Shared wavelength conversion modeling for asynchronous optical packet-switched networks

R. C. Almeida Jr, J. F. Martins Filho and H. Waldman

Resumo— **Contenção de pacotes é um assunto de importância determinante para as redes ópticas comutadas por pacote. Este artigo apresenta um modelo analítico para a avaliação de desempenho de comutadores de pacotes ópticos assíncronos quando conversores de comprimento de onda compartilhados são utilizados para a resolução de contenção. Será mostrada a economia do número de conversores de comprimento de onda em relação à arquitetura que os dispõe dedicados por canal de entrada. O modelo analítico se baseia numa suposição de granularidade infinitamente fina dos canais de entrada, estando em boa conformidade com os resultados obtidos por simulação.**

Palavras-Chave— **Comutação de pacotes ópticos, Resolução de contenção, Conversão de comprimento de onda, Modelos de Markov.**

Abstract— **Packet contention is a very important issue in optical packet-switched networks. This paper proposes a Markovian model to evaluate the performance of asynchronous optical packet switches when shared wavelength conversion is used as contention resolution mechanism. It will be shown the saving in the number of wavelength converters in relation to the switch architecture that presents one tunable optical wavelength converter per input wavelength channel. The analytical model is based on an infinitely fine input granularity assumption and it is shown to approximate quite well the simulation results.**

Keywords— **Optical packet switching, contention resolution, wavelength conversion, Markov modeling.**

I. INTRODUCTION

In a WDM optical packet-switched network, data packets are modulated on a specific wavelength and may travel several hops before reaching their destinations. In each hop, a switching node is used to direct the packet to the correct output fiber link. Output contention occurs when arriving packets on the same wavelength are designed to be at the same output port and overlapped in time. In optical packet switching, there are three ways to handle output contention: delay-line buffering [1], [2]; deflection routing [3], [4]; and wavelength conversion [5], [6], [7], [8], [9]. These techniques exploit respectively the time, space and wavelength domains [10], [11]. Such techniques may still be combined [3], [4], [5], [12], [13].

In the literature, the works that focus on studying and modeling the contending methods are usually based on synchronous networks [3], [5], [6], [7], [8], [12]. In the optical domain, however, maintaining synchronization is not a simple task, since optical signal processing at bit level is not readily available. Additionally, assuming an Internet environment, fixed-length packets imply the need to segment IP datagrams at the edge of the network and reassemble them at the other edge, which can be a problem at very high speeds. For these reasons, it is worth investigating switch block performance in the case where variable-length packets are routed without alignment (asynchronously).

In this paper we focus on the wavelength domain exploitation for contention resolution in asynchronous optical networks. The wavelength domain as contention resolution mechanism appeared in the context of WDM optical networks, where several wavelengths run on the same fiber link that connects two optical switches. Therefore, on the arrival of a new packet, if its wavelength is already being used on the destination output link, it may be converted to some free wavelength, such that the packet can still be transmitted.

In order that any packet from any input fiber may potentially be converted to any free wavelength of the desired output fiber, it is commonly assumed single per channel wavelength converters with full-range wavelength conversion capability. In such a scheme, a packet will be blocked only when there is not any available wavelength on the desired output link, which represents the best performance of the switch equipped with wavelength conversion. However, since for each input wavelength channel one tunable optical wavelength converter (TOWC) will be needed and optical wavelength converters is still a cost element, the number of TOWCs required may become unacceptably high.

An alternative is to share a pool of TOWCs among all input wavelength channels, so that the number of TOWCs may be reduced for a close switch performance with respect to the single per channel architecture. This is possible due to the fact that: a) not all input wavelengths transmit a packet at the same time; b) if a packet is directed to a free output wavelength, it will not need wavelength conversion; and c) a packet directed to an output fiber with no free wavelength does not need wavelength conversion, since it will be blocked.

In this paper we propose a Markovian analytical model that enables the calculation of packet blocking probability for asynchronous optical packet switches equipped with shared wavelength converters. We also conduct simulations to validate our analytical model. The analytical model is based on an infinitely fine input granularity assumption and it is shown to approximate quite well the simulation results. Analytical models are very useful mainly for low packet blocking probability, where simulations become time consuming.

The paper is structured as follows: Section II describes the switch architectures and traffic characteristics used in our analysis. In Section III, we present our analytical modeling for the shared per node tunable optical wavelength converter switch architecture, while the numerical results and model

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validation are presented in Section IV. Finally, in Section V we make our conclusions.

II. BASIC CONSIDERATIONS

The performance of the switch equipped with shared per node wavelength converters will be evaluated by simulating and analytically calculating the packet loss probability as a function of the load per input, number of wavelengths per fiber and number of shared wavelength converters. All these parameters as well as the basic switch architectures are described below.

A. Basic Switch Architecture

Figs. 1 and 2 show the switch architectures compared in this paper. Both consists of an NxN optical switch, i.e., N fibers on the input and output sides of the switch. On each fiber there are W wavelengths that carry independent data. Thus, there is a total of NW input and output wavelength channels. The input fibers are one by one fed into a demultiplexer, where the different wavelength channels are separated from one another. These will become the inputs of the space switch fabric, which is assumed to be capable of realizing every interconnection pattern between the input and output ports (nonblocking switch). Thus, any input wavelength channel may be connected through the space switch to any output fiber. At the output side of the switching fabric there are optical multiplexers, which will multiplex the individual sets of W output wavelengths into each output fiber.

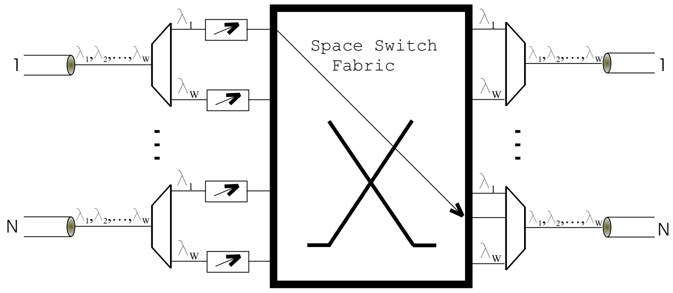

Fig. 1. Single per channel switch architecture.

The first architecture is commonly used in the literature [5], [7], [8], [9], [13]. It is denoted as single per channel (SPC), since each input channel has its own dedicated wavelength converter. The wavelength converters may be limited [7], [8], [9] or full-range [5], [13]. In all the paper, we assume full-range wavelength conversion capability. Therefore, when we refer to SPC architecture, an arriving packet will be blocked only when there is no available wavelength on the desired output link. Such configuration provides the best performance of the switch equipped with wavelength conversion. However, an amount of NW tunable optical wavelength converters is needed, which may become unacceptably high if the number of wavelength channels increases. The performance and modeling of the single per channel switch architecture in asynchronous optical networks have already been treated in [13]. Its results will be used here for comparison purposes.

The second architecture is denoted as shared per node, as there is a pool of TOWCs shared by all input channels. The number of shared TOWCs is represented here as Z. An disadvantage of the shared per node architecture is the need to add switching ports to allow any arriving packet that needs wavelength conversion to reach an empty TOWC of the pool, and further to allow the converted packet to reach the right output fiber. However, it is expected a close performance with the previously cited architecture, with a saving in the number of TOWCs. In Section III, we will present a Markov-based analytical model for the shared per node architecture and investigate the packet blocking probability and saving of TOWCs that are possible to obtain.

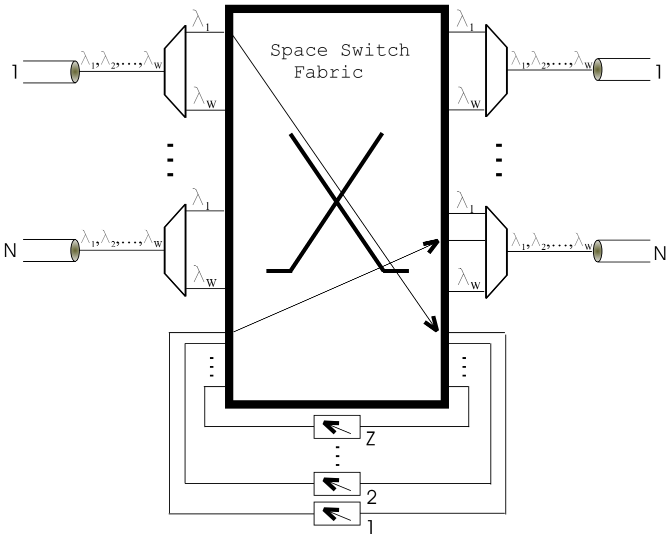

Fig. 2. Shared per node switch architecture used in our analysis.

B. The Traffic Model

For the incoming traffic, it will be assumed that the optical packets arrive to the switch not aligned in time (asynchronously). In addition, it will be assumed that the input channels are independent of each other and that each of them has the same input load (ρ). The traffic partitioning inside the switch will be considered uniform, i.e., a packet arriving at any input fiber has the same probability of being transmitted to any output, which can be written as $p_{i,j} = \frac{1}{N}$, $i, j = 1, 2, \ldots, N$. Finally, the traffic pattern considered in this paper is unicast, i.e., any arriving packet is destined for only one output fiber.

1) Simulations: For the simulations, it will be assumed that each input channel may be in two distinct states, as illustrated in Fig. 3: a) *Active state*, when a packet is present in the input channel under consideration, and thus the mean active state duration is equal to the mean packet length ($\bar{\tau}$); and b) *Waiting state*, with average duration $\bar{T}$ and during which the input channel under consideration is idle. In the simulations presented in this paper, both durations were assumed exponentially distributed. However, as explained in the next Subsection, the performance of the switch is independent of both distributions when the number of inputs is made large enough. Such assumption will be assumed in our analytical modeling and it will be shown to approximate quite well real (finite-input) systems.

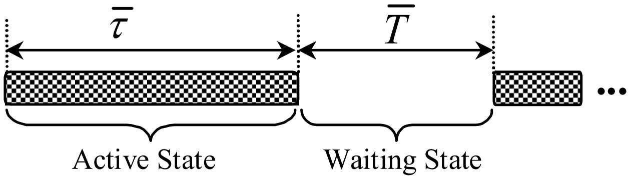

Fig. 3. Input channel traffic characterization

The input load, which is identified as the fraction of time the channel is transferring data, will be given by the ratio of the mean transmission time to the mean inter-arrival time:

$$\rho = \frac{\bar{\tau}}{\bar{\tau} + \bar{T}}. \tag{1}$$

The parallel arrivals of traffic streams at the input ports of the switch are simulated by an asynchronous discrete-event simulator, keeping track of the arrival and departure times of the packets.

2) Analytical Modeling: The analytical model proposed in this paper is based on an infinitely fine input granularity assumption, which is motivated by the fact that, when the number of inputs is made large enough and they are independent, the arrivals to each output fiber become Poissonian. It will be shown that such assumption is a good approximation to finite-input switches. Finally, the fact that $M/G/c$ systems are independent of the service time distribution [14] explains the previously mentioned insensitivity of the switch performance to both the active and waiting state duration distributions.

III. SHARED WAVELENGTH CONVERSION MODELING

For the analytical modeling, as we assume that the destination output port of the packets are uniformly distributed, we may focus on an arbitrary output fiber f_n, $n = 1, 2, \ldots, N$, and thus calculate the packet loss rate of the switch. Due to the infinitely fine input granularity assumption, packets to any output fiber arrive according to a Poisson process with an average rate $\lambda_n = \sum_{n=1}^{N} \sum_{w=1}^{W} \frac{1}{N} \rho/\bar{\tau} = \rho W/\bar{\tau}$. In addition, the duration of the packets may be assumed exponentially distributed, with mean $\bar{\tau} = 1/\mu$. For the calculation of the packet activation and deactivation rates, the following variables will be considered: the number of channels (w_n) that are transmitting a packet on the referred output fiber f_n (busy wavelengths); and the number of shared wavelength converters used by any packet in transmission into the switch (z) and by packets in transmission to the referred output fiber (z_n). Obviously, $0 \leq w_n \leq W$, $0 \leq z_n \leq min(w_n, z)$ and $z_n \leq z \leq Z$, where $min(a, b)$ is the smallest value from a and b.

The idea of the analysis proposed here is to solve the problem in a recursive manner. First, in order to obtain a Markov-chain analytical modeling, we must define the states of the switch. The most trivial would be to define (w_n, z_n, z) as its state, so that w_n and z_n take into account the influence of the packets in transmission to output fiber f_n, while z provides the influence of any packet that occupies one of the shared TOWCs of the switch. A single recursive iteration would be necessary to quantify the transition rates that take into account the variation in z, similar to the one described ahead. Since the state definition as (w_n, z_n, z) provides a three-dimensional Markov chain, it may not be feasible to solve in practice. In fact, considering $Z > W$, it is possible to show that the number of states required by the model would be $(W+1)(W+2)\left[3(Z+1) - W\right]/6$. For values as $W = 16$ and $Z = 32$, the number of states would be 4233.

To deal with this problem, we propose an analytical model with two linked Markov chains that will be shown to interact in a recursive manner. The model divides (w_n, z_n, z) in two sets of states: (w_n, z_n) and (z_n, z). For the first set of states, the number of states will be given by $(W+1)(W+2)/2$, if $Z \geq W$; or $(Z+1)(Z+2)/2 + (Z+1)(W-Z)$, otherwise. For the second set of states, the number of states for $Z > W$ and $Z \leq W$ will be given, respectively, by $(W+1)(W+2)/2 + (Z-W)(W+1)$ and $(Z+1)(Z+2)/2$. Now, if $W = 16$ and $Z = 32$, the number of states will be, respectively, 153 and 425. Notice that, for each recursive step, it will be necessary to solve two Markov systems. However, this will provide faster calculations than for the three-dimensional Markov system previously cited. Obviously, due to the system splitting in two sets of states, there must be a way of relating one Markov chain with the other. So, each recursion step will be composed by three stages:

Stage 1: Here we calculate Q_{w_n,z_n}, the steady-state probabilities of the first set of states. The state transition rates of this Markov chain may be obtained in the following way: on the arrival of a packet, if $w_n < W$, the probability that this packet does not require wavelength conversion to be transmitted is given by $\left[1 - \frac{w_n}{W}\right]$, which represents the probability that the arriving packet is on one of the available (free) output wavelengths. Thus, the transition rate from state (w_n, z_n) to state $(w_n + 1, z_n)$ will be given by $\lambda_n \left[1 - \frac{w_n}{W}\right]$. On the other hand, if the arriving packet is on one of the busy output wavelengths, the packet will be accepted if $z < Z$. Here we define y_{z_n} as the probability that $z = Z$ given that the number of shared wavelength converters used by packets in transmission to output fiber f_n is z_n. Such probability will be obtained in the third stage. Thus, there will be a transition from state (w_n, z_n) to state $(w_n + 1, z_n + 1)$ with transition rate $\lambda_n \frac{w_n}{W}\left[1 - y_{z_n}\right]$. On the other hand, packets will be blocked with transition rate $\lambda_n \frac{w_n}{W} y_{z_n}$. Obviously, if $w_n = W$, any arriving packet will be blocked, which will happen with transition rate λ_n. When one considers the packet deactivation, it is easy to see that when $z_n > 0$, the transition rate from state (w_n, z_n) to state $(w_n - 1, z_n - 1)$ will be $z_n \mu$. Similarly, if $w_n - z_n > 0$, there will be a transition from state (w_n, z_n) to state $(w_n - 1, z_n)$ with rate $(w_n - z_n)\mu$.

Stage 2: calculation of the steady-state probabilities of the second set of states ($Q'_{z_n,z}$). For the transition rate calculation, consider that the switch is at state (z_n, z) and that $z < Z$. This implies that, while $w_n < W$, if a packet headed to output fiber f_n arrives on one of the busy output wavelengths, there will be a transition to state $(z_n + 1, z + 1)$. This happens with transition rate $\lambda_n \sum_{i=z_n}^{W-1} Q_{i,z_n} \frac{i}{W} / \sum_{i=z_n}^{W} Q_{i,z_n}$, as the number of shared wavelength converters used by packets in transmission to output fiber f_n (z_n) is known. Similarly, if a packet headed to any of the remaining $N-1$ output fibers arrives on one of its busy output wavelengths, there will be a transition from state (z_n, z) to state $(z_n, z+1)$. Here, to simplify the analysis, we assume that the number of output wavelengths and shared wavelength converters used by packets in transmission to any of the remaining $N-1$ output fibers do not depend on the state (z_n, z) of the switch. Thus, such a transition will happen with rate $(N-1)\lambda_n \sum_{i=0}^{W-1} \sum_{j=0}^{min(i,Z)} Q_{i,j} \frac{i}{W}$. Obviously, the switch will remain in the same state with arrival rate $N\lambda_n$ minus the previously described rates. If $z = Z$, any arriving packet will be either accepted without needing wavelength conversion or blocked, representing a transition to the same state (z_n, z) with transition rate $N\lambda_n$.

The steady state probabilities of both Markov chains discussed ahead may be calculated by numerically solving the stationary equations for the continuous-time Markov process ($\mathbf{QT} = \mathbf{0}$), where $\mathbf{Q}$ is the steady state probability vector and $\mathbf{T}$ is the matrix of transition rates [14].

Stage 3: Evaluation of y_{z_n} and the switch packet blocking probability. The probability that $z = Z$ given that the number of shared wavelength converters used by packets in transmission to output fiber f_n is z_n is given by: $Q'_{z_n,Z} / \sum_{j=z_n}^{Z} Q'_{z_n,j}$. Finally, the switch packet blocking probability may be obtained as:

$$P_B = \sum_{i=0}^{W-1} \sum_{j=0}^{min(i,Z)} Q_{i,j} \frac{i}{W} y_j + \sum_{j=0}^{min(W,Z)} Q_{W,j}. \qquad (2)$$

Therefore, stages 1,2 and 3 are repeated until the results converge.

IV. NUMERICAL RESULTS AND MODEL VALIDATION

In this section, we evaluate the performance of the shared per node switch architecture in asynchronous optical networks. Fig 4 compares the packet blocking probability calculated through our Markovian model with estimates obtained through simulations, for a switch with $N = 8$ input and output fibers, $W = 16$ wavelengths per fiber and different values of shared wavelength converters (Z) and input load per wavelength (ρ). SPC represents the performance of the switch equipped with single per channel TOWCs, discussed in [13]. The simulations were evaluated for $P_b \geq 10^{-6}$, due to the long time that would be required to obtain reliable results for lower packet blocking probabilities. Notice the good approximation of the analytical model proposed with estimates obtained through simulations. In terms of performance, it can be seen the saving $(1 - Z_{th}/(NW))$ in the number of TOWCs that the sharing architecture allows to obtain, where Z_{th} is the number of shared wavelength converters that provides almost the same performance of the SPC switch architecture. For example, for input loads $\rho = 0.3$, 0.5 and 0.6, the saving in the number of TOWCs is about 75%, 62.5% and 50%, respectively.

The packet blocking probability of the shared per node switch architecture as a function of the number of shared TOWCs Z is shown in Fig. 5 for a switch with $N = 8$ input and output fibers, under a load per input wavelength $\rho = 0.6$ and different values of wavelengths per fiber W. As it can be seen, for low values of shared wavelength converters Z, the packet blocking probability becomes higher when one increases the number of input channels and maintains their loads constant. This occurs due to the fact that the same (small) quantity of wavelength converters is shared for a higher resultant traffic load. This increases the average number of conversion requests and consequently the probability that a TOWC is busy. Notice in addition from the curves that when the switch is under the same load per wavelength ($\rho = 0.6$), the saving in the number of wavelength converters coincides (about 50%) for all cases (Z_{th} is gotten from the figure observing the value of Z beyond which no performance improvement is obtained, which corresponds to the performance of the SPC

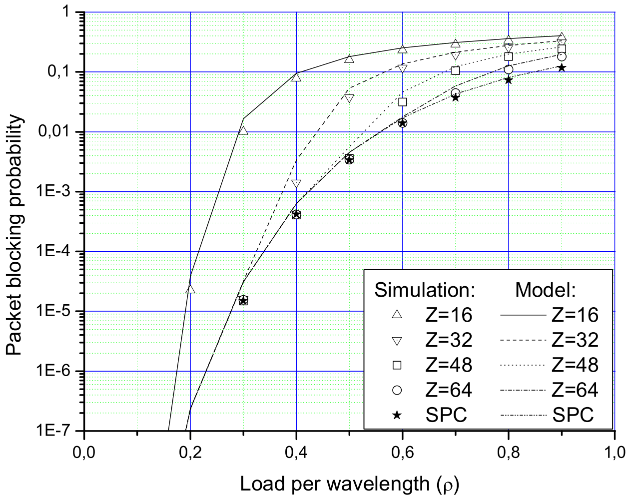

Fig. 4. Packet blocking probability versus load per wavelength (ρ) for a shared per node switch architecture with $N = 8$, $W = 16$ and $Z = 16, 32, 48$ and 64. SPC represents the packet blocking probability of the single per channel switch architecture, which uses 128 wavelength converters.

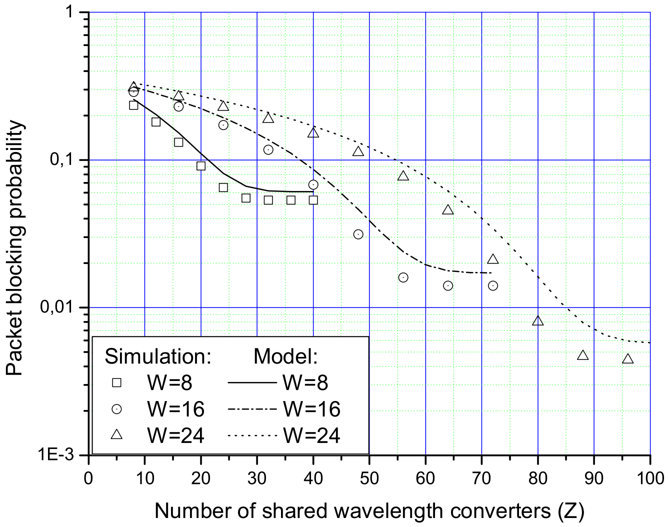

Fig. 5. Packet blocking probability versus number of shared wavelength converters (Z) for a switch with $N = 8$, $W = 8, 16, 24$, under load per wavelength $\rho = 0.6$.

architecture).

On the other hand, if we assume a fixed input load per input fiber (ρW), the switch performance in relation to the number of wavelengths becomes different. For example, Fig. 6 shows the switch packet blocking probability for $N = 8$, $W = 16$, $\rho = 0.6$ and when we expand the number of wavelengths to $W = 24$ and $W = 32$, which implies $\rho = 0.4$ and 0.3, respectively. As it can be seen, both the switch performance and saving in the number of wavelength converters drastically increase if we expand the number of wavelengths per fiber W and fix the total load per fiber ρW. For example, for $W = 16$, $\rho = 0.6$ and $W = 32$, $\rho = 0.3$, the switch packet blocking probability and saving in the number of wavelength converters changes approximately from $2 * 10^{-2}$ and 50% to $7 * 10^{-9}$ and 75%, respectively.

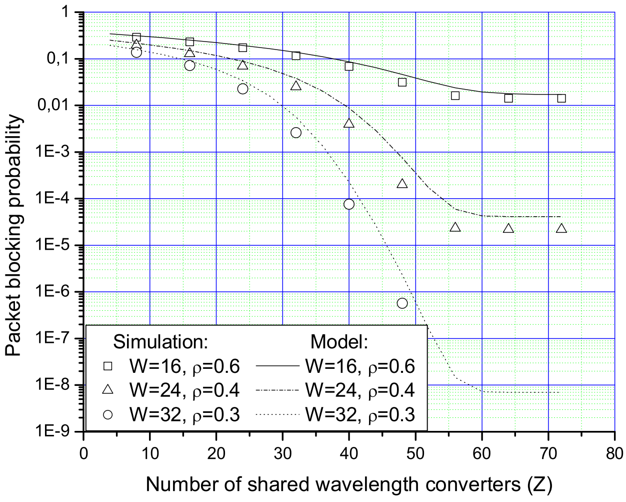

Fig. 6. Packet blocking probability versus number of shared wavelength converters (Z) for a switch with $N = 8$, $W = 16, 24, 32$ and the same load per input fiber $\rho W = 9.6$.

V. CONCLUSIONS

In this paper we presented a Markovian analytical model for evaluating the packet blocking probability of optical packet switches equipped with shared full-range tunable wavelength converters. Comparisons between simulation and theoretical results confirmed the good approximation of our analytical model. The study of the shared per node architecture is of great interest as both: it can save wavelength converters in relation to the single per channel architecture and, as known, wavelength converters still represent one of the most costly elements of an optical packet switch. We showed for example that for practical input load values ($\rho = 0.6$) the saving in the number of wavelength converters in relation to the single per channel architecture is about 50%. In addition, we observed that this saving is higher as lower is the input load ρ, as the probability of packet contention at each output wavelength channel reduces and the average number of packets that request wavelength conversion also reduces when the input load decreases.

As future work, since limited-range wavelength conversion is cheaper and easier to implement when compared to full-range, an interesting topic would be to expand the Markovian model developed in this paper for shared limited-range wavelength converters. Another interesting future work would be to consider differentiated service classes in the modeling.

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