \tag{2}$$

$$CW \times XB_{ij,w} \geq X_{ij,w} \tag{3}$$

$$CB^{sd}_{ij,w} \leq C^{sd}_{ij,w} \tag{4}$$

$$CW \times CB^{sd}_{ij,w} \geq C^{sd}_{ij,w} \tag{5}$$

$$\sum_{w} XB_{ij,w} \leq Wmax \tag{6}$$

$$\sum_{sd} C^{sd}_{ij,w} \leq X_{ij,w} \tag{7}$$

$$X_{ij,w} \leq CW \tag{8}$$

$$\sum_{i} C^{sd}_{ij,w} - \sum_{i} C^{sd}_{ji,w} = \begin{cases} D_{sd,w} & \text{when } j = s \\ -D_{sd,w} & \text{when } j = d \\ zero & \text{otherwise} \end{cases} \tag{9}$$

$$\sum_{w} XB_{ij,w} = W_{ij} \tag{10}$$

$$\sum_{w} D_{sd,w} \leq \text{traf}[s][d] \tag{11}$$

Description:

Equations 2 and 3 are simply used to establish the relation between variables $X_{ij,w}$ and $XB_{ij,w}$.
Equations 4 and 5 are analogous to 2 and 3 relative to variables $C^{sd}_{ij,w}$ and $CB^{sd}_{ij,w}$
Equation 6 is the constraint that limits the number of wavelengths to the maximum permitted.
Equation 7 is a traffic grooming constraint, and it shows that the sum of every demand passing through a link ij and wavelength w can not be greater than $X_{ij,w}$.
Equation 8 restrains the quantity of traffic on a wavelength to its full capacity.
Equation 9 is flow conservation constraint that ensures traffic is only added in a source node and dropped at a destination node.
Equation 10 ensures that the number of wavelengths in a link ij does not exceed the maximal number of the wavelengths.
Equation 11 ensures that traffic demand sd over all wavelengths is bounded to the demand matrix (traf[s][d]).

III. THE IMPACT OF GRWA IN OPTICAL NETWORKING

We propose the use a set of post processing metrics to gain insights into the accommodation of traffic demands over a physical topology with the least number of transceivers. The metrics used to assess how this optimal solution influences other revelant aspects are as follows:

- **Optoelectronic Interface Use**, $NW = \sum_{ij} W_{ij}$, gathers the number of transceivers used to connect electronic nodes to the optical network. Although it has been extensively used as a way to evaluate GRWA solutions, it is not yet clear how this metric, when used as objective function, impacts the rest of the network.
- **Optical Network Use**, U, is the ratio between the quantity of resources used in a network and its total capacity,

$$U = \frac{\sum_{ij,w} X_{ij,w}}{CW \times \sum_{ij} W_{ij}} \times 100\%. \tag{12}$$

  This is a popular metric to evaluate network after an optimization process [11], [4], [2].
- **Link Use**, U_{ij}, is defined as the quantity of traffic through link ij,

$$U^{ij} = \sum_{w} X_{ij,w}. \tag{13}$$

  The most loaded link in the network can be then defined as, $U^{ij}_{max} = max\{U^{ij}\}$, for all links ij. This is often used as network *congestion* objective function [10]. It is expected that virtual topologies with low *congestion* produce better distribution of traffic. The link that accommodates the least amount of traffic is here also defined, $U^{ij}_{min} = min\{U_{ij}\}$. This metric might be useful to identifying spare capacity left in the links.

- **Node Processing** in a node i, P, is defined as the total transit traffic that node i has to handle.

$$P = max\left\{\sum_{sd}\sum_{j}\sum_{w} C^{sd}_{ij,w}\right\} \qquad (14)$$

with $i \neq d$ and $i \neq s$. The higher is P, the more demands are placed on node electronic systems. P also indicates the opacity of the network [12].

- **Node Criticality** is defined as the quantity of traffic lost when a node i fails,

$$C_i = \sum_{j}\sum_{w} X_{ij,w}. \qquad (15)$$

We define $C = max\{C_i\}$ the *Criticality* is related to network protection issues, it is a metric to assess the vulnerability of the grooming solution. The **Variance of Criticality** (VC) is also defined. It can be seen as the dispersion of criticality among nodes (i.e., a second-order metric).

A. Statistical Analysis of Metrics

This paper proposes, for the first time, the use of statistical analysis to look into the solutions found through ILP optimization. This is only possible due to the light ILP formulation proposed, which enables solutions to be found within minutes.

Randomly generated network and traffic instances are used. The seven metrics defined above, namely, NW, U, U^{ij}_{max}, U^{ij}_{min}, P, C, and VC, make up the outcome for each optimization process for a given network/traffic scenario. Cross-correlation coefficient matrix is then taken to find out the relationship between random variables obtained when computing the metrics for stochastic network instances. This allows the first insights into the ILP engine. High correlation should be expected for some pairs of metrics down to the following reasons i) the single-commodity objective function, i.e., the least number of transceiver, lead externalities to be absorbed by other componets of the network, e.g., *Node Processing*; ii) there are different metrics to measure similar things, e.g., *Criticality* and its variance.

Further investigation aimed at obtaining the smallest set of significant metrics is performed through eigenvalue and eigenvector of the cross-correlation coefficient matrix. They reveal, respectively, the dominant components and the orthogonal base to represent them, [13].

IV. RESULTS

The analysis takes 100 random scenarios for each case investigated. Wavelength full capacity is set at 48 units of traffic representing, for instance, OC-48 with OC-1 granularity. The random traffic matrix has demands between all nodes with either 3, 9 or 36 units of traffic with 40%, 40%, and 20% chance, respectively. Note that traffic demands never take wavelength full capacity so that there is room for demands to be groomed. We use CPLEX Linear Optimizer 9.0 [8] to solve the ILP formulation.

Figure 2 shows cross-correlation results for metrics taken for random topologies generated with 5 nodes and 6 edges, while Figure 3, is for 6 nodes and 7 edges and Figure 4 are for networks with 7 nodes and 9 edges. Topologies with 5, 6, and 7 nodes are chosen to observe the evolution of the cross-correlation as networks grow. In addition to the limted number of nodes, due to computational time constraint (the GRWA is a NP-hard problem), only network scenarios (traffic matrices and topologies) that require at most 2 wavelengths per fiber are selected.

The abscissa represents the pair of metrics as follows:

1: $NW \times U^{ij}_{max}$
2: $NW \times U^{ij}_{min}$
3: $NW \times P$
4: $NW \times C$
5: $NW \times VC$
6: $NW \times U$
7: $U^{ij}_{max} \times U^{ij}_{min}$
8: $U^{ij}_{max} \times P$
9: $U^{ij}_{max} \times C$
10: $U^{ij}_{max} \times VC$
11: $U^{ij}_{max} \times U$
12: $U^{ij}_{min} \times P$
13: $U^{ij}_{min} \times C$
14: $U^{ij}_{min} \times VC$
15: $U^{ij}_{min} \times U$
16: $P \times C$
17: $P \times VC$
18: $P \times U$
19: $C \times VC$
20: $C \times U$
21: $VC \times U$

The ordinate represents 2 different entities. On the left, it has the correlation coefficients (represented by a black filled triangle), and its lower and upper bounds for a 95% confidence interval indicated with a vertical segment. On the right hand side, it has the $p - value$, where if p is lower than 0.05 the correlation is significant (p-value is the smallest significance level at which the null hypothesis would be rejected for the given sample).

Note that in Figures 2, 3, 4, NW, $x = 1,..,6$, shows reasonable correlation with most of the metrics, i.e., $x = 1, 3, 4,$ and 5, but low correlation with the metrics related to the the least loaded link U^{ij}_{min} and network use U, i.e., $x = 2 and 6$ respectively. Notice in the former group p-values are below 0.05 (dotted horizontal line) while for the latter they are above this limit. This means that the high correlation found in the first group is trustful while the second group shows not only small but also unreliable values that could have happened by pure chance.

An important outcome is the slight change in pattern observed for the different network scenarios. This might be a good indication of strength of the approach proposed as a means of undertanding ILP inner works. Notice, however, a gradual reduction of correlation coefficient, for $x = 1, 3, 4,$ and 5, as network size is increased from 5 to 7 nodes as seen in Figures 2, and 4. The multiplicity of alternative paths in larger networks is the possible cause for this reduction in the relationship of NW with other metrics.

As it might be expected, P could be well represented by C or VC since they all show high correlation in Figures 2 to 4, while U fails to relate to any other metric as $p - values$ rise above 0.05 or reliable coefficients are low. U^{ij}_{min} also proved to be unrelated to others. The above comments suggest the existence of groupings and orthogonalities between metrics.

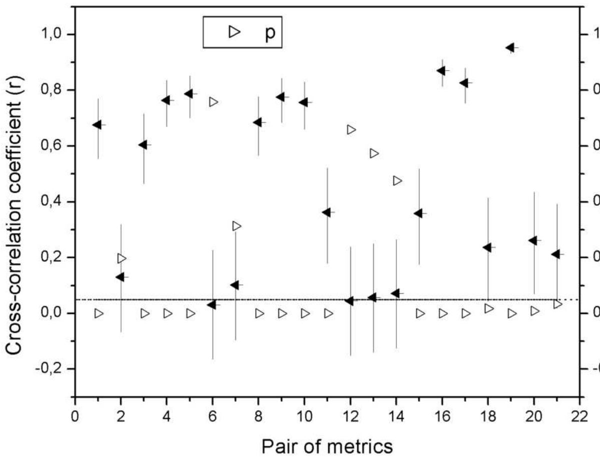

Fig. 2. Cross-correlation coefficients and p-value for networks with 5 nodes

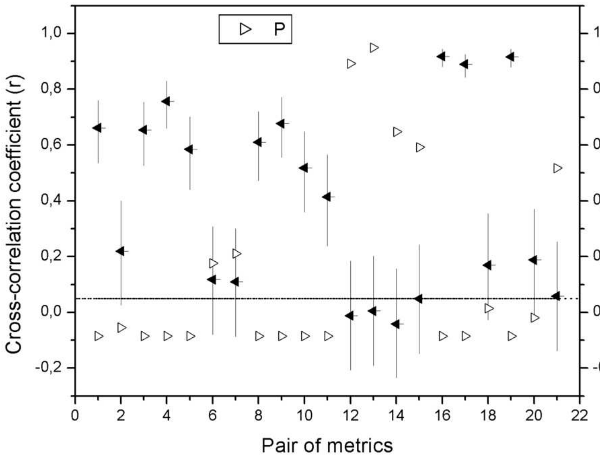

Fig. 3. Cross-correlation coefficients and p-value for networks with 6 nodes

Further investigation regarding orthogonality can be performed using eigenvalue and eigenvector analysis applied on cross-correlation coefficient matrix. Kaiser criteria [13] states that the number of eigenvalues above one indicates the significant components of the data, i.e., the minimal number of dimensions that can be used to represent the data. In addition, the corresponding eigenvectors represent the orthogonal base to be used for projecting the data.

It has been found that our data can be well grouped within just three dimensions. This means that just three metrics, instead of original seven, are sufficient to analyze the results. Each metric can then be represented in a three-dimensional plot using vectors signifying their magnitude and relative position to the axes. A seven-dimensional space would have a unit vector on each axis, but a three-dimensional space already captures most relevant trends of our data. The axes themselves are the groupings sought. Figures 5, 7, and 6 show 2-D projections of normalized metrics on groups identified as *Electronic*, *Optoelectronic*, and *Optical* resources for networks with 7 nodes. This classification was suggested by the nature of the metrics found around these axes.

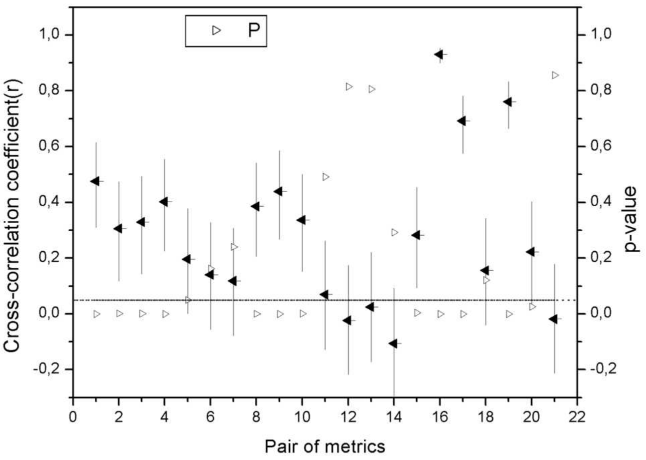

Fig. 4. Cross-correlation coefficients and p-value for networks with 7 nodes

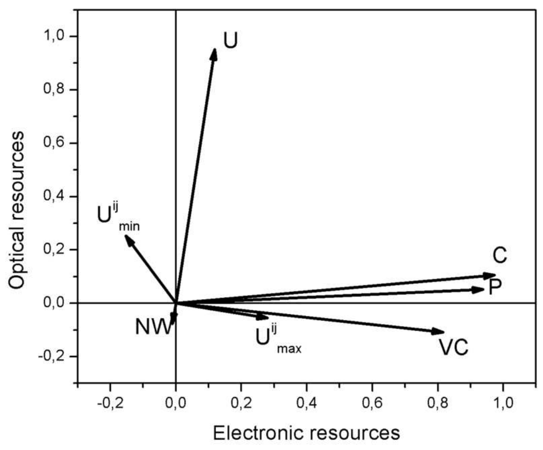

Fig. 5. Optical vs. electronic resources for 7 node network

Metrics grouped around the electronic resources are P, C, and VC (seen in Figure 5); while optical resources encompass U; and optoelectronic is lead (vector module close to unit) by NW, as seen in Figure 6 and 7. Figure 6 also shows very small projections for P, C, and VC meaning that they belong neither to optical nor optoelectronic groups. On the other hand, *Congestion*, i.e., U^{ij}_{max}, seems to fit both electronic and optoelectronic resources category in similar proportions while it has neglible projection on optical resources. However, U^{ij}_{min} has only small projections on the three groups.

Similar results are obtained for networks with 6 nodes. For networks with 5 nodes, the groupings discussed above are visible although metrics are slightly scattered.

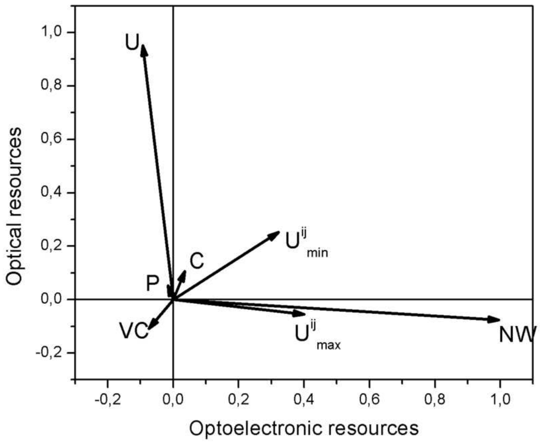

Fig. 6. Optical vs. optoelectronic resources for 7 node network

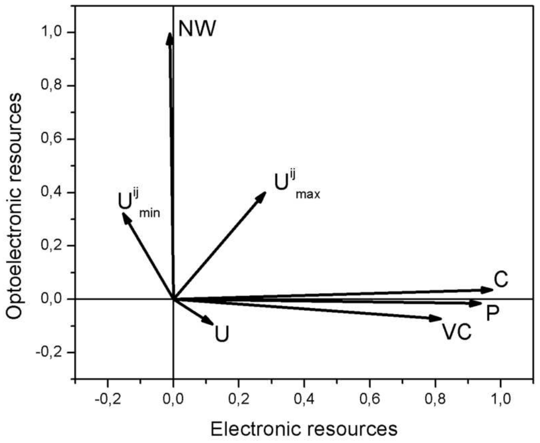

Fig. 7. Electronic vs. optoelectronic resources for 7 node network

V. CONCLUSION

In this work was proposed a statistical study to investigate how metrics relate in networks designed with GRWA aimed at minimal number or transceivers. For this end, we used a light ILP formulation to solve 100 instances for networks with 5, 6, and 7 nodes. In each case a study based on cross-correlation matrix was presented. Is was shown that some metrics are highly correlated, meaning that a metric is possibly inferred by others. On the other hand, some metrics without correlation are also found. This lead us to analyse orthogonality among metrics.

It was interesting to find that metrics are well grouped around distinct network functionalities, i.e., *Electronic*, *Optoelectronic*, and *Optical* resources. The metric used as objective function was found to be orthogonal to both Network utilization and Criticality.

The study might suggest that only few metrics need to be singled out when analysing integrated IP/WDM optical networking. Further investigations will address issues such as re-optimizations, heuristics, and multi-commodity objective function, based on the insights gained from statistical analysis of ILP solutions to GRWA.

VI. ACKNOWLEDGEMENT

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REFERENCES

[1] J. Fang, and A. K. Somani, "Enabling Subwavelength Level Traffic Grooming in Survivable WDM Optical Network Design", *GLOBECOM 2003 - IEEE Global Telecommunications Conference*, vol. 22, no. 1, December 2003, pp. 2761-2766

[2] K. Zhu, and B. Mukherjee, "Traffic Grooming in an Optical WDM Mesh Network", *IEEE Journal on Selected Areas in Communications*, vol. 20, no. 1, January 2002, pp. 122-133.

[3] M. Brunato, and R. Battiti, "A Multistart Randomized Greedy Algorithm for Traffic Grooming on Mesh Logical Topologies", *ONDM 2002*, pp.417-430.

[4] R. Srinivasan, and A. K. Somani, "Dynamic Routing in WDM Grooming Networks", *Photonic Network Communications*, 5:2, 2003, pp.123-135.

[5] L. A. Cox, Jr. and J. Sanchez, "Cost Savings from Optimized Packing and Grooming Optical circuits: Mesh versus Ring Comparisons", *Optical Networks Magazine*, May/june 2001, pp.72-90.

[6] E. Modiano, and P. J. Lin, "Traffic Grooming in WDM Networks", *IEEE Communications Magazine*, July 2001, pp.124-129.

[7] R. Dutta, and G.N. Touskas, "Traffic Grooming in WDM Networks: Past and Future", *IEEE Network*, November/December 2002, pp.46-56.

[8] www.cplex.com

[9] J.Q. Hu, and B. Leida, "Traffic Grooming, Routing, and Wavelength Assignment in Optical WDM Network", *IEEE INFOCOM - 2004*, vol.1, March 2004, pp.495-501.

[10] R. Ramaswami, and K.N. Sivarajan, "Design of Logical Topologies for Wavelength Routed Optical Network", *IEEE J. Sel. Areas Comum*, vol.14, June 1996, pp.840-851.

[11] L.C. Resendo, L.C. Calmon and M.R.N. Ribeiro"Simple ILP Approaches to Grooming, Routing, and Wavelength Assignment in WDM Mesh Networks", *accepted to IMOC 2005*.

[12] R.T.R. Almeida, K. Potratz, L.C. Bueno, M.E.V. Segatto, E. Oliveira and L.C. Calmon, "Addressing the electronic bottleneck to virtual topology design of optical networks", *Microwave and Optoelectronics Conference*, 2003, vol.2, pp.925-930.

[13] D. N. Lawley and A. E. Maxwell, "Factor Analysis as a Statistical Method", American Elsevier Pub. Co., Ed. 2nd, 1971.