Combined PARAFAC-Subspace Approach to Blind Multiuser Equalization

A. L. F. de Almeida, G. Favier, J. C. M. Mota

***Abstract*— This paper presents a tensor decomposition known as PARAFAC (Parallel Factors) and propose a new blind multiuser equalization approach for wireless communication systems, employing an antenna array and oversampling at the receiver. First, a tridimensional PARAFAC approach for modeling the received signal is proposed, the 3 dimensions being *space*, *time* and *oversampling* dimensions. Then, a blind multiuser receiver performing multiuser signal separation and equalization is formulated, combining PARAFAC modeling and a subspace method. The key difference of the proposed approach compared to most of existing ones is on the fact that the inherent tensor structure of the received signal is exploited. The proposed PARAFAC receiver has two blind-processing stages. In the first one, co-channel user signals are separated in the tensorial domain using an alternating least-squares algorithm. In the second stage, a subspace method is used to independently equalize each user sequence. Simulation results are provided to illustrate the performance of the proposed receiver. Our results show that the PARAFAC receiver performs closely to the MMSE (*Minimum Mean Square Error*) and ZF (*Zero Forcing*) receivers.**

***Keywords*— PARAFAC, blind multiuser equalization, oversampling, wireless communications, frequency-selective, alternating least squares, subspace.**

I. INTRODUCTION

The development of advanced signal processing techniques for wireless communications is an attractive research topic. In multiuser (mobile) wireless communication systems, the main task of receiver signal processing is to identify the parameters of the propagation channel and/or to recover the useful transmitted information in the presence of co-channel interference, intersymbol interference and additive noise. The blind multiuser equalization problem is an attractive research topic in the area of signal processing for wireless communications. It consists in recovering the information transmitted by several co-channel users with the assumption of a frequency-selective channel and without the knowledge of training sequences. Most of receiver algorithms deal with matrix (two-dimensional or 2-D) models for the received signal, exploiting its space and time dimensions as well as structural (problem-specific) properties of the transmitted signals (finite-alphabet, constant-modulus, etc..) for signal separation and equalization [1], [2], [3]. Without these additional considerations, it is well known that the low-rank property of matrices is not enough to guarantee a unique model for the received signal. This lack of inherent uniqueness is one of the limitations of a 2-D modeling for the received signal in wireless communication systems.

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Unlike the decompositions of 2-D arrays (matrices), which is generally nonunique for any rank greater than one (for rank one it is unique up to a scalar factor), the decomposition of 3-D arrays (also called third-order *tensors*) can be unique up to a scalar factor for low-enough ranks [4]. One of the most studied low-rank decompositions of 3-D (or higher dimensional) tensors is called PARAFAC (parallel factor) analysis, which was developed by Caroll and Chang [5] and Harshman [6] as a data analysis tool in psychometrics. It has also been widely studied in the context of chemometrics [7]. In the context of wireless communications, PARAFAC has recently appeared as a powerful tool from a receiver signal processing perspective, allowing us to identify channel parameters and to recover user symbols without imperatively utilizing structural properties/constraints. It is also worth noting that a 3-D (tensorial) model for the received signal results from an additional "axis"or dimension in the received signal model instead of the usually considered *space* and *time* dimensions. In wireless communications, this means that diversity can also be exploited in this additional third dimension. Most of research bringing PARAFAC to the context of signal processing for wireless communications were carried out by Sidiropoulos and his co-workers (see [8] and references therein).

In this work, a new approach to the problem of blind multiuser equalization of single-input multiple-output (SIMO) wireless communication systems is introduced. The proposed approach is based on the fact that the received signal can be interpreted as a three-way array or *tensor*, when a receiver antenna array is used together with oversampling. We first show that the received signal can alternatively be represented as a tridimensional (3-D) PARAFAC model, the 3 dimensions being *space*, *time* and *oversampling* dimensions. After presenting the model, we propose a new blind multiuser equalization receiver combining PARAFAC and Subspace decomposition approaches. We consider a single-input multiple-output (SIMO) wireless communication system employing a receiver antenna array together with oversampling. The key difference of the proposed approach compared to other existing ones, is on the fact that the proposed blind multiuser receiver exploits the tensor structure of the received signal instead of treating it as a matrix. The PARAFAC receiver is divided into two processing stages. In the first one, co-channel user signals are separated in the tensorial domain using an alternating least-squares algorithm. In the second stage, a subspace method is used to independently equalize each user sequence. Simulation results are provided to illustrate the performance of the proposed receiver with that of classical ones. Our results also show that the PARAFAC receiver performs closely to the classical MMSE (*Minimum Mean Square Error*) and ZF (*Zero Forcing*) receivers.

This paper is organized as follows. In Section II, some background on the PARAFAC decomposition is given. Section III is dedicated to the signal modeling, where the proposed PARAFAC model is introduced. In Section IV, the proposed PARAFAC receiver for blind multiuser equalization is formulated. Section V contains our simulation results for performance evaluation. The paper is finalized in Section VI with some conclusions and perspectives.

II. PARALLEL FACTORS (PARAFAC) DECOMPOSITION

For an $I \times J \times K$ third-order tensor $\mathcal{X}$, its Q-component PARAFAC decomposition is given by

$$x_{i,j,k} = \sum_{q=1}^{Q} a_{i,q} b_{j,q} c_{k,q}. \quad (1)$$

The standard PARAFAC model for a three-way (3-D) array expresses the original tensor as a sum of rank-one three-way factors, each one of which being an outer product of three vectors. Using tensor notation the PARAFAC decomposition of $\mathcal{X}$ can be stated as

$$\mathcal{X} = \sum_{q=1}^{Q} \mathbf{a}_q \circ \mathbf{b}_q \circ \mathbf{c}_q, \quad (2)$$

where the operator $\circ$ denotes the outer product. Figure (1) illustrates the PARAFAC decomposition of tensor $\mathcal{X}$. By analogy with the definition of matrix rank, the rank of a third-order tensor is defined as the minimum number of rank-one three-way components needed to decompose $\mathcal{X}$. The fundamental difference when going from matrices to tensors are their uniqueness. While rank-R matrix decompositions are not unique for any $R > 1$, rank-R PARAFAC decompositions are essentially unique for a great range of $R > 1$ [4].

The PARAFAC decomposition can also be represented in matrix notation. Define an $I \times R$ matrix $\mathbf{A}$, $J \times R$ matrix $\mathbf{B}$ and $K \times R$ matrix $\mathbf{C}$. Define also a set of $J \times K$ matrices $\mathbf{X}_{i..}$, $i = 1, \cdots, I$, a set of $K \times I$ matrices $\mathbf{X}_{.j.}$, $j = 1, \cdots, J$ and a set of $I \times J$ matrices $\mathbf{X}_{..k}$, $k = 1, \cdots, K$. Based on these definitions, the model (1) can be written in three different ways. For each writing of the model a system of simultaneous matrix equations exists. The three writings of the model are:

$$\begin{aligned} \mathbf{X}_{i..} &= \mathbf{C}D_i[\mathbf{A}]\mathbf{B}^T \quad i = 1, \cdots, I, & (3) \\ \mathbf{X}_{.j.} &= \mathbf{B}D_j[\mathbf{C}]\mathbf{A}^T \quad j = 1, \cdots, J, & (4) \\ \mathbf{X}_{..k} &= \mathbf{A}D_k[\mathbf{B}]\mathbf{C}^T \quad k = 1, \cdots, K, & (5) \end{aligned}$$

where the operator $D_i[\mathbf{A}]$ forms a diagonal matrix formed from the i-th row of $\mathbf{A}$. The matrices $\mathbf{X}_{i..}$, $i = 1, \cdots, I$, $\mathbf{X}_{.j.}$, $j = 1, \cdots, J$, and $\mathbf{X}_{..k}$, $k = 1, \cdots, K$ can be interpreted as slices of the tensor along the first, second and third dimensions, respectively. Stacking the matrix slices $\mathbf{X}_{..k}$, $k = 1, \cdots, K$ into

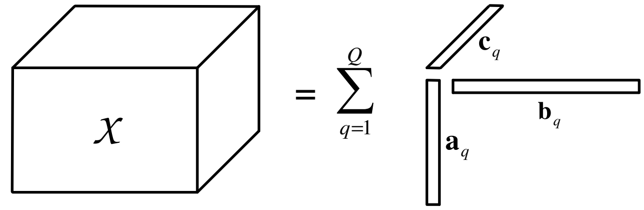

Fig. 1. Q-factor PARAFAC decomposition of a 3-D tensor.

a $IJ \times K$ matrix we have

$$\mathbf{X}_1 = \begin{bmatrix} \mathbf{X}_{..1} \\ \vdots \\ \mathbf{X}_{..K} \end{bmatrix} = \begin{bmatrix} \mathbf{A}D_1[\mathbf{B}] \\ \vdots \\ \mathbf{A}D_K[\mathbf{B}] \end{bmatrix} \mathbf{C}^T = (\mathbf{B} \diamond \mathbf{A})\mathbf{C}^T. \quad (6)$$

Similarly, stacking the matrix slices $\mathbf{X}_{.j.}$, $j = 1, \cdots, J$, into a $JK \times I$ matrix we have

$$\mathbf{X}_2 = \begin{bmatrix} \mathbf{X}_{.1.} \\ \vdots \\ \mathbf{X}_{.J.} \end{bmatrix} = \begin{bmatrix} \mathbf{B}D_1[\mathbf{C}] \\ \vdots \\ \mathbf{B}D_J[\mathbf{C}] \end{bmatrix} \mathbf{A}^T = (\mathbf{C} \diamond \mathbf{B})\mathbf{A}^T. \quad (7)$$

Finally, stacking the matrix slices $\mathbf{X}_{i..}$, $i = 1, \cdots, I$, into a $KI \times J$ matrix

$$\mathbf{X}_3 = \begin{bmatrix} \mathbf{X}_{1..} \\ \vdots \\ \mathbf{X}_{I..} \end{bmatrix} = \begin{bmatrix} \mathbf{C}D_1[\mathbf{A}] \\ \vdots \\ \mathbf{C}D_K[\mathbf{A}] \end{bmatrix} \mathbf{B}^T = (\mathbf{A} \diamond \mathbf{C})\mathbf{B}^T, \quad (8)$$

where $\diamond$ is the Khatri-Rao (columnwise Kronecker) product. Uniqueness of the PARAFAC decomposition was studied by Harshman [6] and the proof was provided by Kruskal [4]. According to Kruskal, a trilinear PARAFAC decomposition over $\mathbb{R}$ is unique, except for the trivial permutation and scaling ambiguity. The uniqueness theorem is now revisited. Consider a set of I matrices $\mathbf{X}_{i..} = \mathbf{B}\mathbf{D}_i[\mathbf{A}]\mathbf{C}^T$ $i = 1, \cdots, I$, where $\mathbf{A} \in \mathbb{R}^{I \times R}$, $\mathbf{B} \in \mathbb{R}^{J \times R}$ and $\mathbf{C} \in \mathbb{R}^{K \times R}$. If

$$k_{\mathbf{A}} + k_{\mathbf{B}} + k_{\mathbf{C}} \geq 2(R+1), \quad (9)$$

the matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are unique, up to common permutation and scaling of columns. This means that, any matrices $\overline{\mathbf{A}}$, $\overline{\mathbf{B}}$ and $\overline{\mathbf{C}}$ satisfying the model $\mathbf{X}_{i..}$, $i = 1, \cdots, I$, are linked to $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ by

$$\overline{\mathbf{A}} = \mathbf{A}\Pi\Delta_1, \;\; \overline{\mathbf{B}} = \mathbf{B}\Pi\Delta_2, \;\; \overline{\mathbf{C}} = \mathbf{C}\Pi\Delta_3, \quad (10)$$

where Π is a permutation matrix and Δ_1, Δ_2 and Δ_3 are diagonal matrices satisfying the condition

$$\Delta_1\Delta_2\Delta_3 = \mathbf{I}. \quad (11)$$

III. SIGNAL MODELING

Let us consider a linear and uniformly-spaced array of M antennas receiving signals from Q co-channel users. Assume that the signal transmitted by each co-channel user is subject to frequency-selective multipath propagation and arrives at the receiver via L specular paths. The length of the channel impulse response is K symbols long. At the output of each receiver antenna, the signal is sampled at a rate that is P

times the symbol rate. Due to temporal oversampling, the resolution of the pulse-shaping filter response is increased by a factor P. Such an increase in the temporal resolution is interpreted here as an addition of a third axis (or dimension) to the received signal, called here the *oversampling dimension*. Let us organize the P oversamples of the signal received at the m-th antenna at the n-th symbol period in a vector $\mathbf{x}_m(n) = [x_m(n)x_m(n+1/P)\cdots x_m(n+(P-1)/P)]^T \in \mathbb{C}^P$. Its discrete-time baseband representation in absence of noise can be factored as

$$\mathbf{x}_m(n) = \sum_{q=1}^{Q}\sum_{l=1}^{L} b_{lq} a_m(\theta_{lq}) \sum_{k=0}^{K-1} \mathbf{g}(k-\tau_{lq}) s_q(n-k), \qquad (12)$$

b_{lq} is the fading envelope of the l-th path of the q-th user, $a_m(\theta_{lq})$ is the phase response of the m-th antenna-element to the l-th path of the q-th user, θ_{lq} being the associated direction of arrival. Similarly, τ_{lq} denotes the propagation delay (in multiples of the symbol period T) and

$$\mathbf{g}(k-\tau_{lq}) = \begin{bmatrix} g(k-\tau_{lq}) \\ g(k-\tau_{lq}+1/P) \\ \vdots \\ g(k-\tau_{lq}+(P-1)/P) \end{bmatrix} \qquad (13)$$

represents the k-th component of the oversampled pulse-shaping filter response evaluated at delay τ_{lq}. The channel length K is such that $K \geq max(\tau_{lq})$. This condition guarantees that all multipath energy is captured in our frequency-selective channel impulse response model. Finally, $s_q(n)$ is the information symbol transmitted by the q-th user at the n-th time symbol period. Depending on the type of signal processing used at the receiver, we may utilize either the above parametric channel model, with explicit description of angles and delays (narrowband assumption), or a non-parametric one, when we are not interested in characterizing angle and delay parameters of the channel. In this work we focus on the parametric model, which means that all the multipath parameters of all users are captured in our tensor models. Define

$$\mathbf{a}_{l,q} = [a_1(\theta_{lq})\, a_2(\theta_{lq}) \cdots a_M(\theta_{lq})]^T \quad \in \mathbb{C}^M \qquad (14)$$

and

$$\mathbf{G}_{l,q} = [\mathbf{g}(0-\tau_{lq}) \cdots \mathbf{g}(K-1-\tau_{lq})] \quad \in \mathbb{C}^{P\times K} \qquad (15)$$

as the spatial and temporal responses of the channel to the l-th multipath of the q-th user, $l = 1,\ldots,L$, $q = 1,\ldots,Q$. In order to rewrite (12) in a more compact form, let us concatenate the LQ spatial and temporal responses into equivalent matrices $\mathbf{A} = [\mathbf{a}_{1,1}\cdots\mathbf{a}_{l,q}\cdots\mathbf{a}_{L,Q}] \in \mathbb{C}^{M\times LQ}$ and $\mathbf{G} = [\mathbf{G}_{1,1}\cdots\mathbf{G}_{l,q}\cdots\mathbf{G}_{L,Q}] \in \mathbb{C}^{P\times KLQ}$, and define $\mathbf{b} = [b_{11}\cdots b_{lq}\cdots b_{LQ}] \in \mathbb{C}^{LQ}$ as a vector of multipath gains. Define also the overall channel impulse response matrix $\mathbf{H} \in \mathbb{C}^{P\times KLQ}$ as

$$\mathbf{H} = \mathbf{G}(diag(\mathbf{b}) \otimes \mathbf{I}_K) \quad \in \mathbb{C}^{P\times KLQ}, \qquad (16)$$

which is nothing but the temporal response matrix scaled by the complex multipath gains. The operator $diag(\cdot)$ forms a diagonal matrix out of its vector argument. Considering that a block of N transmitted symbols is processed at the receiver,

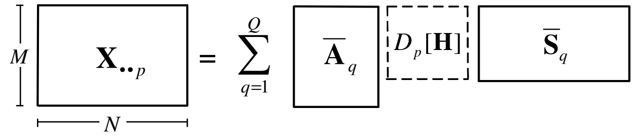

Fig. 2. Proposed PARAFAC decomposition for the 3-D received signal tensor.

we define $\mathbf{S} = [\mathbf{S}_1^T \cdots \mathbf{S}_Q^T]^T \in \mathbb{C}^{KQ\times N}$ a block-Toeplitz matrix concatenating Q Toeplitz symbol matrices, each one of which having its first row and column equal to $\mathbf{s}_q^{(r)} = [s_q(1)\, s_q(2) \cdots s_q(N)]$ and $\mathbf{s}_q^{(c)} = [s_q(1)\, 0 \cdots 0]^T$, respectively. In absence of noise, the received signal is a 3-D tensor $\mathcal{X} \in C^{M\times N\times P}$ that can be expressed as a set of $M\times N$ space-time slices $\mathbf{X}_{..p}$, each one of which admitting the following factorization:

$$\mathbf{X}_{..p} = (\mathbf{A}\Psi) D_p(\mathbf{H}) (\Phi\mathbf{S}), \quad p = 1,\cdots,P, \qquad (17)$$

where

$$\Psi \;=\; \mathbf{I}_{LQ} \otimes \mathbf{1}_K^T \quad \in \mathbb{C}^{LQ\times KLQ}, \qquad (18)$$

$$\Phi \;=\; \mathbf{I}_Q \otimes \mathbf{1}_L \otimes \mathbf{I}_K \quad \in \mathbb{C}^{KLQ\times KQ}, \qquad (19)$$

are constraint matrices, composed of 1's and 0's. The term $\mathbf{1}_K$ being a "all ones"column vector of dimension $K\times 1$ and the operator $\otimes$ defines the Kronecker product. The operator $D_p(\mathbf{H})$ takes the p-th row of its matrix argument and forms a diagonal matrix out of it. Note that (17) follows a tridimensional (3-D) PARAFAC model. With respect to the PARAFAC decomposition in Section II, Equation (17) can be interpreted as the p-th matrix slice of a (M,N,P)-dimensional tensor $\mathcal{X}$. According to (17), the received tensor is completely characterized by a set of three matrix components $\mathbf{A}\Psi$, $\mathbf{H}$ and $\Phi\mathbf{S}$. This tensor model is a PARAFAC model having a constrained structure, the constraints being given by matrices Ψ and Φ. According to (3), the received signal tensor can also be expressed as a set of $P\times M$ matrix slices $\mathbf{X}_{.n.} = \mathbf{H}D_n\left((\Phi\mathbf{S})^T\right)(\mathbf{A}\Psi)^T$, $n = 1,\ldots,N$ or as a set of $N\times P$ matrix slices $\mathbf{X}_{m..} = (\Phi\mathbf{S})^T D_m(\mathbf{A}\Psi)\mathbf{H}^T$, $m = 1,\ldots,M$. The three unfolded matrices $\mathbf{X}_{i=1,2,3}$, containing the full tensor information, are defined as $\mathbf{X}_1 = [\mathbf{X}_{..1}^T \cdots \mathbf{X}_{..P}^T]^T \in \mathbb{C}^{MP\times N}$, $\mathbf{X}_2 = [\mathbf{X}_{.1.}^T \cdots \mathbf{X}_{.N.}^T]^T \in \mathbb{C}^{PN\times M}$ and $\mathbf{X}_3 = [\mathbf{X}_{1..}^T \cdots \mathbf{X}_{M..}^T]^T \in \mathbb{C}^{NM\times P}$, respectively. Figure 2 illustrates the PARAFAC decomposition of the 3-D received signal tensor as a Q-user sum of three-way factors, the q-th three-way factor being decomposed in terms of $\overline{\mathbf{A}}_q = \mathbf{A}_q \otimes \mathbf{1}_K^T$, $\mathbf{W}_q$ and $\overline{\mathbf{S}}_q = \mathbf{1}_L \otimes \mathbf{S}_q$.

IV. Combined PARAFAC-Subspace Algorithm

The proposed blind receiver algorithm is divided in two processing stages. In the first stage, co-channel user signals are separated in the 3-D (space $\times$ time $\times$ oversampling) domain using an alternating least squares (ALS) algorithm that is similar in spirit to the one proposed in [7]. In the second stage, the symbol sequences are independently recovered in the time-domain using a single-user equalization algorithm based on subspace decomposition [9]. Our receiver can be thought of as a blind multiuser equalizer where user separation and

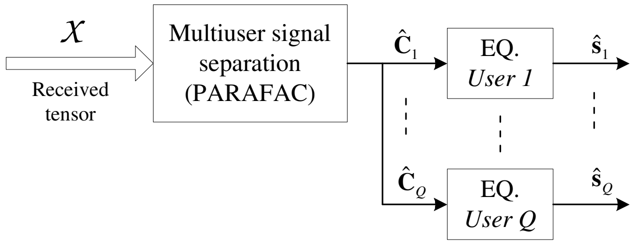

Fig. 3. Block-diagram of the PARAFAC receiver.

equalization are decoupled. Figure 3 depicts the block-diagram of the proposed PARAFAC receiver.

A. User separation stage:

For the received signal tensor $\mathcal{X}$, the user separation stage is represented by the trilinear alternating least squares (ALS) algorithm that consists in estimating three matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$. In the presence of additive Gaussian noise, these matrices optimize a maximum likelihood criterion formulated as a set of three independent nonlinear least squares minimization problems:

$$\begin{aligned}\widehat{\mathbf{C}} &= \underset{\mathbf{C}}{\operatorname{argmin}} \|\mathbf{X}_1 - (\mathbf{B} \diamond \mathbf{A}\Psi)\Phi\mathbf{C}\|^2 \\ \widehat{\mathbf{A}} &= \underset{\mathbf{A}}{\operatorname{argmin}} \|\mathbf{X}_2 - ((\Phi\mathbf{C})^T \diamond \mathbf{B})(\mathbf{A}\Psi)^T\|^2 \\ \widehat{\mathbf{B}} &= \underset{\mathbf{B}}{\operatorname{argmin}} \|\mathbf{X}_3 - (\mathbf{A}\Psi \diamond (\Phi\mathbf{C})^T)\mathbf{B}^T\|^2\end{aligned} \quad (20)$$

where $\mathbf{X}_1$ $(MP \times N)$, $\mathbf{X}_2$ $(PN \times M)$ and $\mathbf{X}_3$ $(NM \times P)$ are matrices formed from the slices of the received signal tensor along the first, second and third dimensions, respectively. Disregarding scaling and permutation ambiguities, these matrices are linked to the matrices $\widehat{\mathbf{H}}$, $\widehat{\mathbf{G}}$ and $\widehat{\mathbf{S}}$ in the following way:

$$\widehat{\mathbf{H}} = \widehat{\mathbf{A}}, \;\; \widehat{\mathbf{G}} = \widehat{\mathbf{B}}\mathbf{T}, \;\; \widehat{\mathbf{S}} = \mathbf{T}^{-1}\widehat{\mathbf{C}}, \quad (21)$$

where $\mathbf{T}$ is a $KQ \times KQ$ square ambiguity matrix that will be found in the second stage of the receiver. The i-th iteration of the ALS algorithm consists of three steps: 1) update $\widehat{\mathbf{C}}_i$ conditioned on $\widehat{\mathbf{A}}_{i-1}$ and $\widehat{\mathbf{B}}_{i-1}$; 2) update $\widehat{\mathbf{A}}_i$ conditioned on $\widehat{\mathbf{B}}_{i-1}$ and $\widehat{\mathbf{C}}_i$; 3) update $\widehat{\mathbf{B}}_i$ conditioned on $\widehat{\mathbf{A}}_i$ and $\widehat{\mathbf{C}}_i$. These three updating steps are repeated until convergence of the algorithm. Several initialization strategies exist. Here, we initialize $\widehat{\mathbf{A}}_0 = \mathbf{A} + \delta\mathbf{E}_a$ and $\widehat{\mathbf{B}}_0 = \mathbf{B} + \delta\mathbf{E}_b$, where $\mathbf{E}_a$ and $\mathbf{E}_b$ are matrices whose entries are randomly generated from a normal distribution with $\delta = 0.01$. In practice, a good initial guess for $\widehat{\mathbf{A}}_0$ and $\widehat{\mathbf{B}}_0$ can be obtained, for example, from prior information (or imprecise knowledge) about the spatial geometry of the receiver antenna array as well as from knowledge about the temporal structure of the pulse-shaping filters. If no prior knowledge is available, initialization can be done from the matrix slices of the received signal via generalized eigenvalue decomposition methods. Other more sophisticated initialization strategies exist but they are beyond the scope of this work.

B. Equalization stage:

At the end of the first stage of the receiver, we are left with matrices $\widehat{\mathbf{A}} = \widehat{\mathbf{H}}$, $\widehat{\mathbf{B}} = \widehat{\mathbf{G}}\mathbf{T}^{-1}$ and $\widehat{\mathbf{C}} = \mathbf{T}\widehat{\mathbf{S}}$. In order to estimate the transmitted sequences we must determine the ambiguity matrix $\mathbf{T}$. According to (21), the transmitted sequences can be recovered by solving $\widehat{\mathbf{C}} = \mathbf{T}\widehat{\mathbf{S}}$, i.e.,

$$\begin{bmatrix} \widehat{\mathbf{C}}_1 \\ \vdots \\ \widehat{\mathbf{C}}_Q \end{bmatrix} = \begin{bmatrix} \mathbf{T}_1 & & \\ & \ddots & \\ & & \mathbf{T}_Q \end{bmatrix} \begin{bmatrix} \widehat{\mathbf{S}}_1 \\ \vdots \\ \widehat{\mathbf{S}}_Q \end{bmatrix}. \quad (22)$$

The ambiguity matrix $\mathbf{T}$ is block-diagonal, i.e. $\mathbf{T} = BlockDiag(\mathbf{T}_1 \cdots \mathbf{T}_Q)$, which means that users' symbol sequences can be independently recovered by solving a set of smaller system of equations $\widehat{\mathbf{C}}_q = \mathbf{T}_q\widehat{\mathbf{S}}_q$, $q = 1,\ldots,Q$, each one of which being a single-input multiple-output (SIMO) blind equalization problem. In other words, the PARAFAC approach decouples a multiuser equalization problem into Q equivalent single-user equalization problems. The $K \times K$ square (non-singular) ambiguity matrix $\mathbf{T}_q$, $q = 1,\ldots,Q$, can be interpreted as an equivalent SIMO channel with K impulse responses of length K each. Several strategies exist that can be used to blindly estimate $\mathbf{T}_q$ and $\mathbf{S}_q$, $q = 1,\ldots,Q$. Here we use the subspace algorithm proposed by Moulines et al. [9], in which $\mathbf{T}_q$ is found by minimizing the following quadratic cost function

$$\mathbf{t}_q = \underset{\mathbf{t}_q}{\operatorname{argmin}}\; \mathbf{t}_q^H \mathcal{F}(\mathbf{U}_q)\mathcal{F}(\mathbf{U}_q)^H \mathbf{t}_q \quad (23)$$

under the constraint $\|\mathbf{t}_q\| = 1$, for each $q = 1,\ldots,Q$, where $\mathbf{t}_q = \text{vec}(\mathbf{T}_q)$ and $\mathcal{F}(\mathbf{U}_q)$ is a block-Toeplitz matrix formed from a basis of the noise subspace $\mathbf{U}_q$ (associated with the smallest left singular vectors) of a convolution matrix formed from $\mathbf{C}_q$. For reasons of space, we report the interested reader to [9] for further details. After estimation of $\mathbf{T}_q$, we calculate $\widehat{\mathbf{S}}_q = \mathbf{T}_q^{-1}\widehat{\mathbf{C}}_q$ for each $q = 1,\ldots,Q$.

V. Simulation Results

The average bit-error-rate (BER) performance of the proposed blind PARAFAC multiuser equalization receiver has been evaluated through computer simulations, considering $Q = 2$ co-channels users. The signal transmitted by each user arrives at the receiver via $L = 2$ independent Rayleigh-faded multipaths. The multipaths of the first user are parameterized by $[\theta_{11}, \theta_{21}] = [0^\circ, 30^\circ]$, $[\tau_{11}, \tau_{21}] = [0, T]$ and $[\beta_{11}, \beta_{21}] = [1, 1]$, while those of the second user are parameterized by $[\theta_{12}, \theta_{22}] = [-20^\circ, 50^\circ]$, $[\tau_{12}, \tau_{22}] = [0, T]$ and $[\beta_{12}, \beta_{22}] = [1, 1]$. The length of the channel impulse response is $K = 2$ symbols. For all the simulations and receivers, the number of receiver antennas is $M = 2$, the oversampling factor is $P = 2$ and the number of received binary-phase shift keying (BPSK) symbols processed is $N = 50$. BER results are plotted as a function of the signal-to-noise ratio (SNR) per receiver antenna and are drawn from 1000 independent Monte Carlo runs.

In Figure 4, we compare the results of the blind PARAFAC receiver with those of previously proposed blind subspace-based receivers. For the multiuser case, the blind PARAFAC receiver is compared with the blind subspace algorithm proposed by Van der Veen et al. [3], which

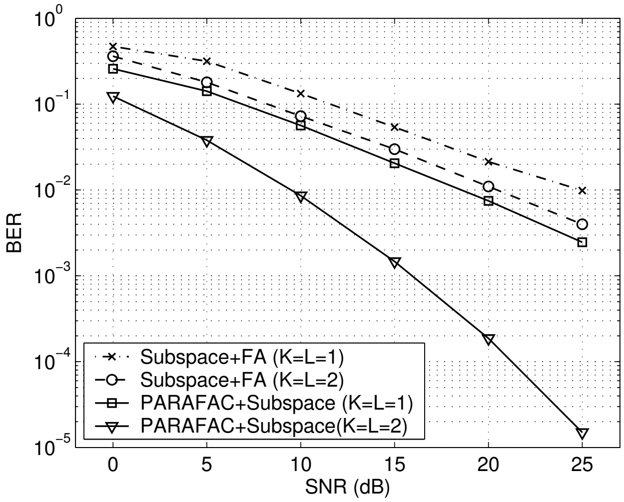

Fig. 4. Performance of the PARAFAC receiver, compared with the blind subspace receiver of [3]. $M = 2$, $P = 2$ and $N = 50$.

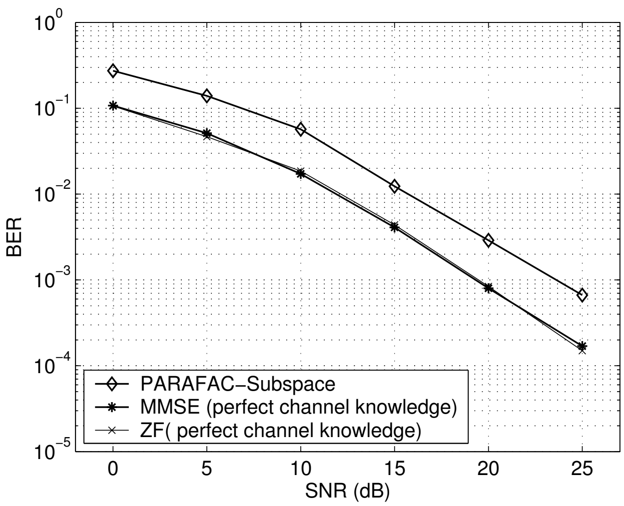

Fig. 5. BER versus SNR. $L = 2$, $K = 2$, $(M,P) = (2,2)$.

combines a subspace method with the exploitation of the Finite-Alphabet (FA) property of the transmitted symbols for multiuser space-time equalization. This receiver is called "Subspace+FA"in the figure. It can be seen that the proposed PARAFAC receiver outperforms the Subspace+FA receiver in the multiuser frequency-selective scenario with $L = 2$ and $K = 2$. For the a single-path flat fading scenario (K=L=1) scenario (only the first multipath of each user is present), the performance improvement of the proposed PARAFAC receiver over the Subspace+FA one is also verified (5dB for 10^{-2} target BER). As a reference for comparison, the performance of the PARAFAC receiver is also compared with those of the Zero Forcing (ZF) and Minimum Mean Square Error (MMSE) receivers. For both the ZF and MMSE receivers, perfect knowledge of all multipath parameters is assumed. We assume $(M,P) = (2,2)$ for all the receivers. Figure 5 shows that the iterative PARAFAC-subspace receiver is close to the ZF/MMSE one, with a performance gap of only 3 dB approximately.

Discussion

Some interesting similarities and differences between the Subspace+FA algorithm of [3] and the PARAFAC-Subspace algorithm exist. Note that both the Subspace+FA algorithm and the PARAFAC-Subspace algorithm perform multiuser signal separation and equalization in two processing stages. The Toeplitz structure of the users' symbol matrices are also exploited in both cases, in the blind equalization stage. However, in our PARAFAC receiver multiuser signal separation is done prior to equalization while in Subspace+FA receiver of [3], equalization precedes multiuser signal separation. In the latter case, multiuser signal separation is only made possible due to the exploitation of the Finite Alphabet (FA) property of symbols. Note, however, that our PARAFAC receiver does not require the FA property as a necessary condition for the separation of users' signals, which is done by exploiting the tensor structure of the received signal instead. To conclude, the fundamental difference between the proposed PARAFAC approach and the other existing ones is that *space* and *oversampling* dimensions are treated as two different dimensions, resulting in a 3-D structure for the received signal, allowing one to benefit from PARAFAC uniqueness for information recovering.

VI. Conclusions and Perspectives

In this work, we have presented a tensor approach for modeling the received signal in wireless communications systems employing receive antenna arrays and oversampling. The PARAFAC approach models the received signal as a 3-D tensor, the dimensions being *space*, *time* and *oversampling* dimensions. Based on this modeling approach, a new blind multiuser equalization receiver combining PARAFAC and subspace decomposition methods was proposed for multiuser signal separation and equalization. The proposed PARAFAC-Subspace receiver operates in a two-stage approach, performing multiuser signal separation in the tensorial domain, followed by an equalization stage in the temporal domain. The performance of the proposed PARAFAC receiver has been evaluated through computer simulation. The results have shown that the combined PARAFAC-Subspace approach performs closely to the MMSE (*Minimum Mean Square Error*) and ZF (*Zero Forcing*) receivers. Future directions of this work include a deeper study on the identifiability conditions of the proposed PARAFAC tensor model as well as the exploitation of the Finite Alphabet (FA) property of symbols within the multiuser signal separation stage of the proposed PARAFAC receiver.

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