A Game-Theoretic Approach to the Distributed Power Control Problem in Wireless Systems and the Application of a Simple Prediction Method

Fabiano de S. Chaves, Raimundo A. de Oliveira N. and Francisco R. P. Cavalcanti

***Abstract*— Game theory is a set of mathematical tools suitable to the modeling and optimization of problems involving agents with con icting interests competing for limited resources. It applies well for a number of problems in wireless communication, including power control. In this work, some basic concepts on game theory are presented and a general framework for the decentralized power control problem in wireless systems is constructed. Finally, a new algorithm based on a simple prediction method is developed and compared to the classical distributed power control.**

***Keywords*— Power control, noncooperative game theory, prediction.**

I. INTRODUCTION

THE main objective of a wireless communication system is to provide reliable communication to the users, i.e., the system has the task of to satisfy QoS (Quality of Service) requirements. Such requirements may be assumed to correspond to target signal-to-interference-plus-noise ratios (SINRs) [1]. Then, in power-controlled systems, each transmitter usually tries to provide a determined SINR to its correspondent receiver.

The application of game-theoretic tools to the power control problem in wireless systems is attractive and powerful due to the need of self-optimization and distributiveness. Game theory is a tool for analyzing the interaction of decision makers with conflicting objectives and limited resources. In recent years, it has been applied to problems in wireless communication, mainly to the power control problem [2], [3], [4], [5].

In this work, we construct a general framework to the decentralized power control problem by means of a game-theoretical approach. In Section II, some basics on game theory are presented. Section III brings the mathematical development of a general solution to the problem, which has as special configuration the well-known Distributed Power Control (DPC) algorithm [6]. In Section IV, a simple and efficient prediction method is used to compose an algorithm which is a good alternative to DPC algorithm, since it approximates the general solution better than DPC. In order to compare the performance of both DPC and proposed algorithms, computational simulations in a cellular system environment are performed in Section V. Conclusions of this work are given in Section VI.

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This work is supported by a grant from Ericsson of Brazil - Research Branch.

II. BASICS ON GAME THEORY

Game theory is composed by a set of mathematical concepts dedicated to the study of situations where interdependent players (decision makers) make decisions according to the actions of the other players. The basic unit of game theory is the game, which has three basic elements: a set of players, a set of possible actions for each player, and a set of objective functions mapping action profiles into real numbers.

Players are the elements found in conflict. Each player has decision rights only over its own decision variables. Players are assumed to be rational, which means they decide for the strategy with the best individual game outcome.

Actions or decisions of players are confined to their strategy space, that is, the set of possible actions. Strategies may be pure (deterministic) or mixed (stochastic). In this work, only pure strategy spaces are considered.

The satisfaction of each player is represented by its objective function. This is a special element in a game, since the objective function of a player must correspond to its interest with respect to the optimization process to be carried out. Objective functions can be classified as cost functions or utility functions. The concept of cost function refers to the pay back of the player as a result of its actions. Utility functions take the place of cost functions when one refers to satisfaction measures of players instead of their pay back.

Game theory can be divided in two classes: noncooperative game theory and cooperative game theory. Noncooperative game theory studies situations where players interact, however negotiations (or agreements) are not possible or allowed. In cooperative game theory, such negotiations are allowed. In this work, only concepts of noncooperative game theory are used.

Games can also be classified in static games and dynamic games. In general, a game where players act simultaneously is static. When the order of player's actions is relevant to the outcome of a game, such game is dynamic.

Finally, a game where the gains of a player represent losses to the other players is called a zero-sum game. In zero-sum games, as the name suggests, the sum of the cost functions of all players is identically zero. Even if this sum is equal to a nonzero constant, the game can be treated within the framework of zero-sum games without any loss of generality. However, in other cases, the gains of a player do not correspond to losses to the other players, i.e., the sum of the cost functions of all players is not a constant. Such games are called nonzero-sum games. A detailed discussion about noncooperative games can be found in [7].

III. Distributed Power Control Game

A number of works has been dedicated to the formulation of the power control problem as a game [2], [3], [4]. All of them define the transmitters as the set of players and the (convex) set of possible power values as strategy space. Then, transmit power is the decision variable. Furthermore, only nonzero-sum noncooperative static games are considered.

Differences between such works are found with respect to objective functions. Most previous works relative to the power control problem formulated as a game fall in two classes with respect to the objective functions: those dependent only on intrinsic properties of the channel (SINR, transmit power) [4] and those dependent also on lower layer decisions such as modulation and coding [2], [3].

We formulate the problem as a static multi-stage nonzero-sum noncooperative game, as in [5]. In order to avoid studies conditioned to some system configurations, the adopted objective functions are not dependent on system parameters, such as modulation and coding.

We denote $G_K = [N_j, \{P_j\}, \{c_j\}]$ the static multi-stage nonzero-sum noncooperative power control game with K stages. The adopted strategy based on such kind of game is justified by the fact that users (players) do not cooperate and gains of a user do not correspond to losses of the others users. The game is composed by K stages, where each stage k corresponds to an actuation of the power control algorithm, which is discrete and has k as its discrete time index.

The transmitters constitute the set of players, with $N_j = \{1, 2, ..., N\}$ as their index set; the continuous set of power values $P_j = [p_{min}, p_{max}]$ is the strategy set of player j; and c_j is the cost function of player j, $j \in N_j$. We emphasize that the j^{th} player has control only over its own power p_j, which is selected such that $p_j \in P_j$. The power vector $\mathbf{p}(k) = [p_1(k), ..., p_N(k)] \in P$ is the outcome of the stage k of the game in terms of the selected power levels of all the players, where $P = P_1 \quad\quad P_N$. The vector consisting of the elements of $\mathbf{p}(k)$ other than the j^{th} element is denoted by $\mathbf{p}\ _j(k)$.

As discussed before, one may assume that QoS requirements are represented by target SINRs. Then, the power control task can be seen as the tracking of a target SINR through power decisions. An intuitive cost function which preserve such characteristics is the squared error between the target SINR and the actual SINR. Note that it is imposed a penalty to the player which obtain SINR values far from the target SINR. Thus, the strategy of each player j in a time instant k of the game G_K is defined below:

$$\min_{p_j(k+1)\in P_j} c_j \quad p_j(k+1), \mathbf{p}\ _j(k+1) \quad = \quad | \ _t - \ _j(k+1)|^2, \tag{1}$$

where $_t$ is a fixed target SINR and $_j(k+1)$ is the SINR of player j at the time instant $k+1$, expressed as:

$$_j(k+1) = \frac{p_j(k+1)g_j(k+1)}{I_j(k+1)}, \tag{2}$$

where $g_j(k+1)$ is the channel gain and $I_j(k+1)$ is the interference-plus-noise power perceived by the receiver j, that is:

$$I_j(k+1) = \sum_{i=1}^{N} (p_i(k+1)g_i(k+1)) + \quad ^2, \ i \neq j, \tag{3}$$

where ^{2} is the average AWGN power.

Then, at the time instant k, each player has as objective to determine its own power level at the next time instant in such a manner that the squared error between the target and the actual SINRs is minimized. Note that the channel gain and the interference-plus-noise power have positive and continuous values. Furthermore, the transmit power that optimizes individual cost function depends on the transmit powers of all other transmitters. Therefore, it is necessary to determine a set of powers where each player is satisfied with the cost that it has to pay, given the power selections of other players. Such an operating point is called equilibrium point.

A suitable solution to this problem is the Nash Equilibrium Point. The Nash Equilibrium concept offers a predictable, stable outcome of a game where multiple agents with conflicting interests compete through self-optimization and reach a point where no player wishes to deviate from. Formally, a power vector $\mathbf{p}\ (k) = [p_1(k), ..., p_N(k)]$ is a Nash Equilibrium Point of G_K if, for each $j \in N_j$, it holds:

$$c_j \quad p_j(k+1), \mathbf{p}\ _j(k+1) \qquad c_j \quad p_j(k+1), \mathbf{p}\ _j(k+1) \quad . \tag{4}$$

A. *Existence and Uniqueness of* $\mathrm{G_K}$ *Equilibrium*

Necessary and sufficient conditions for the existence of a Nash Equilibrium Solution are given by Theorem 1.

Theorem 1: For each $j \in N_j$ let P_j be a closed, bounded and convex subset of a finite-dimensional Euclidian space, and the cost functional $c_j : P_1 \quad\quad P_N \longrightarrow \mathbb{R}$ be jointly continuous in all its arguments and strictly convex in p_j for every $p_i \in P_i$, $i \neq j$. Then, the associated nonzero-sum game admits a Nash Equilibrium.

Proof: A proof of Theorem 1 can be found in [7]. ■

The strategy set $P_j = [p_{min}, p_{max}]$ is a closed, bounded and convex subset of the Euclidian space $\mathbb{R}$, for all j. Thus, in order to prove the existence of a Nash Equilibrium Solution to the presented nonzero-sum game, it is necessary to verify the continuity of the cost function c_j with respect to all its arguments and if it is strictly convex in p_j for all $p_i \in P_i$, $i \in N_j$, $i \neq j$. Then, from (1) and (2), we obtain:

$$c_j = \ _t^2 - 2 \ _t \ \frac{g_j(k+1)}{I_j(k+1)} \ p_j(k+1) + \ \frac{g_j(k+1)}{I_j(k+1)}^{\ 2} p_j^2(k+1). \tag{5}$$

We conclude from (3) and (5) that the cost function c_j is continuous with respect to all its arguments. The cost function strict convexity is considered in the following.

B. Nash Equilibrium Point of $\mathbf{G_K}$

The necessary optimality condition for a differentiable function is that its first-order derivative be equal to zero. The partial derivative of the cost function c_j with respect to p_j is given below:

$$\frac{\partial c_j}{\partial p_j(k+1)} = -2 \;{}_t\; \frac{g_j(k+1)}{I_j(k+1)} + 2 \; \frac{g_j(k+1)}{I_j(k+1)} ^2 p_j(k+1), \tag{6}$$

$$\frac{\partial c_j}{\partial p_j(k+1)} = 0 \Longrightarrow p_j(k+1) = \;{}_t\frac{I_j(k+1)}{g_j(k+1)}. \tag{7}$$

The sufficient optimality condition for a two-time differentiable function is that its second-order derivative be different from zero. The second-order partial derivative of c_j with respect to p_j is shown below to be strictly positive. Then, the strict convexity of c_j is formally guaranteed:

$$\frac{\partial^2 c_j}{\partial p_j^2(k+1)} = 2 \; \frac{g_j(k+1)}{I_j(k+1)} ^2 > 0. \tag{8}$$

Therefore, the presented game admits a unique Nash Equilibrium Solution, given by (7). Furthermore, (8) guarantees that this solution minimizes the cost function c_j for all $j \in N_j$.

C. Stability Analysis of the Nash Equilibrium Point

An important characteristic of an equilibrium solution is the stability. A Nash Equilibrium Solution is stable with respect to a determined deviation of a player, i.e., a choice different from the Nash Equilibrium Solution, if an iterative process converges to the originally satisfatory solution (Nash Equilibrium Point). The following definition can be used to carry out a stability analysis [7], [8].

Definition 1: A Nash Equilibrium Solution $u_j, j \in M = \{1, \quad, m\}$ is stable with respect to the deviation scheme Υ if it may be obtained as:

$$u_j = \lim_{k\to\infty} u_j(k), \tag{9}$$

$$u_j(k+1) = \arg\min_{u_j \in U_j} J_j(u_j(k+1), u^{\Upsilon_k}_{\;j}), \tag{10}$$

where u is the decision variable, $u^{\Upsilon_k}_{\;j}$ is the deviation of players except player j in the time instant k, and J is the objective function.

In $\mathbf{G_K}$, using the expression of interference-plus-noise power in the Nash Equilibrium Point, respectively (3) and (7), the following expression is obtained:

$$p_j(k) = \;{}^{t}_{j}\frac{\sum_{l=1}^{N}[p_l(k)g_l] + \;^2}{g_j}, \tag{11}$$

where $l \neq j$ and g_j is the channel gain of player j, $j \in N_j$. Then, from Definition 1, the Nash Equilibrium Point may be written as follows:

$$p_j = \lim_{k\to\infty} \;{}^{t}_{j}\frac{\sum_{l=1}^{N}[p_l(k)g_l] + \;^2}{g_j}. \tag{12}$$

A special case of deviation corresponds to the situation where players adjust their actions simultaneously in response to the more recent actions of the other players, that is, $u^{\Upsilon_k}_{\;j} = u \;\, _j(k)$. In $\mathbf{G_K}$, such situation is represented by $p^{\Upsilon_k}_{\;j} = p \;\, _j(k)$, which corresponds to:

$$p_j(k+1) = \arg\min_{p_j \in P_j} c_j(p_j(k+1), p \;\, _j(k)) \Longrightarrow$$
$$p_j(k+1) = \;{}^{t}_{j}\frac{\sum_{l=1}^{N}[p_l(k)g_l] + \;^2}{g_j}, \quad l \neq j. \tag{13}$$

Thus:

$$\lim_{k\to\infty} p_j(k+1) = \lim_{k\to\infty} p_j(k) = p_j, \;\; \forall j \in N_j, \tag{14}$$

assuring the stability of the Nash Equilibrium Point of $\mathbf{G_K}$ with respect to the situation where the players adjust their transmit power in response to the status given in the previous power control actuation.

In practice, such a deviation scheme is reasonable, since values of channel gain and interference-plus-noise power at time instant $k+1$ are not available at the time instant k. Consequently, at time instant k each player j has not information about the transmit power choice of the other players for time instant $k+1$. Then, the Nash Equilibrium Solution given by (7) is not realizable. However, from the stability analysis, we conclude that the approximation given by (15) makes (7) to converge to the Nash Equilibrium Solution in an iterative process:

$$\frac{I_j(k+1)}{g_j(k+1)} \approx \frac{I_j(k)}{g_j(k)}. \tag{15}$$

In fact, this Nash Equilibrium Solution approximation is shown to be the well-known Distributed Power Control (DPC) algorithm:

$$p_j(k+1) = \;{}_t\frac{I_j(k)}{g_j(k)} \Longrightarrow p_j(k+1) = \frac{t}{\;_j(k)} p_j(k). \tag{16}$$

The DPC algorithm as an approximation of the Nash Equilibrium Solution motivates the use of more accurate approximations in order to obtain better performance. Low computational complexity and simple implementation are characteristics of the DPC algorithm. Then, in the next section, a simple prediction technique is employed to develop a new power control algorithm which approximates the Nash Equilibrium Solution more accurately than the DPC, however keeping its desirable characteristics.

IV. Taylor Series-based Power Control Algorithm

Values of $I(k+1)$ and $g(k+1)$ can be approximated separately by making use of some past samples in a Taylor's Series [9]. Taylor's Series is used to expand continuous functions $f(x)$ as follows:

$$f(x) = f(x_0) + \sum_{n=1}^{\infty} \frac{f^{(n)}(x_0) \, (x - x_0)^n}{n!}, \qquad (17)$$

where the term $f^{(n)}(x)$ represents the n^{th} derivative of $f(x)$ with respect to x. Due to $(x - x_0)^n$ and $n!$, when x and x_0 are adjacent values, the higher order terms can be neglected [10]. Thus, neglecting the derivative terms of second or higher order, we obtain:

$$f(x) = f(x_0) + f'(x_0) \, (x - x_0). \qquad (18)$$

Now, we transform (18) into a difference equation. For this, we assume that x_0 is the current discrete time instant k and x the next instant $k+1$. Further, $f'(x_0)$ is substituted by $f(k) - f(k-1)$. Thus, we obtain:

$$f(k+1) = 2 \, f(k) - f(k-1). \qquad (19)$$

Therefore, we can use (19) in order to approximate the path gain and the co-channel interference:

$$\hat{g}_j(k+1) = 2 \, g_j(k) - g_j(k-1), \qquad (20)$$

$$\hat{I}_j(k+1) = 2 \, I_j(k) - I_j(k-1). \qquad (21)$$

So, using the approximations given by (20) and (21) in the expression of the Nash Equilibrium Solution, (7), the transmit power for instant $k+1$ is defined by the following proposed algorithm:

$$p_j(k+1) = \gamma_t \left(\frac{2 \, I_j(k) - I_j(k-1)}{2 \, g_j(k) - g_j(k-1)} \right). \qquad (22)$$

Note that when the estimations tend to correct values, that is, $\hat{g}_j(k+1) \approx g_j(k+1)$ and $\hat{I}_j(k+1) \approx I_j(k+1)$, the SINR tends to the target SINR:

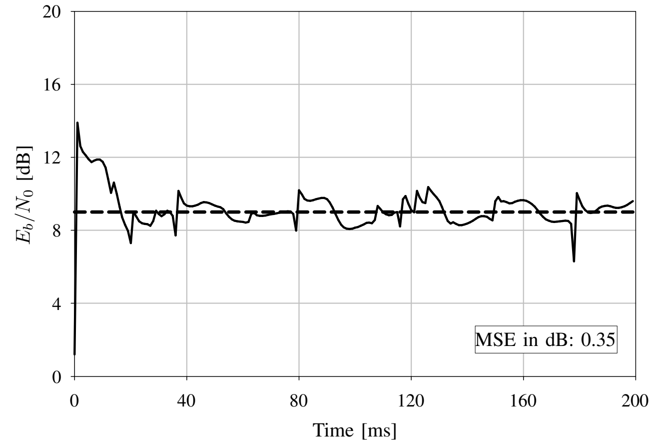
$$\gamma_j(k+1) = \frac{g_j(k+1) \, p_j(k+1)}{I_j(k+1)} \Longrightarrow$$
$$\gamma_j(k+1) = \frac{g_j(k+1)}{\hat{g}_j(k+1)} \, \gamma_t \, \frac{\hat{I}_j(k+1)}{I_j(k+1)} \approx \gamma_t. \qquad (23)$$

It is important to observe that the prediction method employed to develop the new algorithm does not compromise the desirable characteristics of a distributed power control algorithm, such as low computational complexity and simple implementation. The proposed algorithm requires only 2 multiplication operations more than DPC algorithm and a unit of memory.

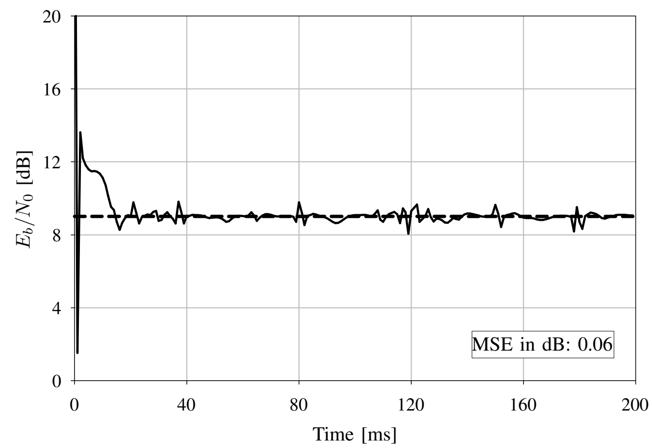
V. Simulation Results

We now illustrate the performance of the proposed algorithm, taking as an example some snapshots extracted from the simulated CDMA system. DPC algorithm is also considered.

Fig. 1 shows a sample of the E_b/N_0 evolution achieved by a given user in a typical snapshot for both the DPC and the proposed algorithm. In this case, ten mobile stations are placed in the cell and the same system configuration and fading realizations are used for both algorithms.



(a) DPC algorithm.



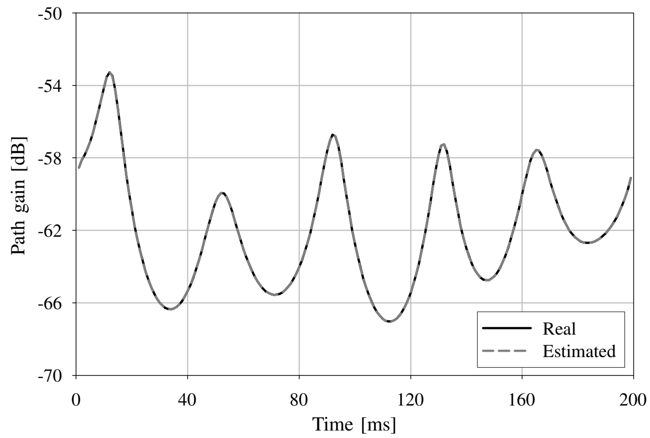
(b) Proposed algorithm.

Fig. 1. Tracking of E_b/N_0 target for the evaluated algorithms.

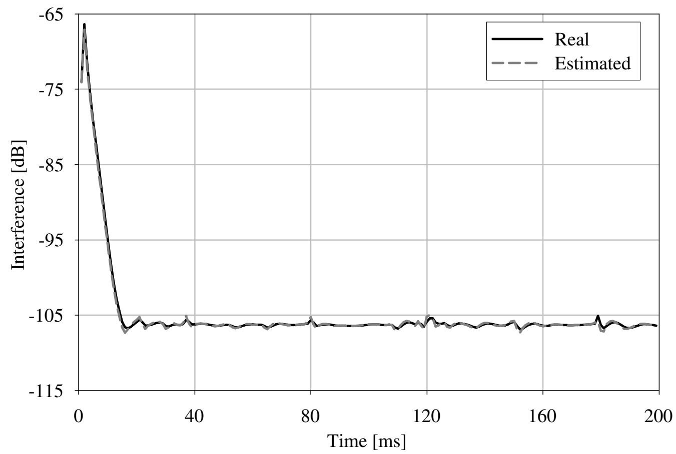
From Fig. 1, it is clearly observable that the proposed algorithm (Fig. 1(b)) is able to stabilize the E_b/N_0 around the target $(E_b/N_0)_t$ better than DPC algorithm (Fig. 1(a)). In other words, the mean squared error (MSE) between the actual and the target $(E_b/N_0)_t$ values is smaller for the proposed algorithm than for the conventional one.

Also for this snapshot and the same mobile station, path gain prediction and interference prediction carried through proposed algorithm are shown. Fig. 2(a) presents the behavior of the path gain and the tracking performance of the path gain prediction. Equivalently, Fig. 2(b) shows interference and its prediction. It can be observed that prediction based on Taylor's Series achieves good performance for both path gain and interference. The same behavior was observed in several snapshots.

In practical systems, it is difficult to keep exactly the E_b/N_0 at the target value, specially in high mobile speeds [11]. Therefore, we assume an E_b/N_0 margin below the target $(E_b/N_0)_t$ in which signal quality is assumed acceptable. We



(a) Path gain prediction



(b) Interference prediction

Fig. 2. Comparison between predicted and correct values using the proposed method.

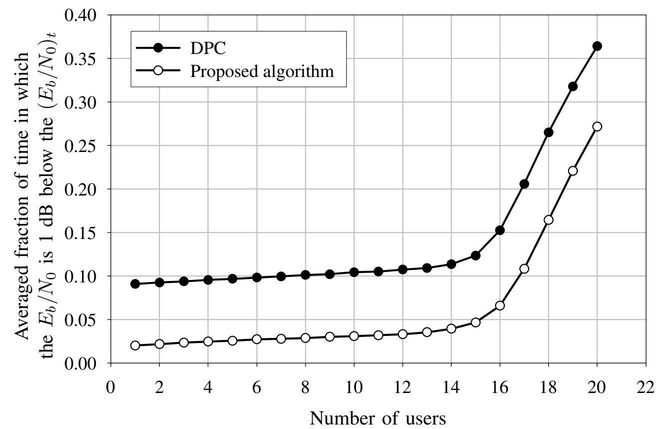


Fig. 3. Average fraction of time in which the E_b/N_0 is 1 dB below the $(E_b/N_0)_t$ for both algorithms and several loads in a CDMA system.

simulate 5000 snapshots and calculate the average fraction of time in which the achieved E_b/N_0 is 1 dB below the target $(E_b/N_0)_t$, for several system loads. In Fig. 3, we illustrate how the superior tracking capability of the proposed algorithm translates into a system-level advantage.

Through these experiments, it can be observed that the employment of this power control algorithm allows for a significant capacity gain when compared to the DPC. The only payoff for this is the need of additional memory when compared with conventional DPC.

VI. Conclusions

A game-theoretical approach was used to construct a general framework to the distributed power control problem. This problem was formulated as a static multi-stage nonzero-sum noncooperative game, with the objective of tracking a given target SINR through transmit power decisions. A unique Nash Equilibrium Solution was obtained, but it was shown to be not realizable.

The stability analysis of Nash Equilibrium Solution showed that approximations of this equilibrium point may converge in an iterative process to the original equilibrium point. The conventional DPC algorithm was shown to be an approximation of the Nash Equilibrium Solution.

A new decentralized power control algorithm was developed by using a simple prediction method. The new algorithm is also an approximation of the Nash Equilibrium Solution, but is more accurate than DPC algorithm. The algorithm preserves the desirable characteristics of a distributed power control algorithm: low computational complexity and simple implementation. It requires only 2 multiplication operations more than DPC and a unit of memory. Computational simulations in a cellular system environment were performed and comproved the better performance of the proposed algorithm compared to DPC algorithm with respect to the tasks of tracking a target SINR and of guarantee of a minimum SINR.

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