Quaternary LDPC Codes on OFDM Systems

Maria Leopoldina M. N. S. Gonçalves and Renato Baldini Filho

Abstract— **This paper evaluates the performance of quaternary LDPC codes associated to OFDM systems. The LDPC codes are based on a ring Z_4 of integers modulo-4. Each OFDM subcarrier is modulated using a 4-PSK modulation, and each output symbol of the encoder is mapped onto that modulation. The performance is evaluated, in terms of *BER* versus E_b/N_0, for AWGN and frequency-selective fading channels. The results show good performance when compared to the channel encoding process of the Brazilian Digital Television System (SBTVD).**

Index Terms—**Quaternary LDPC code, OFDM, frequency-selective fading.**

I. Introduction

Broadband communications based on multiple carriers have attracted enormous amount of attention in recent years.

High data rate transmission standards, such as DVB (Digital Video Broadcasting), ISDB (Integrated Service Digital Broadcasting), Wi-Fi (Wireless Fidelity) and Wi-MAX (Worldwide Interoperability for Microwave Access), use Orthogonal Frequency Division Multiplexing (OFDM) associated to an error-correcting coding scheme to minimize the harmful effects introduced by the wireless channel [1].

Digital TV usually utilizes a concatenated coding scheme based on a Reed-Solomon code (outer coder) with a variable-rate binary convolutional code (inner code). It seems evident that the error-correcting capacity of the inner code can be improved using a more powerful coding scheme, such as LDPC (low density parity check) or turbo codes.

This paper proposes the use of quaternary LDPC codes as inner code in the Brazilian Digital TV System (SBTVD) The analysis is made based on LDPC codes defined over a ring Z_4 of integers modulo-4, associated to an OFDM system with QPSK-modulated subcarriers. The performance is evaluated, in terms of *BER* versus E_b/N_0, for AWGN and frequency-selective fading channels. The results show good performance improvements when compared to the channel encoding process of the Brazilian Digital Television System (SBTVD).

Non-binary encoding schemes applied to OFDM systems may bring some advantages over equivalent binary schemes, such as, a perfect match of the q-PSK modulation to the symbols of the ring Z_q. Moreover, codes over Z_q can be easily made invariant to phase rotation of the carrier.

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II. OFDM System

An OFDM system transforms a high data-rate stream into N low data-rate sub-streams, which are transmitted by N complex subcarriers spaced by

$$\Delta f = R_{mc} = \frac{R_s}{N} \tag{1}$$

where R_{mc} is the symbol rate for each subcarrier and R_s is the overall symbol rate. The transmitted OFDM symbol can be represented as [2]

$$s(t) = \frac{1}{2N}\sum_{n=0}^{N-1}\left[i_n \cos(\omega_n t) - q_n \sin(\omega_n t)\right] \tag{2}$$

where ω_n is the angular frequency of the n^{th} subcarrier and i_n and q_n are the in-phase and quadrature components of a modulated symbol. Notice that $s(t)$ can be seen as the inverse discrete Fourier transform (IFFT) of $c_n = i_n + jq_n$. The sampled baseband OFDM symbol is therefore given by

$$s_m = s(mT_s) = \frac{1}{2N}\sum_{n=0}^{N-1} c_n \exp\left(j\frac{2\pi n}{N}m\right), \tag{3}$$

where T_s is the time interval between adjacent samples of the OFDM symbol and m is the time index of the samples. The passband OFDM symbol is given by

$$s(t) = \frac{1}{2N}\sum_{n=0}^{N-1}\Re\left[c_n \exp(j\omega_n t)\exp(j\omega_r t)\right], \tag{4}$$

where ω_r is the angular frequency of the carrier and $\Re[.]$ means the “real part of”. Usually a guard-time interval is introduced at the beginning of each OFDM symbol to improve the performance on multipath fading channel. A small section of the tail of the OFDM symbol is reproduced at its beginning. This procedure is named cyclic prefix insertion. Therefore, if the duration of the cyclic prefix is ΔT seconds, then the overall data rate is given by

$$R_s = \frac{N}{T + \Delta T}, \tag{5}$$

where T is the period of the OFDM symbol with no insertion of the cyclic prefix. The received signal is translated to baseband and sampled at a sampling rate R_s, resulting in the sampled received signal r_k given by

$$r_k = \frac{1}{2N}\sum_{n=0}^{N-1}\left[c_n \exp(j\omega_n k) * h_k + w_k\right], \tag{6}$$

where h_k is the channel impulse response and w_k is a sample of the complex AWGN. Applying discrete Fourier transform (FFT) in (6) results in the received symbols

$$d_l = \frac{1}{N}\sum_{k=0}^{N-1}\sum_{n=0}^{N-1}\left[c_n \exp\left(j\frac{2\pi n}{N}k\right) * h_k + w_k\right]\exp\left(j\frac{2\pi l}{N}k\right), \tag{7}$$

where $l = 0, 1, \ldots, N-1$.

III. LDPC CODES OVER Z_4

Low-density parity-check (LDPC) codes allows transmission at rates close to Shannon's limit. LDPC codes were introduced by Gallager in 1962, but the lack of powerful processors at the time of invention made them unfeasible for decades [3].

Regular and irregular (n, k) LDPC codes are defined by the parity-check matrix H. The method used to build LDPC codes over Z_4 follows basically the procedure devised by Mackay and Neal [4].

A matrix H is generated from an all-zero $(n-k)\times n$ matrix where w_c non-zero elements from Z_4 are randomly placed on each column. Then the number w_r of non-zero entries in each row is uniformed. The matrix H is generated maximising the code girth. The LDPC codes over Z_4 obtained have length N equal to 204, 504 and 1008 symbols.

IV. DECODING LDPC CODES OVER Z_4

LDPC codes have low density of non-null entries in the H matrix. The complexity of the decoding process is related to this density, therefore a small number of branches between nodes of the bipartite graph allows a simple iterative decoding process. The iterative decoding is performed exchanging information among the symbol nodes and the check nodes through the branches provided by the Tanner graph [4][5][6].

The soft-input soft-output Euclidean distance metric iterative decoder proposed by Farrell and Moreira [7] fit perfectly the decoding process for quaternary LDPC codes. This algorithm uses Euclidean distances in the iterative decoding process which reduces the underflow and overflow effects. Another advantage of this decoding process is that there is no need to know the a priori probabilities of the transmitted symbols as is necessary to the belief propagation algorithm. The Farrell and Moreira algorithm uses only additions and comparisons associated to correction factors provided by look-up tables.

V. RESULTS

A. Channel Models

Three main interferences act on the transmitted signal in a wireless propagation system: path loss, large scale fading generated by shadowing, and low scale fading caused by multiple propagation paths. Signal attenuation and shadowing can be minimized by equalization but the multipath fading is characterized by the fast amplitude fluctuations of the received signal. Together with those interferences the additive white Gaussian noise (AWGN) is always present. The channel models considered for simulation are AWGN and time-varying frequency-selective fading.

B. Performance of Quaternary LDPC Code in a OFDM System

Three distinct regular 1/2-rate LDPC codes are evaluated. The parity check matrixes H have column weight w_c equal to 3 and row weight w_r equal to 6. The number of iterations considered in the simulations is 3. More than 3 iterations do not improve significantly the performance. Moreover, a higher number of iterations increases exponentially the simulation time. The OFDM signal size (N) follows the length of the Z_4-LDPC codes. The symbol synchronization is assumed ideal. The performance of the codes is evaluated in terms of bit-error rate (*BER*) versus energy per bit to unilateral noise power spectral density (E_b/N_0).

Fig. 1 shows the *BER* performance of the quaternary LDPC codes with N = 204, 504 and 1008, in an OFDM system, on AWGN channel. The binary convolutional code used as inner code in the SBTVD system is used as reference. The performance improves with the increase of the code length as expected.

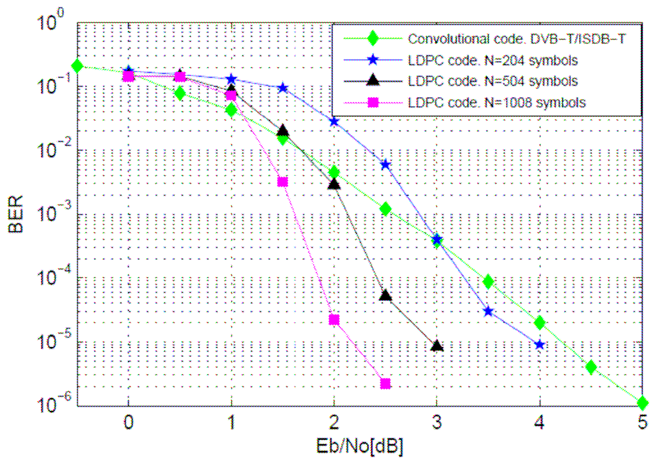

Figure 1: Performance of quaternary LDPC codes on AWGN channel.

Fig. 2 presents a performance comparison of the quaternary LDPC code with N = 1008 in an OFDM system, on a frequency-selective fading channel for 16, 32, 64 and 128 consecutive subcarriers severely attenuated. Notice that the LDPC code presents a very robust performance behavior on frequency-selective fading, i. e. the *BER* degradation is below 0.5 dB for 16 subcarriers and below 1 dB for 32 subcarriers affected in comparison to AWGN performance.

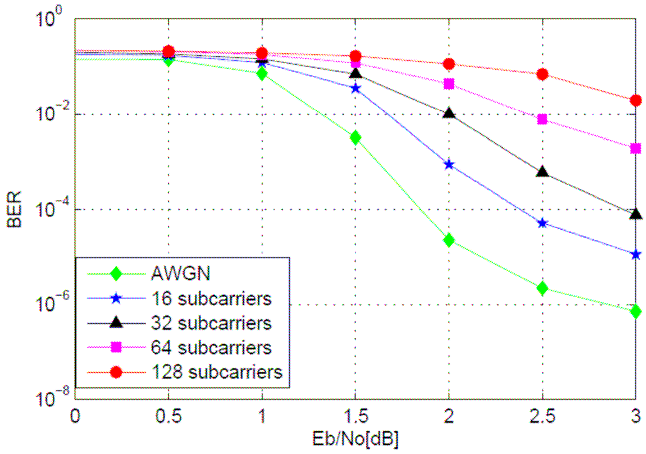

Figure 2: Performance of quaternary LDPC codes on frequency-selective fading channel.

Finally, Fig. 3 shows a performance comparison of the quaternary LDPC code with N = 1008 symbols, a 1/2-rate quaternary turbo code with N = 2048 symbols and 3 decoding iterations [8], a binary (4896, 2448) LDPC code with 50 decoding iterations [9] and the 1/2-rate binary convolutional code used as inner code in the terrestrial DVB and ISDB standards. The channel is AWGN. Notice that the binary LDPC code performs 0.75 dB better than the proposed quaternary LDPC code at $BER = 10^{-5}$. However, the length and the number of iterations for the binary LDPC code are greater than for the quaternary LDPC code. At same BER, the quaternary LDPC code has almost the same performance of the quaternary turbo code, despite of its lower length.

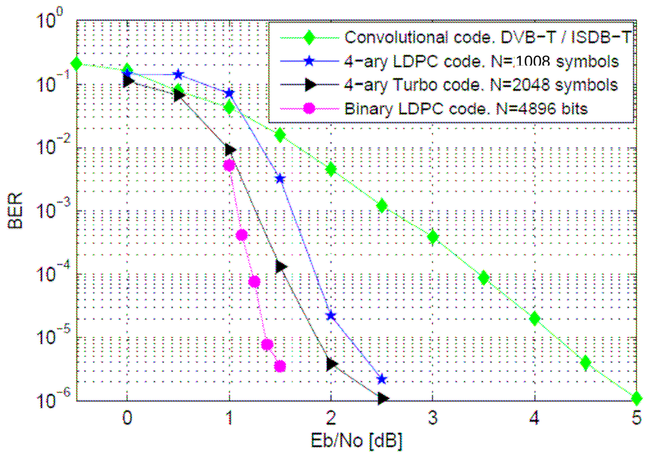

Figure 3: Comparison among quaternary and binary LDPC, quaternary turbo and binary convolutional codes suitable for DVB-T and ISDB-T (SBTVD).

VI. CONCLUSION

Quaternary LDPC codes show better performance than the binary convolutional code used as inner code in the SBDTV, and present slightly worse performance than the binary LDPC code. However, this comparison is unfair since the design parameters of the quaternary LDPC code are smaller than those used to binary LDPC codes. The quaternary LDPC codes have also shown similar performance of the quaternary turbo codes. All codes are analyzed in an OFDM system. It is clear that further improvement can be obtained if a (204, 188, 21) Reed-Solomon code is concatenated to those codes.

Quaternary LDPC codes defined over a ring Z_4 have shown to be a good alternative as inner code for the Brazilian Digital TV system (SBTVD) because of their good BER performance in typical terrestrial transmission scenarios.

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