Blocking Probabilities in Single-Wavelength Multifiber Rings

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***Abstract*— In this paper we present a method for calculating exact values of blocking probabilities in single-wavelength multifiber rings under Poissonian, homogeneous traffic with any spatial profile. It is assumed that the wavelength carried by each fiber is the same in all fibers and links. We also present a new scalable procedure to derive the normalization constant of the Erlang's classical model for ring networks. This procedure eliminates the necessity of listing all possible configurations in order to obtain the normalization constant. Formulas are derived for the utilization rate and blocking probability of the ring.**

***Keywords*— Blocking probability, rings, multifiber, Erlang model.**

***Resumo*— Neste trabalho apresentamos um método para o cálculo exato de probabilidades de bloqueio para anéis de múltiplas fibras com um comprimento de onda por fibra sob tráfego Poissoniano homogêneo de qualquer perfil espacial. Assumimos que o comprimento de onda transportado por cada fibra é o mesmo em todas as fibras e enlaces. Apresentamos também um novo procedimento escalável para a obtenção da constante de normalização do modelo clássico de Erlang para redes em anel. Este procedimento elimina a necessidade de listar todas as configurações possíveis para se obter a constante de normalização. São apresentadas fórmulas para a probabilidade de bloqueio e taxa de utilização no anel.**

***Palavras-Chave*— Probabilidade de bloqueio, anéis, multifibra, modelo de Erlang.**

I. Introduction

Recently, multifiber networks have received much attention as it is cost-effective to make use of the available infrastructure of deployed fiber cables for purposes of, say, network expansions. As deploying fiber in the ground is done once, there is usually much idle capacity in such cables.

For these reasons, performance studies of multifiber networks have been recently proposed in the literature [1], [2], [3]. In [4] the authors discuss the problem of trading wavelengths with fibers, pointing out cost-performance analysis based on analytical models and simulation-based studies previously proposed.

In this paper, we present a method for calculating exactly the blocking probability in single-wavelength multifiber rings under Poissonian, homogeneous traffic with any spatial profile. In order to justify the employment of space division multiplexing, in which multiple fiber pairs are used to connect a pair of nodes, we assume that the wavelength carried by each fiber is the same in all fibers and links. This assumption would be plausible in the scenario where wavelengths were scarce resources, raising new challenges to network designers. Thus, the utilization of space division multiplexing could be an alternative solution.

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The proposed method is based on the Erlang model, which is discussed in Section II. One of the main difficulties in the utilization of this model is that it is a non-trivial problem to calculate the sum in the normalization constant because the sum has many terms fur just moderate link capacities or network sizes. For instance, in [5] the author says that the development of efficient methods for this calculation is one of the major projects of his book. In this paper, however, we present a scalable method for deriving this normalization constant of the Erlang's classical model for rings.

The remainder of the paper is described as follows. Section III contains the matrix-based method that obviates the enumeration of all configurations for single-size path traffic. The exact calculation of utilization rates and blocking probabilities for this case is derived in Sections IV and V, respectively. The extension for multiple-size paths is addressed in Section VI and some examples are shown in Section VII. Section VIII presents the conclusions of our work.

II. The Erlang Model

The Erlang model applies to networks formed by links on which a certain number of circuits is defined [6]. This number, which may or may not vary from link to link, is called the capacity of each link. Routes are defined as sequences of adjacent routes onto which traffic is bound. Each call demands one circuit per each link of the requested route, and will be blocked if and only if no circuit is available in at least one of the component links. Call arrivals are Poissonian, while call durations are independent and may follow any distribution with finite mean.

Let τ_r be the traffic intensity on route r, C_r be the capacity of route r, and $m_r \leq C_r$ be the number of calls in progress at some time t on route r. Under Poissonian call arrivals, the probability of any feasible configuration $\mathbf{n} = \{m_r\}$ will be:

$$p(\mathbf{n}) = G^{-1} \prod_r \frac{\tau_r^{m_r}}{m_r!}, \tag{1}$$

where G^{-1} is the probability of the "empty"configuration in which $m_r = 0$ for all r, which may be obtained from $\sum_{\mathbf{n}} p(\mathbf{n}) = 1$, where the summation is performed over all feasible $\mathbf{n}$.

In principle, the network utilization rate and blocking probability may be derived from averaging them over all configurations using (1). In practice, however, this is often

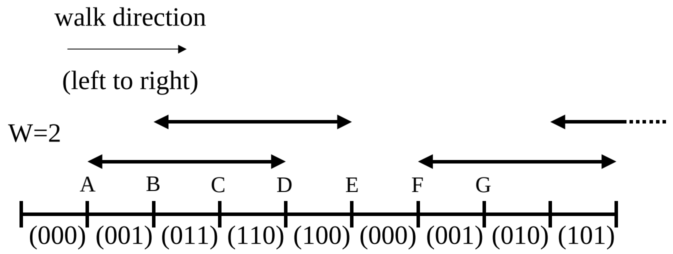

Fig. 1. Link states.

not feasible, as the number of configurations runs away to extremely large values for even moderately sized networks, thus hurdling the determination of G. In this paper, however, we will show a method whereby the Erlang model may be applied to single-wavelength multifiber rings with only a modest computational effort.

III. Probability Generating Matrices (PGM's)

Let us consider a single-wavelength multifiber ring in which all N links have the same capacity (number of fibers) F. For the sake of simplicity, we shall assume that all traffic is bound to H-hop routes, so all paths have the same size H. Furthermore, traffic is assumed to have single granularity and to be homogeneous, meaning that all H-hop routes have the same traffic intensity ν. Extension to multiple sizes will be addressed in Section VI.

Let the state of a link be defined as the H-ple $(a_1, a_2, \ldots, a_H)$ where a_i is the number of active calls in the i-th route in a given direction – say from left to right (w.l.o.g.) – that contains the link. The set of all allowed states is then given by:

$$\mathcal{S} = \{(a_1, a_2, \ldots, a_H) |\, a_i \in \mathbb{N},\, 0 \leq a_i \leq F, \quad 0 \leq \sum_{i=1}^{H} a_i \leq F\}. \quad (2)$$

Notice that link utilization, i.e. the number of active calls in the link, is $\sum_{i=1}^{H} a_i$. Any call arriving in any route that contains a given link will have to be blocked if this summation is F. For this reason, in order for a request to be accommodated, all requested links must be in one of the following set of non-blocking states:

$$\mathcal{R} = \{(a_1, a_2, \ldots, a_H) |\, a_i \in \mathbb{N},\, 0 \leq a_i \leq F, \quad 0 \leq \sum_{i=1}^{H} a_i \leq F - 1\}. \quad (3)$$

When a linear topology is walked through from left to right, states of adjacent links are strongly correlated. If a link is in state $(a_1, a_2, \ldots, a_H)$ and the next link (from left to right) is in state $(b_1, b_2, \ldots, b_H)$, then the following constraint applies:

$$b_i = a_{i+1}, \quad i = 1, 2, \ldots, H - 1, \quad (4)$$

leaving only b_H to be constrained by $\mathcal{S}$. As an example, consider link CD in Fig. 1. The arrowed segments represent 3-hop paths ($H = 3$). The first route (from left to right) that contains CD is AD, the second is BE, and the third (and last) is CF. Since $a_{AD} = a_{BE} = 1$ and $a_{CF} = 0$, then link CD is in state (110). Next link state might then be 101 or 100, corresponding to route DG in active or inactive state, respectively. In the instance shown on Fig. 1, link DE state is (100) because route DG is inactive.

The transition from a link to its right neighbor entails the existence of a nonnegative b_H new active paths starting at the new link from left to right. In the configuration probability expression given by (2), this corresponds to a factor of $\nu^{b_H}/(b_H!)$ in the product form. For this reason, we shall label the transition from $(a_1, a_2, \ldots, a_H)$ to $(a_2, \ldots, a_H, b_H)$ with label $\nu^{b_H}/(b_H!)$. Forbidden transitions are labeled zero.

Let us take all link states in order of increasing link utilization first, with lexicographical order among states with the same link utilization. The transition labels define a $|\mathcal{S}| \times |\mathcal{S}|$ Universal Probability Generating Matrix (UPGM) $\mathbf{D}(\nu)$. If $i = (a_1, a_2, \ldots, a_H)$ and $j = (b_1, b_2, \ldots, b_H)$, then $d_{ij}(\nu)$ is the corresponding factor in the product form at (1).

In an open linear topology, $G^{-1}[\mathbf{D}^N(\nu)]_{ij}$ is the sum of the probabilities of all configurations that lead from link state i to link state j in N steps. In an N-node ring, however, N steps will necessarily lead back to the initial state, so only the diagonal elements $G^{-1}[\mathbf{D}^N(\nu)]_{ii}$ correspond to probabilities of allowed configurations. Therefore, $G^{-1}[\mathbf{D}^N(\nu)]_{ii}$ is the probability of state i in a randomly chosen link of the ring. Summing over all states, we have:

$$G(\nu) = \mathrm{Tr}[\mathbf{D}^N(\nu)], \quad (5)$$

where $\mathrm{Tr}(\cdot)$ is the trace of the argument-matrix, i.e. the sum of all its diagonal elements. Equation (5) allows the calculation of G without requiring the enumeration of all possible path configurations in the ring, which is the main hurdle in the practical application of (1) for generic topologies.

In order to calculate blocking probabilities in Section V, we will also need to consider a PGM that leads only to non-blocking states. Let $\mathbf{r}$ be a $|\mathcal{S}| \times 1$ vector with 1's in all positions that index states that belong to $\mathcal{R}$, and zeros elsewhere, and let $\mathrm{diag}(\mathbf{r})$ be the $|\mathcal{S}| \times |\mathcal{S}|$ matrix with the elements of $\mathbf{r}$ in the diagonal and zeros elsewhere. The non-blocking PGM $\mathbf{F}(\nu)$ is defined as:

$$\mathbf{F}(\nu) = \mathbf{D}(\nu)\mathrm{diag}(\mathbf{r}). \quad (6)$$

Notice that $\mathbf{F}(\nu)$ assigns nonzero labels to and only to all those transitions that lead onto non-blocking links. All remaining transitions are labeled zero.

IV. Utilization Rate

The calculation of $G(\nu)$ through (5) generates a polynomial in ν:

$$G(\nu) = g_0 + g_1\nu + g_2\nu^2 + \ldots + g_i\nu^i + \ldots \quad (7)$$

Since each new link is labeled $\nu^{b_H}/(b_H!)$, where b_H is the number of active H-hop paths starting there, then each path contributes one unit to the power of ν in (1). Therefore, $G^{-1}g_i\nu^i$ is the sum of the probabilities of all ring configurations that contain exactly i active paths, i.e. that have utilization rate exactly $\frac{iH}{NF}$. The mean utilization rate is then given by:

$$\overline{\rho} = \sum_i \frac{iH}{NF} G^{-1}(\nu) g_i \nu^i = \frac{H}{NF} \frac{\sum_i i g_i \nu^i}{\sum_i g_i \nu^i} = \frac{H}{NF} \frac{\nu G'(\nu)}{G(\nu)}. \quad (8)$$

V. Blocking Probability

In order for a H-hop path request not to be blocked, all requested links must be in non-blocking states, i.e. they must be in $\mathcal{R}$. All constrained labels for these links are captured by the non-blocking probability generating matrix $\mathbf{F}(\nu)$. Since the traffic is assumed to be spatially homogeneous, the blocking probability over any given route is the same as in any H-hop route in the ring. Hence the probability of a sequence being accommodated (i.e. not blocked) is the sum of the probabilities of all ring configurations that do not generate blocking states over H successive links. This sum is generated by the trace of a matrix that represents successive H non-blocking transitions followed by successive $(N-H)$ generic transitions:

$$1 - \overline{P_b} = G^{-1} \mathrm{Tr}[\mathbf{F}^H(\nu) \mathbf{D}^{N-H}(\nu)]. \quad (9)$$

The mean blocking probability is then given by:

$$\overline{P_b} = 1 - \frac{\mathrm{Tr}[\mathbf{F}^H(\nu) \mathbf{D}^{N-H}(\nu)]}{\mathrm{Tr}[\mathbf{D}^N(\nu)]}. \quad (10)$$

Equations (8) and (10) yield the mean utilization rate and the mean blocking probability for any given traffic ν, thus allowing the exact blocking performance characterization of the ring.

VI. Extension to Multiple Sizes

Let us now assume that paths are requested with sizes $1, 2, \ldots,$ up to some maximum size M, and let ν_k be the intensity of traffic for paths with k hops, assumed to be the same for all source and destination nodes (homogeneous traffic). Total traffic is then described by a vector $\boldsymbol{\nu} = (\nu_1, \nu_2, \ldots, \nu_M)$ with intensity $\nu = \sum_{k=1}^{M} \nu_k$ Erlangs per node.

Any link will now be contained in 1 single-link route, two 2-link routes, three 3-link routes,..., and M M-link routes, so the state of the link will now be described by the numbers of active paths in a total of $(M+1)M/2$ interfering routes. Each state is represented by the following $(M+1)M/2$-ple:

$$(a_{11}, a_{12}, a_{22}, a_{13}, a_{23}, a_{33}, \ldots, a_{1M}, \ldots, a_{M-1,M}, a_{MM}),$$

where a_{ij} is the number of active paths in the i-th route from left to right, with size j, that contains the link.

The set of all states will be given by:

$$\mathcal{S} = \{ \{a_{ij}\},\ 1 \le j \le M,\ 1 \le i \le j \mid a_{ij} \in \mathbb{N}, \\ 0 \le a_{ij} \le W,\ 0 \le \sum_{i=1}^{j} \sum_{j=1}^{M} a_{ij} \le F \}, \quad (11)$$

and the set of all non-blocking states is:

$$\mathcal{R} = \{ \{a_{ij}\},\ 1 \le j \le M,\ 1 \le i \le j \mid a_{ij} \in \mathbb{N}, \\ 0 \le a_{ij} \le W,\ 0 \le \sum_{i=1}^{j} \sum_{j=1}^{M} a_{ij} \le F-1 \}. \quad (12)$$

The transition from a link in state a_{ij} to its next right neighbor in state b_{ij} is constrained by:

$$b_{ij} = a_{i+1,j}, \quad i = 1, 2, \ldots, j-1, \quad (13)$$

and will be labeled by $\prod_{k=1}^{M} \frac{\nu^{b_{kk}}}{b_{kk}!}$.

These labels define a universal probability-generating matrix $\mathbf{D}(\boldsymbol{\nu})$. The same arguments used to derive (5) may now be invoked to write:

$$G(\boldsymbol{\nu}) = \mathrm{Tr}[\mathbf{D}^N(\boldsymbol{\nu})]. \quad (14)$$

Likewise, the non-blocking probability-generating matrix will be:

$$\mathbf{F}(\boldsymbol{\nu}) = \mathbf{D}(\boldsymbol{\nu}) \mathrm{diag}(\mathbf{r}). \quad (15)$$

Let ρ_k be the rate of partial utilization by paths with size k, meaning that $\rho_k N/k$ is the number of such paths. Then, the mean of ρ_k is given by:

$$\overline{\rho_k} = \frac{k}{NF} \frac{\nu_k \frac{\partial G(\boldsymbol{\nu})}{\partial \nu_k}}{G(\boldsymbol{\nu})}. \quad (16)$$

Summing over all k, the following expression is obtained for the mean of the total utilization rate:

$$\overline{\rho} = \frac{\sum_{k=1}^{M} k \nu_k \frac{\partial G(\boldsymbol{\nu})}{\partial \nu_k}}{NFG(\boldsymbol{\nu})}. \quad (17)$$

The mean blocking probability of a request for a k-hop path will be:

$$\overline{p_{bk}} = 1 - \frac{\mathrm{Tr}[\mathbf{F}^k(\boldsymbol{\nu}) \mathbf{D}^{N-k}(\boldsymbol{\nu})]}{\mathrm{Tr}[\mathbf{D}^N(\boldsymbol{\nu})]}. \quad (18)$$

Finally, the mean blocking probability will be:

$$\overline{P_b} = \frac{\sum_{k=1}^{M} \nu_k p_{bk}}{\nu} = 1 - \frac{\sum_{k=1}^{M} \nu_k \mathrm{Tr}[\mathbf{F}^k(\boldsymbol{\nu}) \mathbf{D}^{N-k}(\boldsymbol{\nu})]}{\nu \mathrm{Tr}[\mathbf{D}^N(\boldsymbol{\nu})]}. \quad (19)$$

VII. Some Examples

A. Single-size Paths

Let us assume a 7-node ring in which all traffic is formed by H-hop paths, with $H = 3$. Consider also that there are $F = 2$ fibers connecting each fiber pair. The set of all allowed states is then given by:

$$\mathcal{S} = \{000, 001, 002, 010, 011, 020, 100, 101, 110, 200\}, \quad (20)$$

which gives rise to the following set of non-blocking states:

$$\mathcal{R} = \{000, 001, 010, 100\}. \quad (21)$$

Therefore, the Universal Probability Generating Matrix is:

$$\mathbf{D}(\nu) = \begin{bmatrix} 1 & \nu & \frac{\nu^2}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \nu & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & \nu & \frac{\nu^2}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \nu & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \nu & 0 & 0 & 0 \\ 1 & \nu & \frac{\nu^2}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (22)$$

Vector $\mathbf{r}$ is a $|\mathcal{S}| \times 1$ vector with 1's in all positions that index states that belong to $\mathcal{R}$, and zeros elsewhere. Matrix $\mathrm{diag}(\mathbf{r})$ is the $|\mathcal{S}| \times |\mathcal{S}|$ matrix with the elements of $\mathbf{r}$ in the diagonal and zeros elsewhere. Therefore, using both $\mathbf{D}(\nu)$ and $\mathrm{diag}(\mathbf{r})$, the nonblocking matrix $\mathbf{F}(\nu)$ is then obtained by equation 6:

$$\mathbf{F}(\nu) = \begin{bmatrix} 1 & \nu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & \nu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & \nu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{23}$$

Notice that $\mathbf{F}(\nu)$ results from $\mathbf{D}(\nu)$ when all its columns indexed by states that are not in $\mathcal{R}$ are zeroed.

Using equation 5, we obtain the following expression for $G(\nu)$:

$$G(\nu) = 1 + 7\nu + \frac{49\nu^2}{2} + 35\nu^3 + \frac{77\nu^4}{4}. \tag{24}$$

Using both equations (8) and (10), we obtain, respectively, the utilization rate and the mean blocking probability for any given traffic profile ν.

B. Multiple-size paths

Paths are now allowed to have sizes $1, 2, \ldots,$ up to some maximum size M. For simplicity, consider $M = 2$, i.e., paths may be of size 1 or 2, with traffic intensity given by ν_1 and ν_2, respectively. Again, we consider a ring with $N = 7$ nodes and $F = 2$ fibers. The set of all allowed states is then given by:

$$\mathcal{S} = \{000, 001, 002, 010, 011, 020, 100, 101, 110, 200\}, \tag{25}$$

which provides the set of nonblocking states:

$$\mathcal{R} = \{000, 001, 010, 100\}. \tag{26}$$

The similarity between (20) and (25), as well between (21) and (26), may be misleading and is merely formal, since the same state representation will have different meanings in the two examples. The first digit now represents the occupation of the only single-hop route that contains the link, and will be "forgotten" in the transition to the next rightmost link; while the second and third digits represent the occupations of the first and second 2-hop routes (from left to right) that contain the link. Labeling each allowed allowed transition by $(\nu_1^{b_{11}} \nu_2^{b_{22}})/(b_{11}!b_{22}!)$, where b_{11} and b_{22} are the first and third digits of the next state representation, and keeping all forbidden transitions labeled by zero, we obtain the Universal Probability Generating Matrix:

$$\mathbf{D}(\boldsymbol{\nu}) = \begin{bmatrix} 1 & \nu_2 & \frac{\nu_2^2}{2} & 0 & 0 & 0 & \nu_1 & \nu_1\nu_2 & 0 & \frac{\nu_1^2}{2} \\ 0 & 0 & 0 & 1 & \nu_2 & 0 & 0 & 0 & \nu_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & \nu_2 & \frac{\nu_2^2}{2} & 0 & 0 & 0 & \nu_1 & \nu_1\nu_2 & 0 & \frac{\nu_1^2}{2} \\ 0 & 0 & 0 & 1 & \nu_2 & 0 & 0 & 0 & \nu_1 & 0 \\ 1 & \nu_2 & \frac{\nu_2^2}{2} & 0 & 0 & 0 & \nu_1 & \nu_1\nu_2 & 0 & \frac{\nu_1^2}{2} \\ 1 & \nu_2 & \frac{\nu_2^2}{2} & 0 & 0 & 0 & \nu_1 & \nu_1\nu_2 & 0 & \frac{\nu_1^2}{2} \\ 0 & 0 & 0 & 1 & \nu_2 & 0 & 0 & 0 & \nu_1 & 0 \\ 1 & \nu_2 & \frac{\nu_2^2}{2} & 0 & 0 & 0 & \nu_1 & \nu_1\nu_2 & 0 & \frac{\nu_1^2}{2} \\ 1 & \nu_2 & \frac{\nu_2^2}{2} & 0 & 0 & 0 & \nu_1 & \nu_1\nu_2 & 0 & \frac{\nu_1^2}{2} \end{bmatrix}. \tag{27}$$

Using equation 14, we obtain the following expression for $G(\boldsymbol{\nu})$:

$$\begin{aligned} G(\boldsymbol{\nu}) = \frac{1}{128}(&14\nu_1^{13} + \nu_1^{14} + 14\nu_1^{12}(7 + 2\nu_2) + \\ &+ 112\nu_1^{11}(4 + 3\nu_2) + 14\nu_1^{10}(106 + 142\nu_2 + 21\nu_2^2) + \\ &+ 28\nu_1^9(134 + 270\nu_2 + 105\nu_2^2) + 28\nu_1^8(266 + 730\nu_2 + \\ &+ 507\nu_2^2 + 52\nu_2^3) + 128\nu_1^7(92 + 322\nu_2 + 336\nu_2^2 + 91\nu_2^3) + \\ &+ 56\nu_1^6(266 + 1132\nu_2 + 1606\nu_2^2 + 778\nu_2^3 + 63\nu_2^4) + \\ &+ 112\nu_1^5(134 + 668\nu_2 + 1206\nu_2^2 + 878\nu_2^3 + 189\nu_2^4) + \\ &+ 448\nu_1^3(16 + 100\nu_2 + 256\nu_2^2 + 322\nu_2^3 + 187\nu_2^4 + 35\nu_2^5) + \\ &+ 112\nu_1^4(106 + 600\nu_2 + 1314\nu_2^2 + 1300\nu_2^3 + 502\nu_2^4 + 35\nu_2^5) + \\ &+ 112\nu_1(8 + 56\nu_2 + 180\nu_2^2 + 320\nu_2^3 + 320\nu_2^4 + 162\nu_2^5 + 29\nu_2^6) + \\ &+ 56\nu_1^2(56 + 376\nu_2 + 1092\nu_2^2 + 1664\nu_2^3 + 1312\nu_2^4 + 442\nu_2^5 + 29\nu_2^6) + \\ &+ 16(8 + 56\nu_2 + 196\nu_2^2 + 392\nu_2^3 + 476\nu_2^4 + 322\nu_2^5 + 105\nu_2^6 + 8\nu_2^7)). \end{aligned} \tag{28}$$

The nonblocking matrix $\mathbf{F}(\boldsymbol{\nu})$ is then obtained using equation 15:

$$\mathbf{F}(\boldsymbol{\nu}) = \begin{bmatrix} 1 & \nu_2 & 0 & 0 & 0 & 0 & \nu_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & \nu_2 & 0 & 0 & 0 & 0 & \nu_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & \nu_2 & 0 & 0 & 0 & 0 & \nu_1 & 0 & 0 & 0 \\ 1 & \nu_2 & 0 & 0 & 0 & 0 & \nu_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & \nu_2 & 0 & 0 & 0 & 0 & \nu_1 & 0 & 0 & 0 \\ 1 & \nu_2 & 0 & 0 & 0 & 0 & \nu_1 & 0 & 0 & 0 \end{bmatrix} \tag{29}$$

Using both equations (17) and (19), we obtain, respectively, the utilization rate and the mean blocking probability for any given traffic profile $\boldsymbol{\nu}$.

Fig. 2 shows the comparison among our Erlang model calculations and simulations for a 7-node ring with $F = 2$ fibers in the following cases: a) single-size paths with $H = 3$ hops; and b) paths of size 1 and 2 ($M = 2$) with uniform traffic ($\nu_1 = \nu_2 = \nu/2$). We can see that our Erlang model calculations fit exactly the simulations dots.

VIII. CONCLUSION

We have shown that the classical Erlang model may be used scalably to yield exact calculations of utilization rate

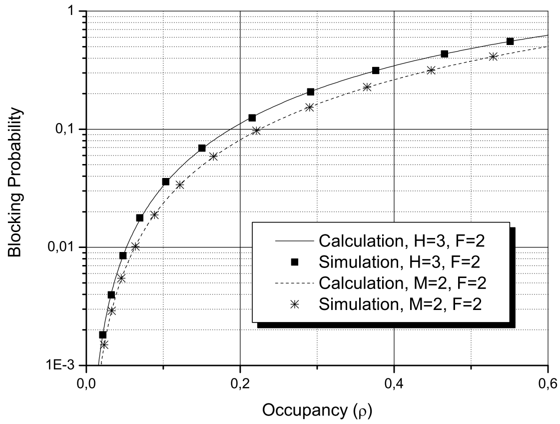

Fig. 2. Comparison between calculation and simulation.

and blocking probability in single-wavelength multifiber rings through a new matrix-based analytical formulation introduced in this paper. Although scalable with respect to the ring size, the size of the matrices increases rather rapidly with the number of fibers and variability of the hop count of requested paths. We have also presented a new scalable method for deriving the normalization constant of the Erlang's classical model. The new procedure obviates the enumeration of all configurations for ring networks.

We are currently extending this method to allow for more than one wavelength in multifiber rings, and also multigranular traffic in WDM rings.

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